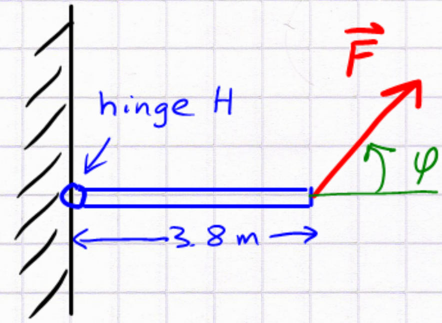
Week 9 8.19

$\tau_H = 18$ Nm $F = 10$ N a) $\varphi = ?$

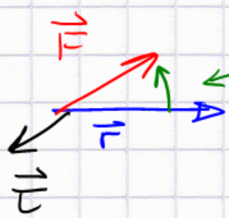
b) Sign of torque ?



Solution

b) The sign of $\vec{\tau}$?

RH rule : $\vec{\tau} = \vec{r} \times \vec{F}$ $\tau_z = r F \sin(\measuredangle \vec{r}, \vec{F})$

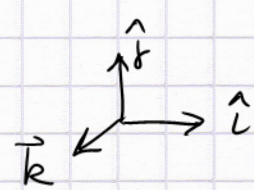


$\leftarrow \varphi = \measuredangle \vec{r}, \vec{F}$!

$\vec{\tau}$ is out of the plane by the RH rule

$\varphi \approx 45°$

$\sin\varphi > 0 \quad \therefore \tau_z > 0$



$\hookrightarrow$ is aligned with $\hat{k}$

τ_z is positive

a) $\tau_H = r F \sin\varphi$

In SI : $18 = 3.8 \cdot 10 \cdot \sin\varphi \quad \therefore \sin\varphi = \frac{1.8}{3.8}$

$\varphi = 28.3°$

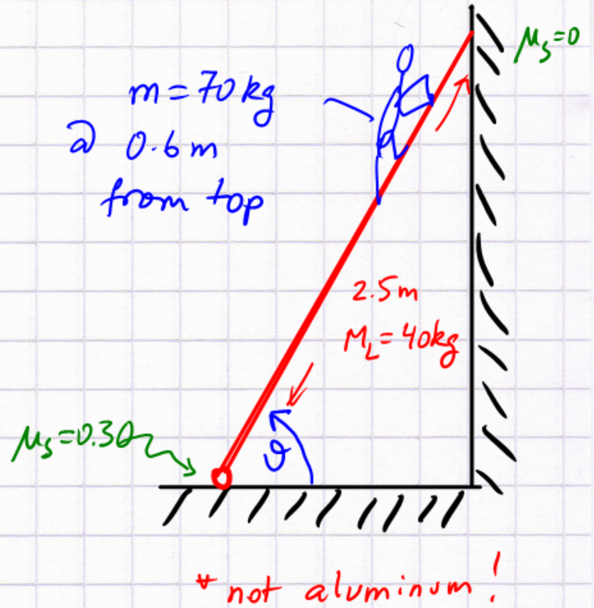
$\therefore$ The angle $\varphi = 28°$

8.30

a) sketch with all forces on ladder

b) translational + rotational equilibrium

c) $\vartheta = 60°$: will the ladder slip?

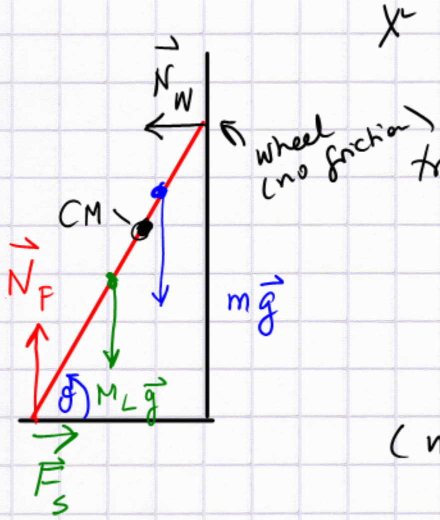


Solution.

Q1 : where is the CM ?

$$(M_L + m)\, x^c = M_L \cdot 0 + m \cdot (1.25 - 0.6)$$

$$x^c = \frac{70}{110}(0.65) = 0.414\ m \qquad (\text{from CM of the ladder})$$

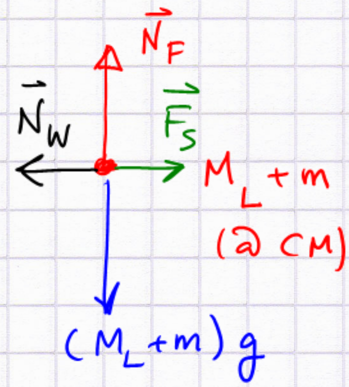


translational equilibrium:

$$N_W = F_S$$

$$N_F = (M_L + m)\, g \rightarrow 1.08 \times 10^3\ N$$

(magnitudes!)

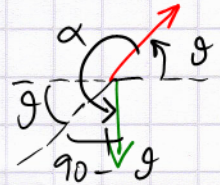


Rotation : using the floor contact as a pivot :
(no torque from $\vec{N}_F$; zero arm length)

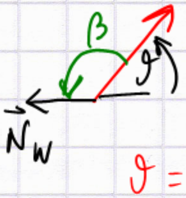
$$\vec{r}_{CM} \times (M_L + m)\,\vec{g} + \vec{r}_W \times \vec{N}_W = \vec{0}$$

LADDER SLIPS!

$$(1.25 + 0.414) \cdot 110 \cdot 9.8 \sin\alpha + 2.5 \cdot N_W \sin\beta = 0$$



$$\alpha = 180 + (90 - \vartheta) = 270 - \vartheta$$



$$\beta = 180 - \vartheta$$

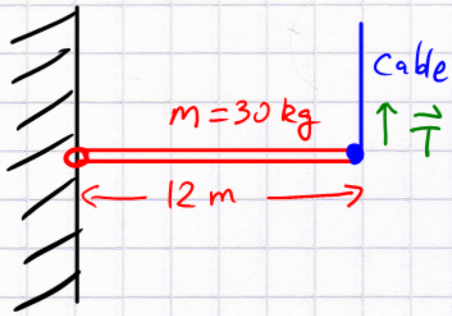
$$1.79 \times 10^3 \left(-\sin(90-\vartheta)\right) = 2.5\, N_W \sin\vartheta$$

$$N_W = 717.5 \cot\vartheta$$

NO!

$\vartheta = 60°$: $N_W = 414\ N$; $F_s = 414 N \leq 0.3\, N_F$?

8.38 flagpole : hinged @ wall
+ supported by cable



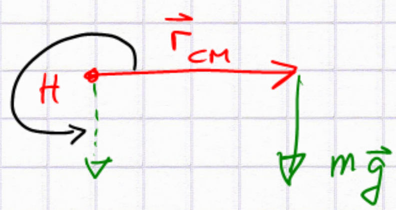
$T = ?$

Solution.

The CM of the pole is located at 6.0 m

The gravitational torque at the hinge :

$\alpha = 270°$



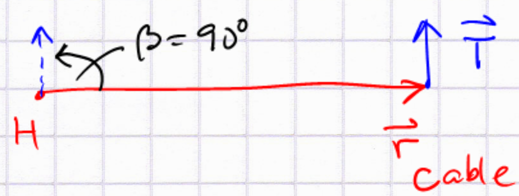
$$\tau_z^{grav} = (r_{CM})(mg)\sin(270°)$$

$$= -mg\,r_{CM} = -30 \cdot 9.8 \cdot 6.0 \text{ Nm}$$

$$= -1.76 \times 10^3 \text{ Nm}$$

(would cause a CW rotation, $\vec{\tau}$ is into the plane by RH rule $\therefore$ consistent)

The counter-torque :



$$\tau_z^{(cable)} = T \cdot r_c \cdot \sin(90°)$$

$$= T \cdot 12 \text{ m} > 0$$

Rotational equilibrium : $|\tau_z^{cable}| = |\tau^{grav}|$

$$T = \frac{1.76 \times 10^3}{12} \text{ N} = 147 \text{ N}$$

$$T = 150 \text{ N}$$

8.72 sports car $\to$ $v_{max} = 70\ \frac{m}{s}$ $r_{wheel} = 30\text{ cm} = 0.30\text{ m}$

uniform acceleration to v_{max} in 9.0 sec wow!

$\alpha_w = ?$

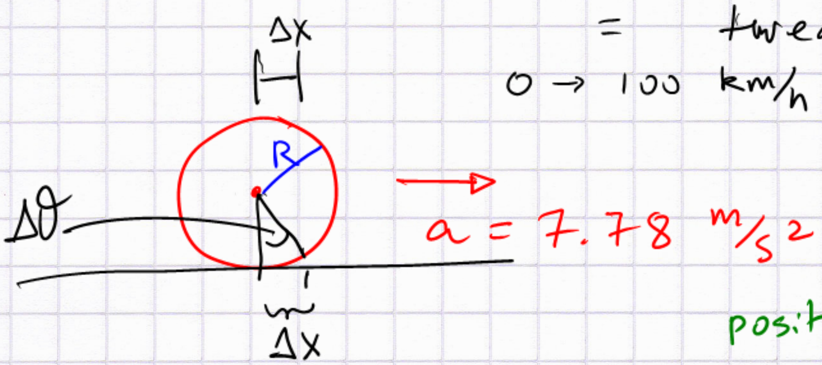
Solution

$$a = \frac{v_{max}}{\Delta t} = \frac{70\ m/s}{9.0\ s} = 7.78\ \frac{m}{s^2}$$

(almost "free-fall" conditions for 9 sec.)

= tweaked Corvette ?

0 → 100 km/h in ≈ 4-5 sec. = $200,000 car



Δx is also how much the CM advances!

positive acc. ⇒ CCW rotation

$$-R\,\Delta\vartheta = \Delta x \quad \Big|\ :\Delta t$$

$$-R\,\frac{\Delta\vartheta}{\Delta t} = \frac{\Delta x}{\Delta t} = v_x$$

$$-R\,\Delta\omega = \Delta v_x \quad \Big|\ :\Delta t$$

$$-R\,\frac{\Delta\omega}{\Delta t} = \frac{\Delta v_x}{\Delta t} = a_x$$

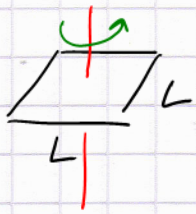
$$-R\,\alpha = a_x$$

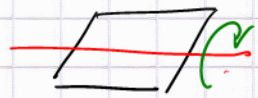
$$|\alpha| = \frac{a_x}{R} = \frac{7.78\ m/s^2}{0.30\ m}$$

$$= 25.9\ \frac{rad}{s^2} \qquad \therefore \quad \alpha = 26\ \frac{rad}{s^2}$$

8.49 square plate $L \times L$, mass M

$I = \frac{M}{6} L^2$

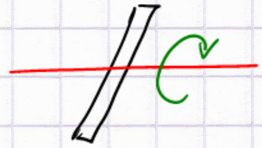




$I = ?$

same as a rod of length L spun about the middle

find:



$I = \frac{1}{12} M L^2$

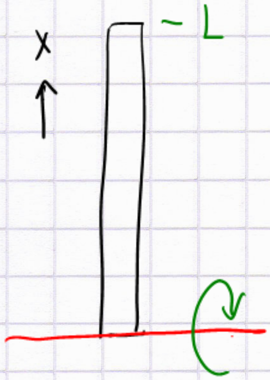
$I = \frac{M}{3} L^2$

(book answer)

Likewise: the desired problem

= rod (length L) spun about end

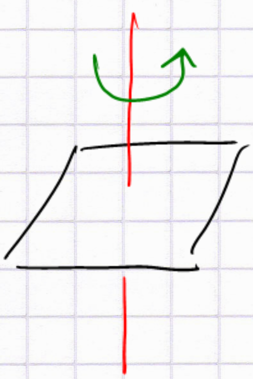
$= \frac{M}{3} L^2$ (given in book)



$$I = \int_0^L x^2 dm = \int_0^L x^2 \underbrace{\left(\frac{M}{L}\right)}_{\text{linear mass density}} dx$$

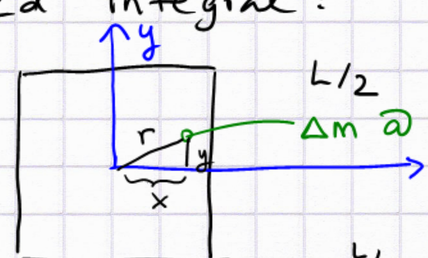
$$= \left(\frac{M}{L}\right) \times \frac{1}{3} x^3 \Big|_0^L = \frac{M}{3L} (L^3 - 0)$$

$$= \frac{1}{3} M L^2$$



requires a 2d integral:

top view



Δm @ (x,y) has $r^2 = x^2 + y^2$

$$I = \int r^2 dm = \iint_{-L/2}^{L/2} r^2 \underbrace{\left(\frac{M}{L^2}\right)}_{\text{area density}} dx\,dy = \frac{M}{L^2} \iint_{-L/2}^{L/2} (x^2 + y^2)\, dx\, dy$$

$$= \frac{M}{L^2} \left[\int x^2 dx \int dy + \int dx \int y^2 dy \right] = \frac{2M}{L^2} \left[L \cdot \frac{1}{3} x^3 \Big|_{-L/2}^{L/2} \right] = \frac{4}{3 \cdot 8} M L^2$$