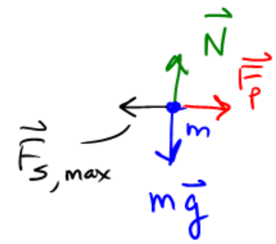
LAST NAME: STUDENT NR:

PHYS 1010 6.0: CLASS TEST 1

Time: 50 minutes; Calculators & formulae provided at the end = only aid; Total = 20 points.

1 a)[3] The coefficient of static friction between a crate and the (level) floor is given as $\mu_s = 0.4$0, while the kinetic friction coefficient is $\mu_{\cancel{s}\,k} = 0.15$. The mass of the crate is $m = 55$kg. You apply a force that is just enough to make the refrigerator move. What is the acceleration? Explain your steps. start with a free-body diagram.

0.5



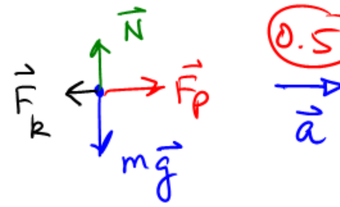
starts to move

vertical direction: $N = mg$ (magnitudes)

$F_{s,max} = \mu_s N = \mu_s mg$

1 $\therefore$ $F_{push} = \mu_s mg = 0.40 \times 55 \times 9.8$ N $(\approx 216 \text{ N})$

0.5



$F_k = \mu_k mg$. $ma_x = F_{net,x}$

$ma_x = F_{push} - F_k = (\mu_s - \mu_k) mg$ 0.5

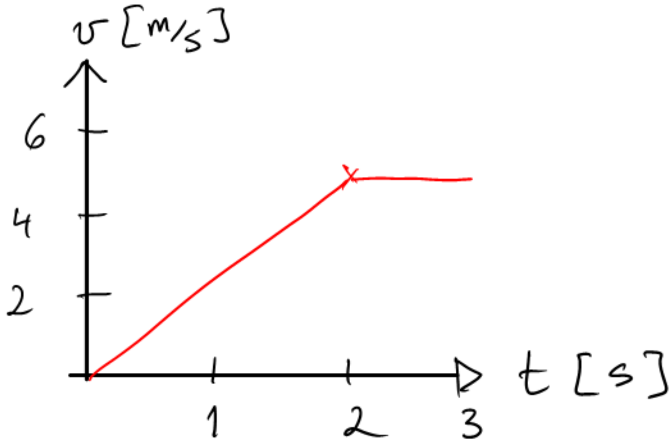
$a_x = (\mu_s - \mu_k) g = 0.25 \times 9.8 \frac{m}{s^2} = 2.45 \frac{m}{s^2}$ 0.5

(2.5 is also OK)

1 b)[2] The maximum speed at which you will push this crate is $v_f = 5.0$m/s. At which time will you reach this speed? Provide a velocity vs time graph from the start, and past the point of reaching v_f

$v = at$ $v_f = at_f$ $t_f = \frac{v_f}{a} = \frac{5.0}{2.45} \frac{m}{s} = 2.0$ s

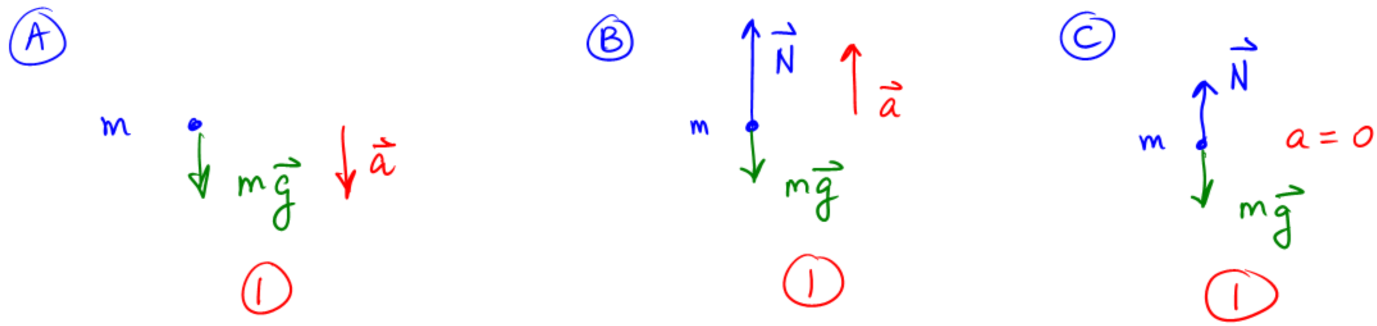
0.5 0.5



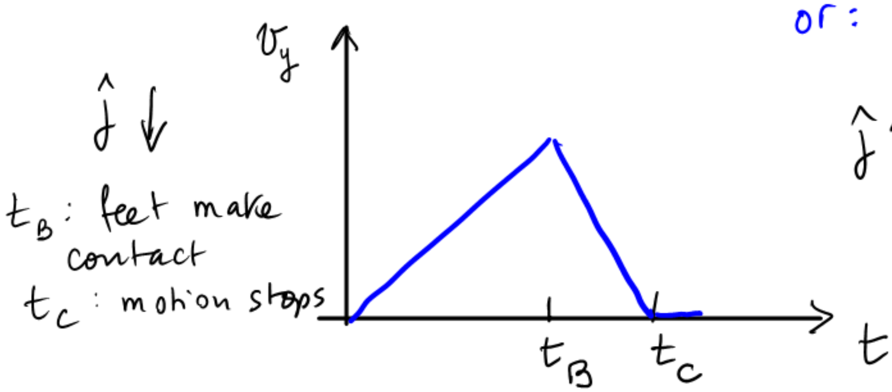
1 (includes proper labeling)

2 a) [3] A girl (mass m) is jumping from a ladder. Draw free-body (FB) diagrams for three situations: *(A)* she is in the air; *(B)* her feet have just made contact with the ground (her legs are straight); *(C)*her centre of gravity has reached the lowest point (her legs are bent at the knees) and the motion is stopped. Indicate the forces with arrows of appropriate respective length in the FB diagrams, and also the acceleration of the centre of gravity (beside the diagrams).

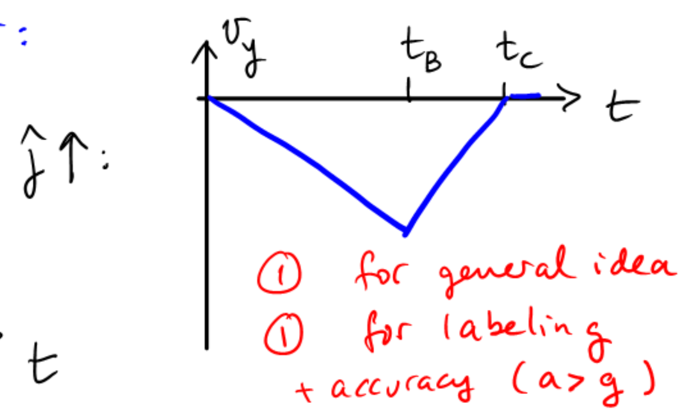
Ignore drag.



2 b) [2] Draw a qualitative velocity vs time diagram and indicate the phases (free fall and breaking the fall).



or:



3) [5] You are given the following densities ρ in kg/m^3: air = 1.3, water = 1.0×10^3, steel = 7.8×10^3, glass = 2.5×10^3, lead = 1.1×10^4. A ball of radius r_B = 10cm filled with water, is dropped from a high tower. Derive the formula for terminal velocity assuming quadratic air drag. Calculate the impact velocity for the ball. Compare this to the free-fall velocity (no drag) for a tower of height h = 100m. Assume the ball to be a sphere made of water, with radius r_B.

sphere volume = $\frac{4}{3}\pi R^3$, sphere area = $4\pi R^2$, max. cross section = πR^2

$$\left.\begin{aligned} m &= \rho_{water} \cdot \frac{4}{3}\pi r_B^3 \\ &= 10^3 \times 4.19 \times 0.1^3 \text{ kg} = 4.19 \text{ kg} \end{aligned}\right\} 1$$

$F_{grav} = F_{drag} \quad \therefore \quad mg \underset{1}{=} 0.5\, \rho_{air} A_{cr} v_t^2 \quad ; \quad A_{cr} = \pi R^2$

$$v_t^2 = \frac{2.0\, mg}{1.3 \times 3.14 \times 0.01} = \frac{2.0 \times 4.19 \times 9.8}{1.3 \times 3.14 \times 0.01} \frac{m^2}{s^2} = 2012 \frac{m^2}{s^2}$$

$$v_t = \sqrt{\frac{2.0 m g}{\rho_{air} A_{cr}}} = 45 \frac{m}{s} \quad 1 \quad (\simeq 160 \text{ km/h})$$

1

= impact velocity.

Free fall (no drag) $\rightarrow$ $v_f^2 = 2gh$ (0.5)

$v_f^2 = v_0^2 + 2a\Delta x$

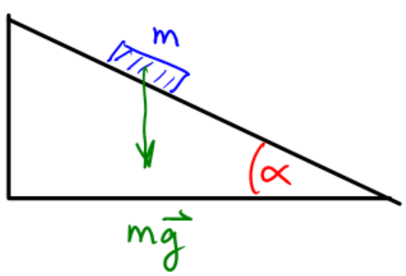
$h = 100\ \text{m} \rightarrow v_f^2 = 200 \times 9.8\ \frac{\text{m}^2}{\text{s}^2} = 1960.$

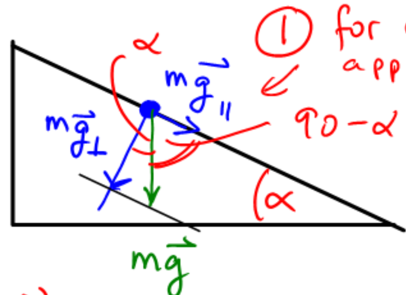
$v_f = 44\ \frac{\text{m}}{\text{s}}$ (0.5)

It takes the water-filled ball about 100 m of free fall to reach v_t.

(In reality a bit more, since drag will kick in sooner and start slowing it down a bit)

4) [5] Consider a metal bar (mass m) resting on an incline made of wood. You can adjust the angle of the incline. At some critical angle α_c the metal bar starts sliding. Show a free-body diagram for the metal bar indicating all forces acting on it for a situation well below the critical angle $\alpha << \alpha_c$ (bar is not sliding, $\mu_s = 0.40$). How big is the static friction force? (symbolic answer).

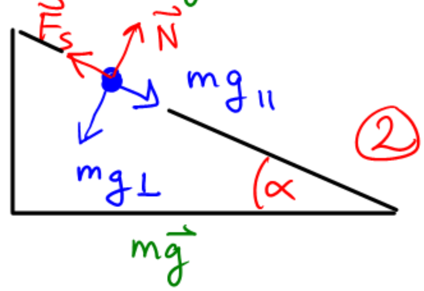




(1) for explaining how α appears

$g_\perp = g\cos\alpha$

$g_\parallel = g\sin\alpha$ (1)



(2)

$F_s = mg_\parallel = mg\sin\alpha$ (1)

Bonus: $mg\sin\alpha_c = \mu_s mg\cos\alpha_c$

$\frac{\sin\alpha_c}{\cos\alpha_c} = \mu_s \qquad \alpha_c = \tan^{-1}(\mu_s)$ extra (+1)

FORMULA SHEET

$v(t_\mathrm{f}) = v(t_\mathrm{i}) + \int_{t_\mathrm{i}}^{t_\mathrm{f}} a(t)\ dt \quad s(t_\mathrm{f}) = s(t_\mathrm{i}) + \int_{t_\mathrm{i}}^{t_\mathrm{f}} v(t)\ dt$

$v_\mathrm{f} = v_\mathrm{i} + a\Delta t \quad s_\mathrm{f} = s_\mathrm{i} + v_\mathrm{i}\Delta t + \frac{1}{2}a\Delta t^2 \quad v_\mathrm{f}^2 = v_\mathrm{i}^2 + 2a\Delta s \qquad g = 9.8\ \mathrm{m/s^2}$

$f(t) = t \quad \frac{df}{dt} = 1 \quad F(t) = \int f(t)\ dt = \frac{t^2}{2} + C$

$f(t) = a \quad \frac{df}{dt} = 0 \quad F(t) = \int f(t)\ dt = at + C \quad F(t) = \text{anti-derivative} = \text{indefinite integral}$

area under the curve $f(t)$ between limits t_1 and t_2: $F(t_2) - F(t_1)$

$x^2 + px + q = 0$ factored by: $x_{1,2} = -\frac{p}{2} \pm \sqrt{\frac{p^2}{4} - q}$

$\exp' = \exp$; $\sin' = \cos$; $\cos' = -\sin$. $\quad \frac{d}{dx}[f(g(x))] = \frac{df}{dg}\frac{dg}{dx}; \quad (fg)' = f'g + fg'$

$m\vec{a} = \vec{F}_\mathrm{net}; \quad F_G = \frac{Gm_1m_2}{r^2}$; $g = \frac{GM_E}{R_E^2}$; $R_E = 6370$ km; $G = 6.67 \times 10^{-11}\frac{\mathrm{Nm^2}}{\mathrm{kg^2}}$; $M_E = 6.0 \times 10^{24}$kg

$f_\mathrm{s} \leq \mu_\mathrm{s}n; \quad f_\mathrm{k} = \mu_\mathrm{k}n; \quad \mu_\mathrm{k} < \mu_\mathrm{s}$.

$\vec{F}_\mathrm{d} \sim -\vec{v}$; linear: $F_\mathrm{d} = dv$; quadr.: $F_\mathrm{d} = 0.5\rho Av^2$; $\quad A$ = cross s'n area; ρ = density of medium