LAST NAME: STUDENT NR:

PHYS 1010 6.0: CLASS TEST 3

Time: 50 minutes; Calculators & formulae provided at the end = only aid; Total = 20 points.

1) [5] Alice ($m = 50$ kg) is on a skateboard ($M = 2$ kg) moving with $v = 5$ m/s on a flat stretch of road. She jumps off backwards such that the board is propelled forward to a speed of 8 m/s. What is her velocity with respect to the ground as she lands? Ignore friction and drag, and note that magnitude and direction are required (the board is moving in the $+x$ direction).

A = Alice and S = skateboard before the jump have momentum : $P_i = (m+M)\,v$ 1.0

The total momentum is conserved during the jump: 0.5

$$P_f = m v_f + M V_f = P_i = (m+M)\,v \quad 1.0$$

(no external forces act during the jump) - optional

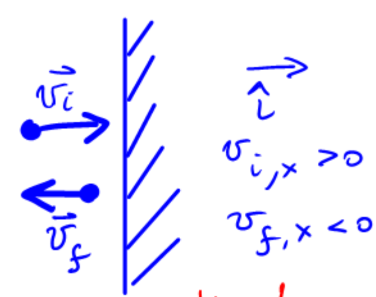
Calculate v_f :

$$\left.\begin{aligned} m v_f + M V_f &= (m+M)\,v \\ v_f &= v + \frac{M}{m}(v - V_f) \\ \text{In SI:}\quad v_f = 5 + \tfrac{2}{50}(5-8) &= 5 - \tfrac{6}{50} = 4.88 \simeq 4.9\ \tfrac{m}{s} \end{aligned}\right\} 2$$

which is still in the forward direction relative to ground . 0.5

2) [5] A squash ball ($m = 100$ g) flies very fast horizontally towards a wall ($v_x = 40$ m/s), and makes a perfect rebound (no mechanical energy loss). Use the impulse-momentum theorem to calculate the impulse provided by the wall in SI units (be careful with the sign). Then sketch a possible $F_x(t)$ curve together with an average force estimate assuming the collision took $\Delta t = 10$ ms.

$$\Delta p_x = J_x = \text{area under } F_x(t)$$



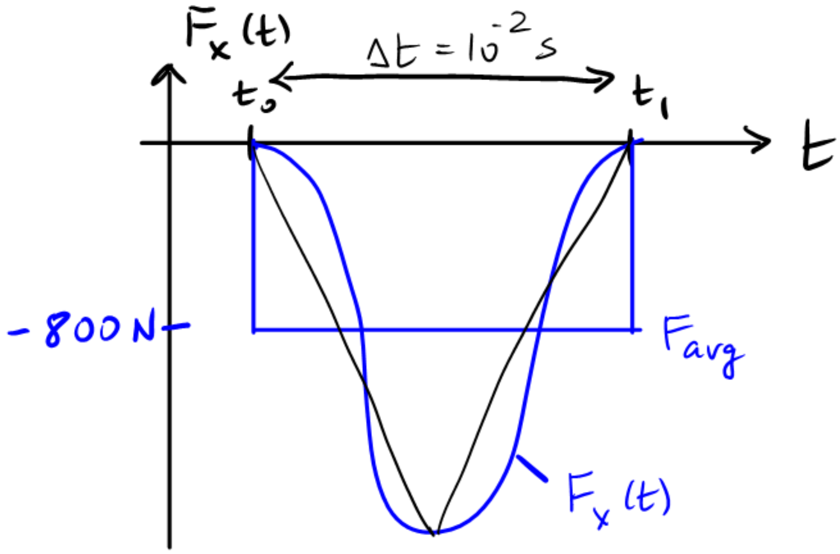
optional

$$\left.\begin{aligned} p_x^{fin} - p_x^{in} &= J_x \\ \text{In SI:}\quad 0.1\,\big(-40 - (+40)\big) &= J_x \\ J_x &= -8.0\ \tfrac{kg\,m}{s} \end{aligned}\right\} 2.0$$

$$J_x = F_{av}\,\Delta t \qquad \therefore \quad F_{av} = \frac{J_x}{\Delta t} = \frac{-8.0}{10^{-2}}\ \text{N} = -800\ \text{N}$$

0.5 0.5

...



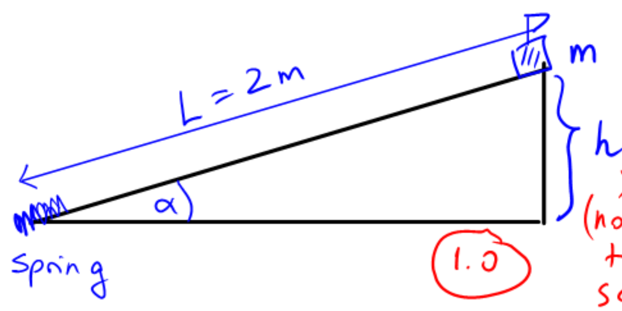
2

different shapes are possible

area under curve $\approx$ $F_{avg} \cdot \Delta t$

neg. sign is important.

3) [5] A glider ($m = 200$ g) is held at the top end of a tilted air track (angle with horizontal $\alpha = 30°$) of length $L = 2$m. At the other end is a short spring with constant $k = 100$ N/m. By how much will this spring be maximally compressed when the glider is released and reaches the bottom? Ignore gravity during the spring compression. Start with a drawing of the situation and state the laws used to solve the problem.



(not to scale is OK)

$h = L \sin\alpha = 2m \cdot (\frac{1}{2}) = 1$ m

Initial energy : $PE = mgh$

$PE = 0.2 \times 9.8 \times 1.0$ Nm

$= 1.96$ Nm $(\approx 2{,}0$ Nm$)$ 1.0

Mechanical energy is conserved. At the bottom of the track (just as the glider touches the spring) the gravitational PE is fully converted into $KE = \frac{1}{2} m v_f^2$ 0.5

The spring is maximally compressed when the total energy is entirely spring-PE : $PE_{spr} = \frac{1}{2} k \Delta x^2$ 0.5

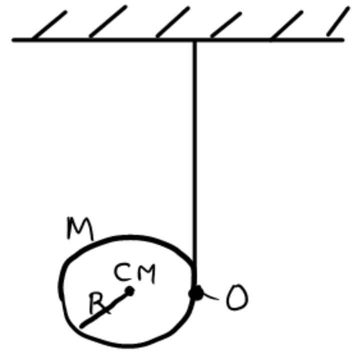
$\therefore \frac{1}{2} k \Delta x^2 = 2.0$ Nm $\therefore \Delta x^2 = \frac{4.0}{k}$ Nm

$= 4.0 \times 10^{-2}$ m^2

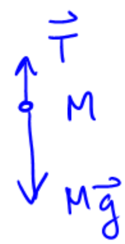
$\Delta x = 2.0 \times 10^{-1}$ m $= 0.2$m $= 20$ cm is the maximal spring compression

2

4) [5] A simple yo-yo is made from a (thin) disk of radius R and mass M ($I_{\mathrm{CM}} = 0.5MR^2$) with a thread wound around it. The downward motion can be understood as a superposition of a fall of the CM and a rotation of the disk about the point where the string connects with the disk ($I_{\mathrm{O}} = 1.5MR^2$). Formulate the laws of motion for the fall (don't forget to specify the net force on the CM and the torque about the pivot point O). Then, calculate the acceleration of the CM, and the tension in the string.



CM motion: free-body diagram



$$M a_{CM} = Mg - T \quad 1$$

Rotation about pivot O:

$$I_0 \alpha = \tau = MgR \quad 1$$ (out of plane, leads to ccw rotation; pos. sense, $\tau > 0$)

$$\frac{3}{2} MR^2 \alpha = MgR$$

$$\alpha = \frac{2}{3} \frac{g}{R}$$

To relate rotation and CM translation use the constraint $R\vartheta = y_{CM} \therefore R\omega = v_{CM} \therefore R\alpha = a_{CM}$

$$\alpha = \frac{a_{CM}}{R} \quad \therefore \quad a_{CM} = \frac{2}{3} g \quad 2$$

$$T = Mg - M a_{CM} = M\left(g - \frac{2}{3}g\right) = \frac{1}{3} Mg \quad 1$$

FORMULA SHEET

$v(t_\mathrm{f}) = v(t_\mathrm{i}) + \int_{t_\mathrm{i}}^{t_\mathrm{f}} a(t)\ dt \quad s(t_\mathrm{f}) = s(t_\mathrm{i}) + \int_{t_\mathrm{i}}^{t_\mathrm{f}} v(t)\ dt$

$v_\mathrm{f} = v_\mathrm{i} + a\Delta t \quad s_\mathrm{f} = s_\mathrm{i} + v_\mathrm{i}\Delta t + \frac{1}{2}a\Delta t^2 \quad v_\mathrm{f}^2 = v_\mathrm{i}^2 + 2a\Delta s \qquad g = 9.8\ \mathrm{m/s^2}$

$f(t) = t \quad \frac{df}{dt} = 1 \quad F(t) = \int f(t)\ dt = \frac{t^2}{2} + C$

$f(t) = a \quad \frac{df}{dt} = 0 \quad F(t) = \int f(t)\ dt = at + C \quad F(t) =$ anti-derivative = indefinite integral

area under the curve $f(t)$ between limits t_1 and t_2: $F(t_2) - F(t_1)$

$x^2 + px + q = 0$ factored by: $x_{1,2} = -\frac{p}{2} \pm \sqrt{\frac{p^2}{4} - q}$

uniform circular m. $\vec{r}(t) = R(\cos\omega t\ \hat{\mathbf{i}} + \sin\omega t\ \hat{\mathbf{j}})$; $\vec{v}(t) = \frac{d\vec{r}}{dt} = ...$; $\vec{a}(t) = \frac{d\vec{v}}{dt} = ...$

$\exp' = \exp$; $\sin' = \cos$; $\cos' = -\sin$. $\quad \frac{d}{dx}[f(g(x))] = \frac{df}{dg}\frac{dg}{dx}$; $\quad (fg)' = f'g + fg'$

$m\vec{a} = \vec{F}_\mathrm{net}$; $\quad F_G = \frac{Gm_1m_2}{r^2}$; $g = \frac{GM_E}{R_E^2}$; $R_E = 6370$ km; $G = 6.67 \times 10^{-11}\,\frac{\mathrm{Nm^2}}{\mathrm{kg^2}}$; $M_E = 6.0 \times 10^{24}$kg

$f_\mathrm{s} \leq \mu_\mathrm{s} n$; $\quad f_\mathrm{k} = \mu_\mathrm{k} n$; $\quad f_\mathrm{r} = \mu_\mathrm{r} n$; $\quad \mu_\mathrm{r} << \mu_\mathrm{k} < \mu_\mathrm{s}$. $\qquad F_H = -k\Delta x = -k(x - x_0)$.

$\vec{F}_\mathrm{d} \sim -\vec{v}$; linear: $F_\mathrm{d} = dv$; quadratic: $F_\mathrm{d} = 0.5\rho A v^2$; $\quad A =$ cross sectional area

$W = F\Delta x = F(\Delta r)\cos\theta \quad$ For $F(x)$ the work is given as area under the F_x vs x curve.

$\Delta\vec{p} = \vec{J} = \int \vec{F}(t)dt$; $\Delta p_x = J_x =$ area under $F_x(t) = F_x^\mathrm{avg}\Delta t$; $\quad \vec{p} = m\vec{v}$; $K = \frac{m}{2}v^2$

$\Delta\vec{p}_1 + \Delta\vec{p}_2 = 0$; $K_1^\mathrm{in} + K_2^\mathrm{in} = K_1^\mathrm{fin} + K_2^\mathrm{fin}$ for elastic collisions. $\quad \vec{a}_\mathrm{CM} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2}{m_1 + m_2}$

$\vec{\tau} = \vec{r} \times \vec{F}$; $\quad \tau_z = rF\sin(\alpha)$ for $\vec{r}$, $\vec{F}$ in xy plane. $\quad I = \sum_i m_i r_i^2$; $\quad I\alpha_z = \tau_z$; ($\hat{k}$ = rot. axis)

$K_\mathrm{rot} = \frac{I}{2}\omega^2$; $L_z = I\omega_z$; $\frac{d}{dt}L_z = \tau_z$; $\vec{L} = \vec{r} \times \vec{p}$; $\frac{d}{dt}\vec{L} = \vec{\tau}$