Week 22 12.59 Two guitar notes ① and ②

with $f_2 / f_1 = \underbrace{5/4}$ where f_2 and f_1 are fundamental freq.'s

major third interval note 1 → note 2

Suppose the strings are from same material, same thickness, same tension. What is the ratio of string lengths?

(not the same guitar?)

Solution.

same string material + thickness → $\mu = \frac{M}{L}$ will be the same! $M \sim L$; $\mu_1 = \mu_2$

wave propagation speed: $v_w = \sqrt{\frac{F_t}{\mu_1}} = \sqrt{\frac{F_t}{\mu_2}}$ = the same,

since F_t is the same;

The fundamental standing wave satisfies: $\lambda = 2L$

$\lambda_1 = 2L_1$; $\lambda_2 = 2L_2$

For both strings: $f_{1/2} = \frac{v_w}{\lambda_{1/2}}$ since v_w is the same

$$\frac{f_2}{f_1} = \frac{v_w/\lambda_2}{v_w/\lambda_1} = \frac{\lambda_1}{\lambda_2} = \frac{5}{4} \quad \therefore \frac{2L_1}{2L_2} = \frac{5}{4}$$

$\therefore \frac{L_1}{L_2} = \frac{5}{4}$

The lower-tone string ① is 1.25 as long as ②

$L_1 = 1.25\, L_2$

12.66 Home-made string phone vs shouting vs cell phone via tower 5.5 km away ?

$L = 35\,m$, $M_s = 18\,g$, $F_t = 10\,N$

Solution.

Assume that the string phone signal speed is given in analogy to transverse wave propagation

$$v_W = \sqrt{\frac{F_S}{\mu}} = \sqrt{\frac{10}{0.018/35}} \; SI \quad \therefore \; v_W = 139\,\frac{m}{s}$$

a) $$\Delta t_{sp} = \frac{L}{v_W} = 0.25\,s = 250\,ms$$

b) sound (@ 20°C) propagates @ $v_S = 343\,\frac{m}{s}$

$$\Delta t_S = \frac{L}{v_S} = 0.10\,s = 100\,ms$$

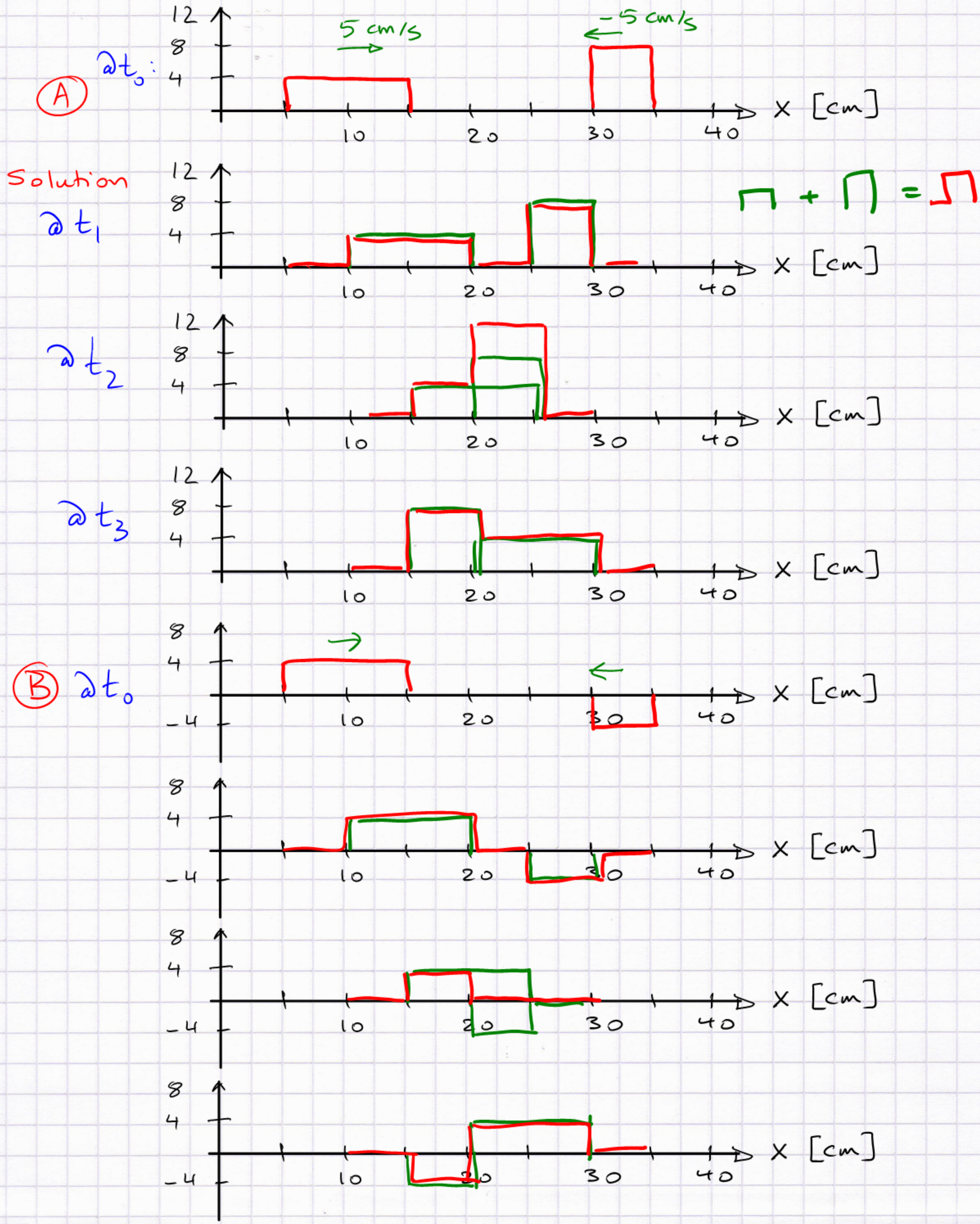
c) cell : total distance $s = 2 \times 5.5 \times 10^3\,m$

$$c = 3.0 \times 10^8\,\frac{m}{s}$$

$$\Delta t_c = \frac{1.1 \times 10^4\,m}{3.0 \times 10^8\,m/s} = 3.7 \times 10^{-5}\,s = 37\,\mu s$$

12.70 Wave pulses ; $v_1 = +5\,\mathrm{cm/s}$; $v_2 = -5\,\mathrm{cm/s}$

$t_n = \{1, 2, 3\}$ s



13.12 Car wheel: $d = 2r = 0.50\ m$ emits sound @ $f = 10\ Hz$

$v_{car} = ?$

Solution

The idea is that the hum from rolling tires, motor axles, etc., is caused by air disturbance at the rotation rate $f = \frac{1}{T}$

Thus, the car wheel is rotating with $f = 10\ Hz$, or 10 revolutions per second.

The wheel advances $2\pi r = \pi d$ per revolution (under perfect rolling).

$$v_{car} = \pi d \cdot f = 10\pi d\ \frac{m}{s} = 5 \cdot 3.14\ \frac{m}{s} = 15.7\frac{m}{s}$$

$$v_{car} = 16\ \frac{m}{s} = 56\ \frac{km}{h}$$

13.54 Siren approaches observer with $v = 35 \frac{m}{s}$

$f_0 = 1100$ Hz. Stops moving. What is f_s now?

Solution

Stationary observer, moving source:

$$f_{obs} = \frac{f_{src}}{1 - \frac{v_{src}}{v_{sound}}}$$

"−" for approaching source $\therefore$ $f_{obs} > f_{src}$

When the siren stops moving, observer records f_{src}.

$$f_{src} = \left(1 - \frac{v_{src}}{v_{sound}}\right) f_{obs}$$

$$= \left(1 - \frac{35}{343}\right) 1100 \text{ Hz}$$

$$= 988 \text{ Hz} \simeq 990 \text{ Hz}$$

Message: • source moved @ 10% of v_{sound}

• Doppler effect (frequency increase) $\simeq$ 10%

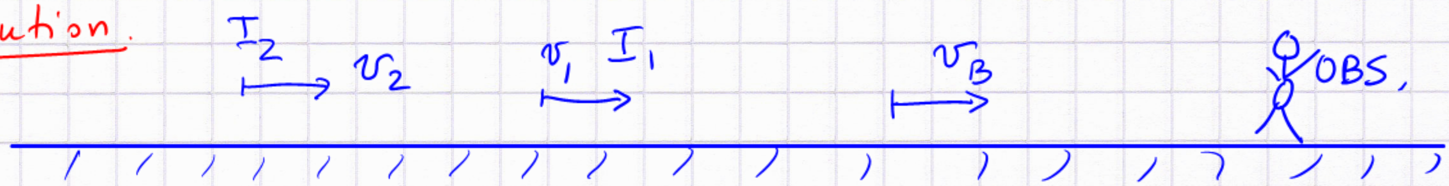
13.58 2 interceptors chase bad guy ; $f_{src} = 500\ Hz$ siren
① + ② Ⓑ

$f_1 = 600\ Hz$ $f_2 = 700\ Hz$ for stationary observer

a) which interceptor moves faster? b) $v_1 = ?$ $v_2 = ?$

c) Ⓑ observes 650 Hz from ② → is ② catching up? d) $v_B = ?$

Solution.



a) ② yields a higher frequency for OBS → is faster

b) I_1: $$f_1^{obs} = \frac{f_{src}}{1 - \frac{v_1}{v_s}} \quad \therefore \quad 1 - \frac{v_1}{343} = \frac{f_{src}}{f_1^{obs}}$$

$$\frac{v_1}{343} = 1 - \frac{f_{src}}{f_1^{obs}}$$

$$\therefore \quad v_{1/2} = 343\left(1 - \frac{f_{src}}{f_{1/2}^{obs}}\right)$$

$$v_1 = 343\left(1 - \frac{500}{600}\right) = 57.2\ \frac{m}{s} = 206\ \frac{km}{h} \quad \text{fast!}$$

$$v_2 = 343\left(1 - \frac{500}{700}\right) = 98\ \frac{m}{s} = 353\ \frac{km}{h} \quad \text{impossible}$$

c) combine Doppler formulae for moving source & observer

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Gio., Eq. 13.22

$$f_B = f_{src}\,\frac{1 - v_B/v_s}{1 - v_2/v_s}$$

← obs moves away → lowers f

← v_2 is approaching, smaller denom. → larger f

He hears an increased f ∴ I_2 is catching up

$$1 - \frac{v_B}{343} = \frac{f_B}{f_{src}}\left(1 - \frac{v_2}{343}\right) = \frac{650}{500}\left(1 - \frac{98}{343}\right) = 0.929$$

$$v_B = 343(1 - 0.929) = 24.5\ \frac{m}{s} = 88\ \frac{km}{h}$$

bad choice of frequencies

SM: $f_{src} = 550\ Hz$ is better!