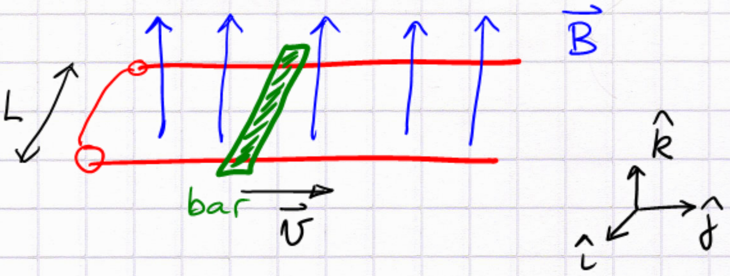
Week 19 21.22



a) $I_{induced}$ through bar
→ which loop?

b) direction of flux?

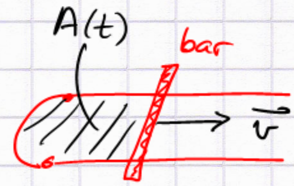
c) flux magnitude increasing or decreasing?

d) direction of $\vec{B}_{ind}$?

e) $R_{loop} = 1.5\ k\Omega$ → magnitude and direction of I_{ind}?

Solution.

a) a closed loop permeated by $\Phi_M(t)$:



b) flux direction: not unique! we can choose $\Phi_M > 0$, i.e., area vector $\vec{A}$ is aligned with $\vec{B}$ $\therefore$ $\Phi_M = \vec{A}\cdot\vec{B} = A\,B > 0$

c) $\frac{d}{dt}|\Phi_M| > 0$ since $A(t)$ grows

d) $\vec{B}_{ind}$ will counteract $\vec{B}$, since $\frac{d}{dt}|\Phi_M| > 0$

$\therefore$ $\vec{B}_{ind} \sim -\hat{k}$, since $\vec{B} = B_0\,\hat{k}$.

e) By Ohm's law: $|I_{ind}| = \frac{|\Delta V_{ind}|}{R} = \frac{1}{R}\left|\frac{d}{dt}\Phi_M\right| = \frac{1}{R}B\frac{dA}{dt}$

$$= \frac{1}{R}\cdot B\cdot\frac{d}{dt}(L\cdot x(t)) = \frac{LB}{R}\underbrace{\frac{dx}{dt}}_{v}$$

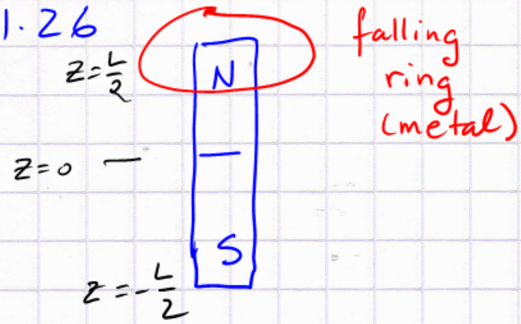
$L = 1.5\ m$, $B = 2.3\ T$, $v = 0.54\ \frac{m}{s}$, $R = 1.5\ k\Omega$

$I_{ind} = 1.24\ mA \rightarrow 1.2\ mA$

orientation: RH rule → CW current as viewed from above

Remark: while deriving Faraday's EMF as produced by the moving bar we found + charges accumulating at the bottom → CW current. The bar = EMF generator. If the bar was at rest, but $B(t)$ was ramping up (→ $\Phi_M(t)$ increasing) we would have no (+, −) "terminals" at bar ends. A tricky source of EMF!

21.26



a) sketch $I_{ind}(z)$ for $-nL < z < nL$ (n = large)

b) plot $F_{M \text{ on ring}}(z)$ and understand Lenz' rule

Solution

a) For $z \gg \frac{L}{2}$ $|\vec{B}|$ is very weak $\therefore \phi_M^{ring} \approx 0$

$\rightarrow$ as z approaches $\frac{L}{2}$ ϕ_M^{ring} grows large, since $B(z)$ grows

$\rightarrow$ between $z=\frac{L}{2}$ and $z=-\frac{L}{2}$ ϕ_M maintains its sign!

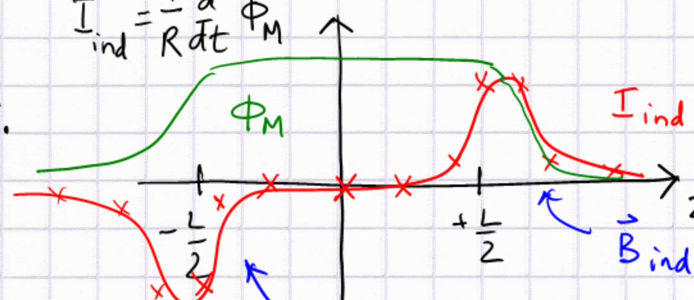
the number of field lines passing through the loop is about constant $\therefore$ @ $z=0$ $\frac{d}{dt}\phi_M = 0$

$\rightarrow$ for $z < -\frac{L}{2}$ ϕ_M^{ring} decreases, i.e., $\frac{d}{dt}\phi_M < 0$

(we picked $\phi_M > 0$ initially)

$I_{ind} = \frac{-1}{R}\frac{d}{dt}\phi_M$

$\therefore$



I_{ind} (really shown as if loop was moving with $v_z > 0$, i.e., moving upward)

$\vec{B}_{ind}$ counteracts $\vec{B}_{PM}$ $\therefore$ CW current as viewed from above

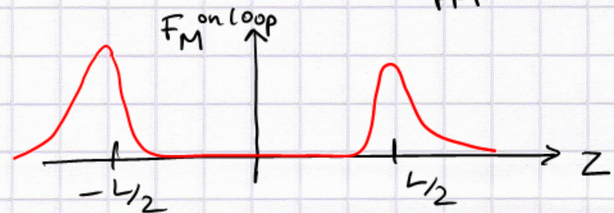
$\vec{B}_{ind}$ supports $\vec{B}_{PM}$ $\therefore$ CCW current

b) $F_M^{\text{on loop}}$?

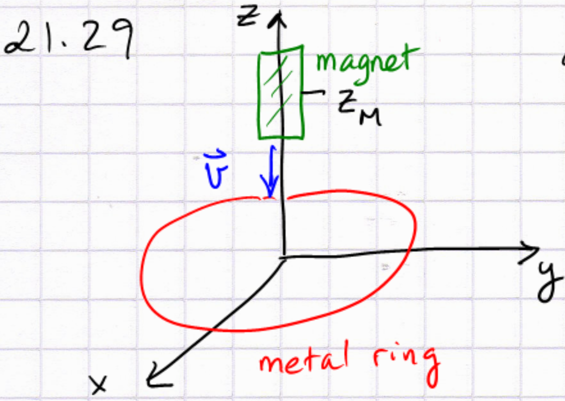
For $z \gtrsim \frac{L}{2}$ the loop is repelled ($\vec{B}_{ind}$ opposes $\vec{B}_{PM}$)

For $z \lesssim -\frac{L}{2}$ the loop is attracted ($\vec{B}_{ind}$ and $\vec{B}_{PM}$ are aligned)

Thus, $\vec{F}_M^{\text{on loop}} \sim \hat{k}$ for both situations



21.29



a) for $z_M > 0$: CCW I_{ind} (viewed from above)

which way is the magnet oriented ?

b) which way is $\vec{F}_M^{\text{on PM}}$?

c) suppose the magnet was flipped :

which way is $\vec{F}_M^{\text{on PM}}$?

Solution.

a) CCW current (bird's view) $\rightarrow$ $\vec{B}_{ind}$ is $\uparrow^{N}_{S}$

Since $\frac{d}{dt}|\Phi_M| > 0$ as the PM approaches, we

have anti-parallel (counteraligned) $\vec{B}_{ind}$ and $\vec{B}_{PM}$,

thus $\vec{B}_{PM}$ is $\downarrow^{S}_{N}$ $\boxed{\begin{array}{c} S \\ N \end{array}}$ $\downarrow \vec{v}$

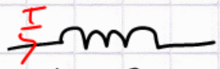
b) $\vec{F}_M^{\text{on PM}}$ is repulsive, i.e., it is directed along $\hat{k}$

c) Flipping the PM : $\vec{B}_{PM}$ is $\uparrow^{N}_{S}$ now $\boxed{\begin{array}{c} N \\ S \end{array}}$ $\downarrow \vec{v}$

To oppose the rise in $|\Phi_M|$ $\vec{B}_{ind}$ is $\downarrow^{S}_{N}$, and

again $\vec{F}_M^{\text{on PM}}$ is repulsive, i.e., points upward.

Remark: $\vec{F}_M^{\text{on PM}}$ is the force that acts on a magnet falling through a copper plumbing tube (class demo). It cancels gravity and the PM falls with terminal velocity. The orientation of the PM doesn't matter

21.36 Inductor  $L = 5.0\ mH$

ϕ_M changes: $\frac{d\phi_M}{dt} = .0045 \frac{Tm^2}{s}$

what is $\frac{d}{dt} I$?

(no external field)

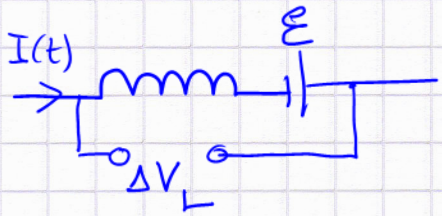
Solution

$$\mathcal{E} = - \frac{d\phi_M}{dt} \quad \text{(Faraday's law)}$$

voltage drop across L : $\Delta V_L = L \frac{dI}{dt}$ (eq. 21.23)

(no − sign)

self-inductance is visualized as



$$|\mathcal{E}| = \left|\frac{d\phi_M}{dt}\right| = L\left|\frac{dI}{dt}\right|$$

$$\therefore \quad \frac{dI}{dt} = \frac{1}{L}\left|\frac{d\phi_M}{dt}\right| = \frac{.0045}{5.0 \times 10^{-3}} = \frac{.0009}{10^{-3}} = 0.90\ \frac{A}{s}$$

is the rate of change of the current.

We could also argue:

$\phi_M = L\,I$ from eqs 21.20 and 21.21

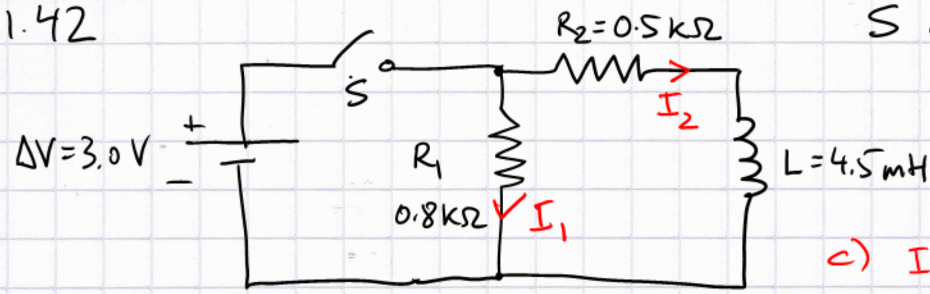
$\therefore \frac{d}{dt}\phi_M = L\frac{dI}{dt}$ (L is constant)

$$\therefore \quad \frac{dI}{dt} = \frac{1}{L}\frac{d\phi_M}{dt}$$

$\nearrow \frac{dI}{dt} > 0$ ϕ_M ramps up

$\searrow \frac{dI}{dt} < 0$ ϕ_M ramps down

21.42



S closes @ $t=0$, was open forever

a) $I_1(0)=?$ $I_2(0)=?$

b) $\Delta V_L(0)=?$

c) $I_1(t\to\infty)=?$ $I_2(t\to\infty)=?$

$\Delta V_L(t\to\infty)=?$

d) t_{open}: $I_1(t_{open})=?$ mag + dir'n!

e) $\Delta V_L(t_{open})=?$

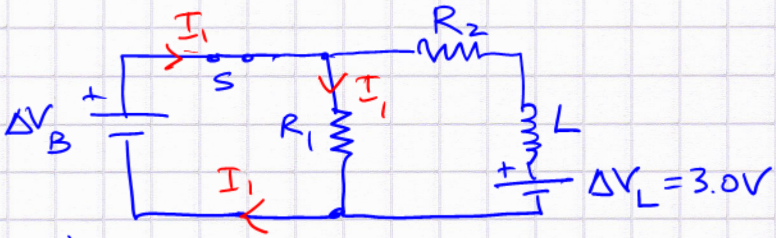
Solution.

a) $I_1(0) = \frac{\Delta V_B}{R_1} = \frac{3.0}{0.8}\,\text{mA} = 3.8\,\text{mA}$; $I_2(0)=0$ due to counter EMF from L

b) to counteract $\frac{d}{dt}\Phi_M$ ($\Phi_M = 0$ at $t=0$)

the inductor self induces $\Delta V_L = 3.0\,V$ with + @ top

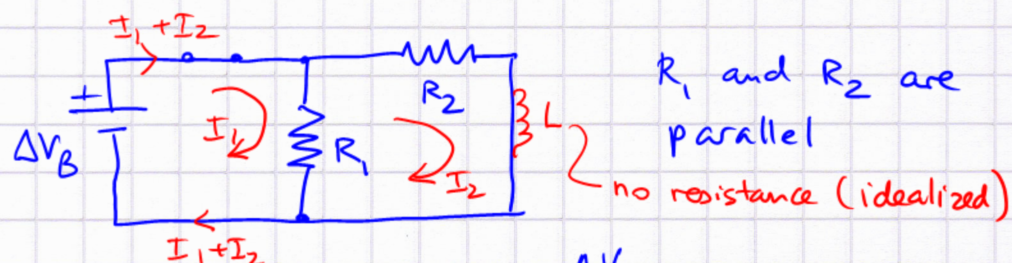
∴ equivalent circuit @ $t=0$



Note that ΔV_{R_1} doesn't change, and $\Delta V_{R_2}=0$

c) for t large $\Delta V_L \to 0$ since $\frac{d}{dt}I_2 \to 0$

∴ equivalent circuit

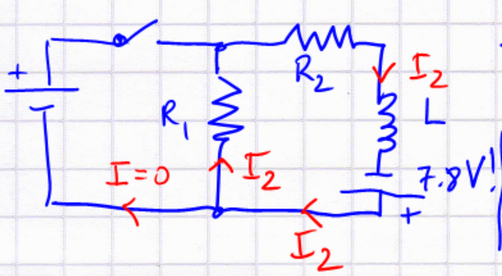


R_1 and R_2 are parallel

$I_1 = \frac{\Delta V_B}{R_1}$ (unchanged) $I_2 = \frac{\Delta V_B}{R_2} = 6.0\,\text{mA}$

e) + d) @ t_{open}: I_2 is unchanged for an instant $= 6.0\,\text{mA}$ to keep Φ_M

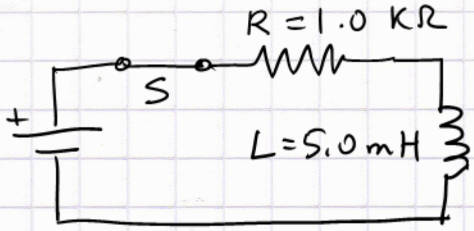
ΔV_B is disconnected. For I_2 to keep going we need $\Delta V_L = (R_1+R_2)I_2 = 1.3 \times 6.0\,V = 7.8\,V$



The current through R_1 switches from 3.8 mA to -6.0 mA (opposite). We neglected inductances in the wires + resistors

21.46

$\Delta V_B = 3.0\,V$



S closed long time.

energy stored in L ?

Solution.

$$PE_{magn} = \frac{1}{2} L I^2$$

- L is idealized as having no resistance
- S was closed long $\therefore \frac{dI}{dt} = 0 \quad \therefore \Delta V_L = 0$

$$I = \frac{\Delta V_B}{R} = \frac{3.0}{1.0}\ mA = 3.0\ mA$$

$$PE_{magn} = \frac{1}{2}\, 5.0 \times 10^{-3} \times (3.0 \times 10^{-3})^2 \ J$$

$$= 2.5 \times 9.0 \times 10^{-3} \times 10^{-6} \ J$$

$$= 2.25 \times 10^{-8} \ J \qquad (= 22.5\, nJ)$$

Remark: Energy is stored in $\vec{B}$.

When S is opened, current will continue to flow through L (for some time) since L will release the stored energy