Week 20 21.50

single circular loop, $R = 2.0$ cm, rotates $f = 60$ Hz

in $\vec{B}$ field ($B = 1.5$ T, constant, uniform)

Maximum EMF = ?

Solution.

This is a case where $\Phi_M = B \cdot A \cdot \cos\varphi$ is changing

in time since $\varphi = \omega t = 2\pi f t$; $B_0 = 1.5$ T, $A = \pi R^2$

$$\therefore \quad \Phi_M(t) = B_0 A \cos(2\pi f t)$$

Now, Faraday : $\text{EMF} = -\dfrac{d\Phi_M}{dt} \quad \therefore \quad |\text{EMF}| = \left|\dfrac{d\Phi_M}{dt}\right|$

Maxima occur when $\sin\varphi = \pm 1 \quad \therefore \varphi = (n+\frac{1}{2})\pi$

$n = 0, 1, 2, \ldots$

why? $\left|\dfrac{d\Phi_M}{dt}\right| = \left|2\pi f \, B_0 A \sin(2\pi f t)\right|$

$$\therefore \quad \text{EMF}_{max} = 2\pi f \, B_0 A$$

$$= 6.28 \cdot 60 \cdot 1.5 \cdot 3.14 \cdot (2.0 \times 10^{-2})^2 \text{ V}$$

$$= 7.10 \times 10^3 \times 10^{-4} \text{ V}$$

$$= 0.71 \text{ V} = 710 \text{ mV}$$

NB: solution manual doesn't use calculus and gets a hand-wavy estimate of 0.92 V (sine wave → triangle wave)

21.58 Current loop sits in $B(t)$ magnetic field, perpendicular to the loop

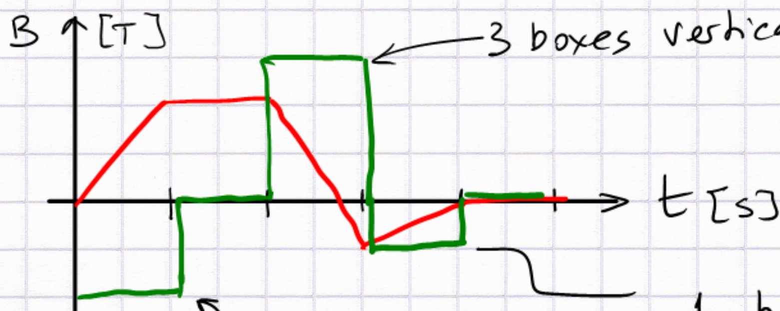
graph the EMF induced

Solution

Faraday: $\text{EMF} = -\dfrac{d\Phi_M}{dt} = -\dfrac{d}{dt}\left(B(t)\,A\right)$ ← $\cos\varphi = 1$ since $\varphi = \pi/2$

$$= -A\frac{dB}{dt}$$

EMF [V]

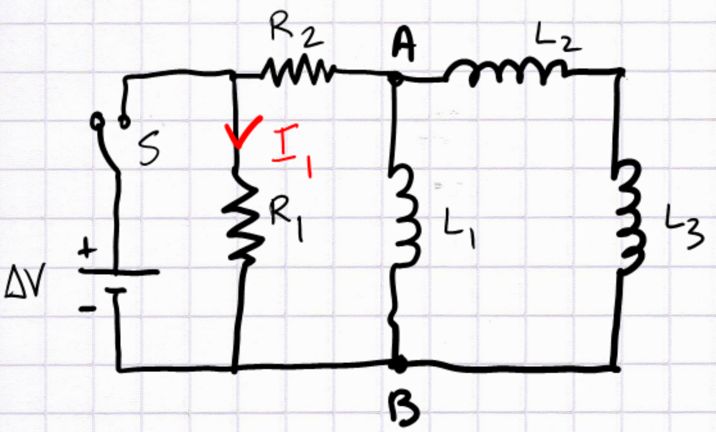


3 boxes vertically / 2 boxes horizontally

1 box vertically / 2 boxes horizontally

$\dfrac{\text{2 boxes vertically}}{\text{2 boxes horizontally}}$

21.62 S was open a long time, $R_1 = R_2 = 50\,\Omega$



$L_1 = L_2 = L_3 = 40$ mH

$\Delta V = 12$ V

a) $L_{eq.}^{AB} = ?$

b) at $t=0$ close S

$I_1 = ?$

c) $I_1(t\to\infty) = ?$

Solution.

L_{eq}^{AB} comes from L_1 in parallel with L_2 & L_3 in Series

Two coils in series : adding the turns ; means

$$L \equiv \frac{\mu_0 N^2 A}{\ell} \qquad \ell = \text{length}\ ; \ \frac{N}{\ell} = \text{const}$$

$$L = \underbrace{\left(\frac{\mu_0 N}{\ell} A\right)}_{\text{const.}} \cdot N$$

$$\therefore\ L_{23}^{eq} = L_2 + L_3$$

Also by Kirchhoff: $\Delta V_{L_{eq}} = \Delta V_{L_2} + \Delta V_{L_3}$

$$= L_2 \frac{dI}{dt} + L_3 \frac{dI}{dt} = (L_2 + L_3)\frac{dI}{dt}$$

Parallel inductors: current splits, is not the same,

but voltage drop is the same

$$\Delta V_{AB} = L_{eq}^{AB} \underbrace{\frac{dI_2}{dt}}_{\text{total current } I_2 \text{ derivative}} = L_1 \frac{dI_{L_1}}{dt} = L_{23}^{eq} \frac{dI_{L_{23}}}{dt}$$

where $I_2 = I_{L_1} + I_{L_{23}}$

$$\therefore\ L_{eq}^{AB}\left(\frac{dI_{L_1}}{dt} + \frac{dI_{L_{23}}}{dt}\right) = \Delta V_{AB}$$

$$\therefore \quad L_{eq}^{AB}\left(\frac{\cancel{\Delta V}_{AB}}{L_1} + \frac{\cancel{\Delta V}_{AB}}{L_{23}}\right) = \cancel{\Delta V}_{AB} \qquad 1$$

$$\therefore \quad \frac{1}{L_{eq}^{AB}} = \frac{1}{L_1} + \frac{1}{L_{23}} \qquad \text{(like parallel resistors)}$$

$$L_{eq}^{AB} = \frac{L_1 L_{23}}{L_1 + L_{23}}$$

(a) $L_{eq}^{AB} = \frac{40 \cdot 80}{40+80}\ \mathrm{mH}$

$= \frac{320}{12}\ \mathrm{mH} = 26.7\ \mathrm{mH}$

b) L_{eq}^{AB} will pass no current @ $t=0$ due to counter-EMF

$$\therefore \quad I_1(0) = \frac{\Delta V_B}{R_1} = \frac{12V}{50\Omega} = \frac{24}{100}A = 0.24A$$

c) In steady state the current in the loop made up by the battery, R_2, and L_{eq}^{AB} is given by Ohm's law

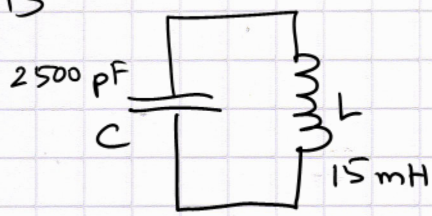
$$I_2^{ss} = \frac{\Delta V_B}{R_2} = \frac{12}{50}A = 0.24 A$$

The current through R_1 doesn't change: the single resistor R_1 is parallel to R_2, which is followed in series by L_{eq}^{AB}.

Both branches pass equal amounts of current in steady state, since R_1 and R_2 are the same.

Note: opening S' will result in $I_1 \rightarrow -I_1$, since L_{eq}^{AB} will continue to pass I_2, but the battery is now disconnected.

22.43



At $t = 0$ we have $q = 0$ on C, and $I = 45\,mA$

a) energy stored in inductor @ $t=0$?

b) @ $t = t_1$ we have $I(t_1 = 0)$. How big is q on the plates @ t_1?

c) what is t_1?

d) $I(3t_1 = ?)$, $q(3t_1) = ?$

Solution.

a) $PE_L = \frac{L}{2} I^2 = \frac{15}{2} \times 10^{-3} \times 45^2 \times 10^{-6}\ J = 15.2\ \mu J$

b) when $I = 0$ we have no B field, all energy is in the capacitor's E field

$PE_C = \frac{C}{2} \Delta V^2$ where $\Delta V = \frac{q}{C}$ $\therefore PE_C = \frac{1}{2C} q^2$

$\therefore q^2 = 2C\ PE_L = 2 \times 2.5 \times 10^3 \times 10^{-12} \times 15.2 \times 10^{-6}\ C^2$

Coulomb

$= 75.9 \times 10^{-15}\ C^2 = 7.59 \times 10^{-14}\ C^2$

$q = 2.76 \times 10^{-7}\ C = 280\ nC$

c) $t_1 = ?$ charge $q(t) = q_0 \sin(\omega t)$ where $\omega = \frac{1}{\sqrt{LC}} = 1.63 \times 10^5\ 1/s$

$\omega t_1 = \pi/2$ $\therefore t_1 = \frac{3.14}{2 \cdot 1.63 \times 10^5}\ s = 9.6\ \mu s$

d) $t_2 = 3t_1$ corresponds to $\omega t_2 = 3\frac{\pi}{2}$; this should be max. charge / zero current again, but charge is reversed $q_2 = 280\ nC$, opposite. $I_3 = I(t_2) = 0$

22.46 same circuit. $q(t_3) = 50\,nC$, $PE_L = PE_C$

What is $I(t_3)$?

Solution.

$I = I(t_3)$

$q = q(t_3)$

@ $t = t_3$: $\frac{1}{2} L \cdot I^2 = \frac{1}{2} C \cdot \Delta V^2 = \frac{q^2}{2C}$

$$I^2(t_3) = \frac{2}{L}\,\frac{q^2}{2C} = \frac{q^2}{LC} = \omega^2 q^2$$

$$\therefore \quad I(t_3) = \omega q = 1.63 \times 10^5 \times 50\,\frac{nC}{s} = 8.15\ mA$$

we can check that $PE_C = PE_L = 0.50\ \mu J$ at this time.

22.66 $120V \rightarrow 12V$ transformer $N_{out} = 4{,}000$

What is N_{in}? We need $\frac{N_{out}}{N_{in}} = \frac{1}{10}$ $\therefore N_{in} = 40{,}000$

Connected in reverse:

$120V \rightarrow 1{,}200\ V$ since now

the effective $N_{in} = 4{,}000$ and $N_{out} = 40{,}000$

i.e., it steps up AC by a 10 : 1 ratio now.