Week 21 12.10 wave on a string: $y = A \sin(35t - .025x)$

in SI and $A = 0.15\,m$. Max speed of a point on string? (Q 12.11)

Find f, λ, v_W!

Solution.

A point on the string moves in the y-direction (even though the wave moves along x).

We need $v_y(x,t)$, then find maximum (magnitude)

$$v_y(x,t) = \frac{dy}{dt} = A\cos(35t - .025x) \cdot 35$$

$$= 35\,A\cos(35t - .025x)$$

$$v_y^{max} = 35A = 0.15 \cdot 35\ \frac{m}{s} = 5.25\ \frac{m}{s}$$

Find f: $y(x,t) = A\sin\left(2\pi f t - \frac{2\pi x}{\lambda}\right)$

$$\therefore\quad f = \frac{35}{2\pi} = 5.6\ \frac{1}{s} = 5.6\ Hz$$

$$\lambda = \frac{2\pi}{0.025} = 6.28 \cdot 40\ m = 250\ m$$

(2 digits)

$$v_W = \lambda f = 250\ m \cdot 5.6\ \frac{1}{s} = 1400\ \frac{m}{s}$$

Note: the propagation speed exceeds the largest transverse speed by orders of magnitude.

12.14 A wave has $f = 3.0$ kHz, $\lambda = 0.10$ m.

Find an equation for it.

Solution.

$$y = A \sin\left(2\pi f t - \frac{2\pi}{\lambda} x\right)$$

A is unspecified. $A = 1$m ? (any value is ok)

$$y = 1 \sin\left(2\pi \cdot 3.0 \times 10^3\, t - \frac{2\pi}{0.1} x\right) \quad \text{in SI}$$

$$= 1.0 \sin\left(19 \times 10^3\, t - 63\, x\right)$$

Eqn is OK for all positions x and times t.

12.24 wave on a string. $v_W = 200\ \frac{m}{s}$.

Increase string diameter by a factor of 2, while the tension remains the same. What is the new v_W?

Solution

start with $v_W = \sqrt{\frac{F_t}{\mu}}$ F_t is unchanged

$\mu = \frac{M}{L}$

L is unchanged

String $\approx$ cylinder $V = \pi\left(\frac{D}{2}\right)^2 L$; $\rho = \frac{M}{V} = \frac{M}{\pi\left(\frac{D}{2}\right)^2 L}$

$$\mu = \frac{M}{L} = \frac{\rho V}{L} = \rho\pi\left(\frac{D}{2}\right)^2 = \pi\rho\frac{D^2}{4}$$

- Doubling D implies $\rightarrow$ mass increases by factor of 4

$\therefore \mu = \frac{M}{L}$ increases 4-fold

$\therefore \tilde{v}_W = \sqrt{\frac{F_T}{4\mu}}$ since F_T is the same (by assumption)

$$\tilde{v}_W = \frac{1}{2}\sqrt{\frac{F_T}{\mu}} = \frac{1}{2} v_w = 100\ \frac{m}{s} \simeq \frac{1}{2} v_W^{\text{original}}$$

thicker string / same tension

$\rightarrow$ lower wave propagation speed.

12.26 guitar string: $v_w = 120 \frac{m}{s}$

originally $F_T = 70\,N$ $\longrightarrow$ increase to $110\,N$

$v_w^{new} = ?$

Solution.

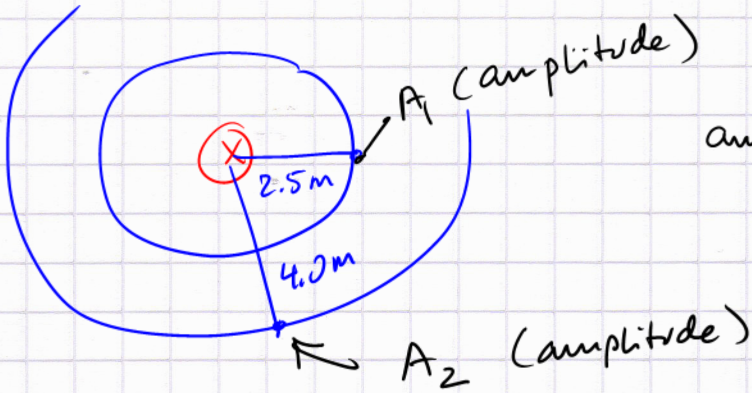
$$v_w = \sqrt{\frac{F_T}{\mu}} \qquad \tilde{v}_w = \sqrt{\frac{\tilde{F}_T}{\mu}}$$

$$\frac{\tilde{F}_T}{F_T} = \frac{110}{70} \quad \therefore \quad \frac{\tilde{v}_w}{v_w} = \sqrt{\frac{110}{70}} \simeq 1.25 \simeq \frac{5}{4}$$

$$\therefore \tilde{v}_w = 120 \frac{m}{s} \cdot \frac{5}{4} = \frac{600}{4} \frac{m}{s} = 150 \frac{m}{s}$$

Note: This analysis ignores the fact that an increase in tension is usually associated with a small change in $\mu = M/L$. As the tuning peg is tightened some mass ΔM is removed from the string between 0 and L, i.e., the nylon (or metal - to a lesser extent) string gets a little thinner!

12.36 small lightbulb, emits spherical wave



amplitude ratio ?

Solution

The intensity drops : $I \sim \frac{1}{r^2}$

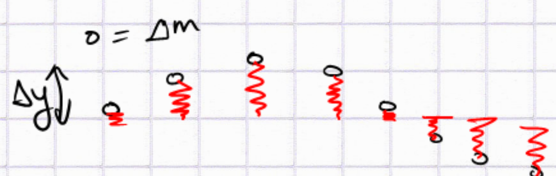
why? same power through surfaces $4\pi r^2$

$I = \frac{\text{power}}{\text{area}}$ power $= \frac{\text{energy}}{\text{time}}$

cannot disappear

$I \sim \text{amplitude}^2$ why? p. 390 slinky example

String :



$PE_{spring} = \frac{1}{2} k \Delta y^2$ energy of transverse oscillations

$\sim$ max. displacement - squared

$=$ amplitude squared

intensity $\sim$ power $= \frac{\text{energy}}{\text{time}} \sim A^2$ $\therefore A \sim \sqrt{I}$

$$\frac{A_2}{A_1} = \sqrt{\frac{I_2}{I_1}} = \sqrt{\frac{r_1^2}{r_2^2}} = \frac{r_1}{r_2} = \frac{2.5\text{ m}}{4.0\text{ m}} = 0.63$$

12.38 Two counterpropagating waves, equal frequency

constructive interference spots separated by $\Delta x = 1.5$ m

Q: a) longest possible λ ?

b) two other possible λ values?

Solution.

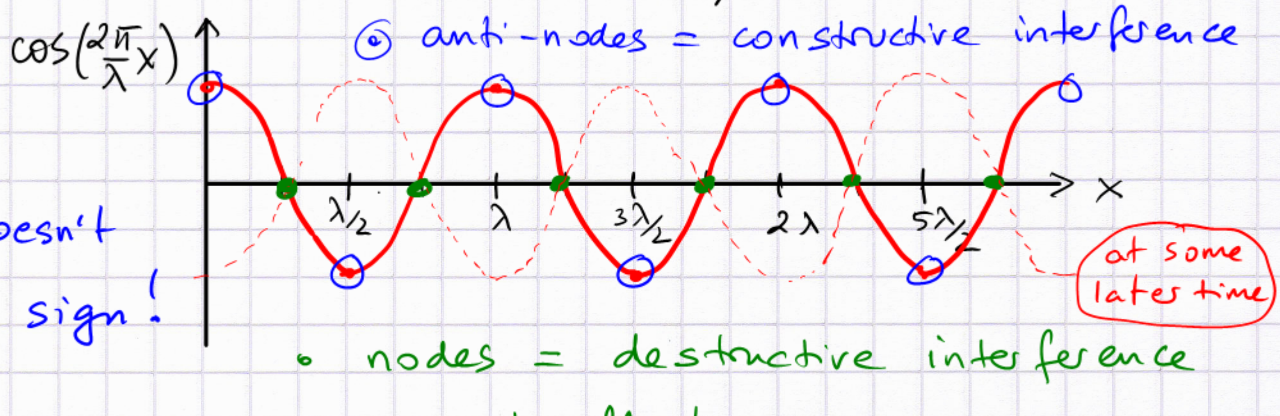
Two equal-amplitude counterpropagating waves:

$$\left.\begin{aligned} y_1 &= A \sin\left(2\pi f t - \frac{2\pi}{\lambda} x\right) \\ y_2 &= A \sin\left(2\pi f t + \frac{2\pi}{\lambda} x\right) \end{aligned}\right\} \text{ same frequency } f \text{ and wavelength } \lambda$$

$$y_{comb} = y_1 + y_2 = A \cdot 2 \sin(2\pi f t) \cos\left(\frac{2\pi}{\lambda} x\right)$$

from $\sin(a+b) + \sin(a-b) = 2 \sin a \cos b$

The nodal structure follows from $\cos\left(\frac{2\pi}{\lambda} x\right)$!



intensity $I \sim A^2$ doesn't care about sign!

Neighboring peak-to-peak distance is $\lambda/2$

$\therefore \quad \Delta x = 1.5 \text{ m} = \lambda/2 \quad \therefore \quad \lambda_1 = 3.0$ m

Non-adjacent bright spots separated by $\Delta x = 1.5$m:

E.g., $\Delta x = \lambda \quad \therefore \quad \lambda_2 = 1.5$ m ; $\Delta x = \frac{3\lambda}{2} \quad \therefore \quad \lambda = 1.0$ m, etc.