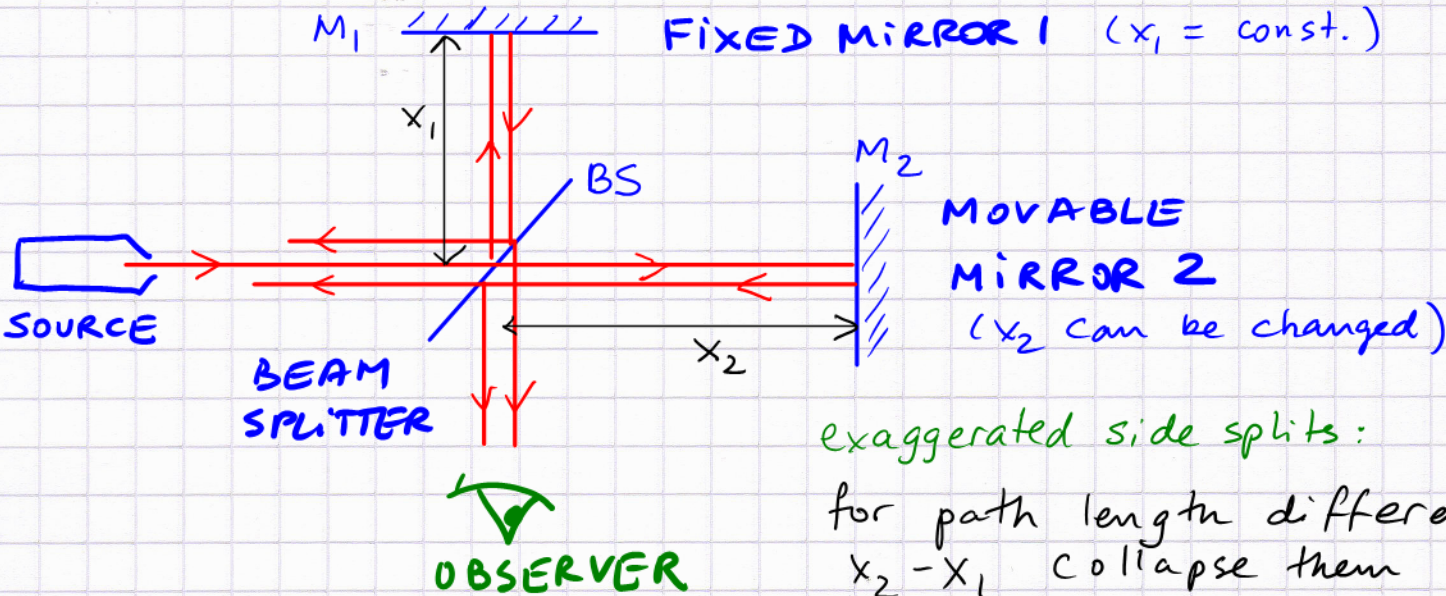
Week 23   25.10 is solved as Interference A4.pdf

it is the basis for 25.12.



exaggerated side splits: for path length difference $x_2 - x_1$ collapse them into one!

Here we have $x_1 = x_2$, arms are tubes that can be evacuated. Problem: When air is allowed into one arm, and the pattern shifts by 4,000 fringes, while orange light (600 nm) is used, what is $x_1 = x_2$?

Solution. $\quad x = x_1 = x_2 =$ physical path length

$OPL_1 = x \quad ; \quad OPL_2 = n_{air}\, x \quad$ where $n_{air} = 1.0003$

Table 24.1, p. 801

phase difference

$$\frac{2\pi}{\lambda_{vac}} x - \frac{2\pi}{\lambda_{air}} x = \frac{2\pi x}{\lambda_{vac}} \left(1 - n_{air}\right) = \frac{2\pi x}{\lambda_{vac}} \cdot 3 \times 10^{-4}$$

This should equal $m \cdot 2\pi$ (m passages of dark-bright alternations)

$\therefore \; 3 \times 10^{-4}\, x = m\, \lambda_{vac} \quad \therefore \; x = \frac{4 \times 10^3 \times 6. \times 10^2}{3 \times 10^{-4}} = 8 \times 10^9 \text{ nm} = 8 \text{ m}$!? need a good laser!

25.16 is solved as Interference A5. pdf
25.19 is solved as Interference A6. pdf
} see below.

28.10 How many photons of orange-red light (600 nm) does it take to have 1 J of energy?

Solution.

$E = hf$ is the energy of a single photon

$f = \frac{c}{\lambda}$; $h = 6.63 \times 10^{-34}$ Js

$$f = \frac{3.0 \times 10^{8}\ \text{m/s}}{600 \times 10^{-9}\ \text{m}} = 0.5 \times 10^{15}\ \text{Hz} = 0.5\ \text{THz}$$

$$E_\gamma = 6.63 \times 10^{-34} \times 0.5 \times 10^{15}\ \text{J}$$

$$= 3.3 \times 10^{-19}\ \text{J} \qquad (2\ \text{eV - range})$$

$$TE = 1\ \text{J} = n E_\gamma \quad \therefore \quad n = \frac{1\ \text{J}}{E_\gamma} = \frac{1}{3.3} \times 10^{19}$$

$$= 3.0 \times 10^{18}\ \text{photons}$$

28.16 How many photons does a green laser ($\lambda = 510\,nm$) with a power rating of 2.0 W emit in 1 μs?

Solution.

$$P = 2.0\ W = 2.0\ \frac{J}{s} = 2.0 \times 10^{-6}\ \frac{J}{\mu s}$$

$\therefore$ Question asks: how many photons in 2.0×10^{-6} J?

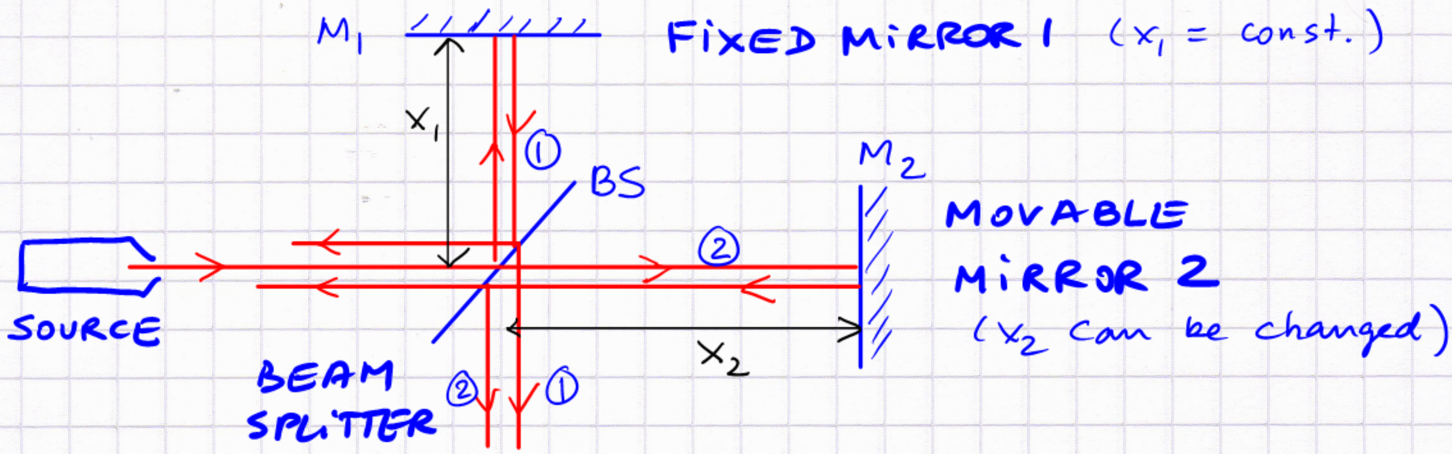
$$E_\gamma = h \cdot \frac{c}{\lambda} = 6.63 \times 10^{-34}\ Js \cdot \frac{3.0 \times 10^{8}\ m/s}{510 \times 10^{-9}\ m}$$

$$= \frac{6.63 \cdot 3.0}{510}\ 10^{-34} \cdot 10^{17}\ J$$

$$= 0.039 \cdot 10^{-17}\ J = 3.9 \times 10^{-19}\ J$$

$$n_\gamma = \frac{E}{E_\gamma} = \frac{2.0 \times 10^{-6}\ J}{3.9 \times 10^{-19}\ J} = 5.1 \times 10^{12}\ \text{photons}$$

25.10



Remark : The BS can be splitting to any ratio, e.g., 10 : 90 %, the intensity will still be 50 : 50.

Why? each recombining beam ① and ② underwent one reflection and one transmission of the B.S. !

∴ a simple glass plate would work !

25.10 : mirror 2 was moved by 2.0 mm. The interference pattern moved through 7000 dark fringes. What was λ?

Solution : path length difference $d = 2(x_2 - x_1)$

$x_1 = \text{const}, \quad x_2 \to x_2' = x_2 + \Delta x \qquad \Delta x = 2.0$ mm

Whether we count dark or bright fringes doesn't really matter but initially : (optical path length $n \cdot d \simeq d$, $n_{air} \approx 1$)

$$\left.\begin{aligned}(m_1 + \tfrac{1}{2})\lambda &= 2(x_2 - x_1)\\ (m_2 + \tfrac{1}{2})\lambda &= 2(x_2 + \Delta x - x_1)\end{aligned}\right\} -$$

$m_2 - m_1 = 7000$

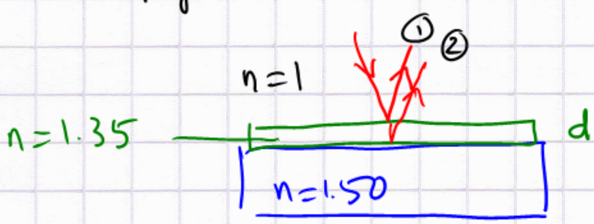
$$(m_1 - m_2)\lambda = 2\Delta x \qquad \therefore \lambda = \frac{2\Delta x}{7000} = \frac{2 \cdot 2.0}{7{,}000} \text{ mm}$$

$\lambda = 0.571\ \mu m = 571$ nm yellow

25.16 Soapy water film ($n = 1.35$) sits on top of a flat glass plate ($n = 1.50$) and has a red color ($\lambda_{red} \simeq 600$ nm) when viewed at normal incidence. What is the film's thickness?

Solution. 1) assume constructive interference for $\lambda = 600$ nm

2) figure out the condition (reflected light)



Both reflections (for beams ① and ② are for passages $n_1 < n_2$, i.e., phase jumps do occur

Since both do undergo $\lambda/2$ jumps we ignore them

physical path length difference $\simeq 2d$

optical " " " $\simeq 2d\, n_1$ ($n_1 = 1.35$)

Constructive interference: $2dn_1 = m\lambda$

try $m = 1$ $\therefore$ $d = \dfrac{\lambda}{2n_1} = \dfrac{600 \text{ nm}}{2 \cdot 1.35} = 222$ nm

Film thickness $\simeq 220$ nm $= 0.22\,\mu$m

seems plausible.

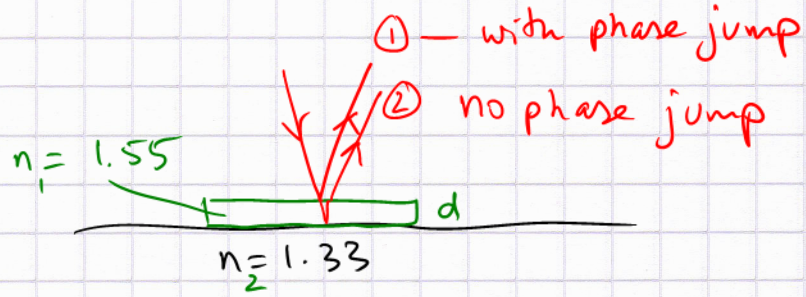
NB: 25.17 → film is on $n_2 = 1.1$ material → no phase jump for beam ② $\therefore$ constructive IF: $2n_1 d = (m + \frac{1}{2})\lambda$

$m = 0$ $\therefore$ $2n_1 d = \dfrac{\lambda}{2}$ $\therefore$ $d = 0.11\,\mu$m

25.19 Very thin glass sheet $n = 1.55$ floats on water ($n = 1.33$). White light at normal incidence $\rightarrow$ reflection is strong at 560 and 400 nm

Glass thickness = ?

Solution.



Constructive IF: $2 d n_1 = (m_1 + \frac{1}{2}) \lambda_1$

$$= (m_2 + \tfrac{1}{2}) \lambda_2$$

question: can we find small m_1, m_2 to make this work?

$$(m_1 + \tfrac{1}{2})\, 560 = (m_2 + \tfrac{1}{2})\, 400 \qquad \text{clearly } m_2 > m_1$$

$(\Delta m = m_2 - m_1)$

$$560\, m_1 + 280 = 400\, m_2 + 200$$

$$400\, m_2 - 560\, m_1 = 80$$

$$5\, m_2 - 7\, m_1 = 1$$

Q: which integers (m_1, m_2) will satisfy this?

try

m_1	m_2
0	×
1	$5 m_2 = 8$ ×
$\underline{\underline{2}}$	$5 m_2 = 15$ ✓ $\quad \underline{\underline{m_2 = 3}}$

$\rightarrow$ now $(2 + \frac{1}{2}) \lambda_1 = 2 d n_1$

$$d = \frac{2.5 \cdot 560 \text{ nm}}{2 \cdot 1.55}$$

$d = 452$ nm $= 450$nm