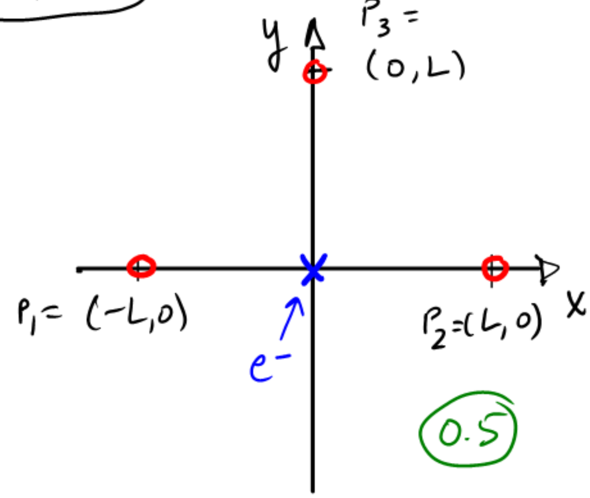
LAST NAME: STUDENT NR:

PHYS 1010 6.0: CLASS TEST 4

Time: 50 minutes; Calculators & formulae provided at the end = only aid; Total = 20 points.

1) [5] Three identical charges with $Q = +7.5\mu$C are located at $P_1 = (-L, 0)$, $P_2 = (L, 0)$ and $P_3 = (0, L)$ in the xy plane. What are the magnitude and direction of the force on an electron located at the origin $O = (0, 0)$ of the plane? Begin with a figure which shows the location of the three charges, and the electron. $\boxed{\text{for } L = 1.0 \text{ cm}}$

(b) calculate the e^- acceleration.



0.5

We add the force vectors.

The contributions from Q_1 and Q_2 cancel 0.5

By Coulomb's law:

$$|\vec{F}_E| = \frac{k|q_e Q|}{r^2} = \frac{k e Q}{L^2} \quad 1$$

in the $+\hat{j}$ direction 1

$$F_E = |\vec{F}_E| = \frac{9.0 \times 10^9 \times 1.6 \times 10^{-19} \times 7.5 \times 10^{-6}}{(10^{-2})^2} \quad \frac{N \frac{m^2}{C^2} C^2}{m^2}$$

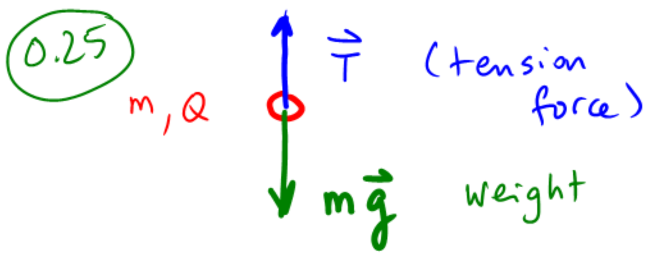
$$= 9.0 \times 1.6 \times 7.5 \times 10^{-12} \text{ N} = 1.1 \times 10^{-10} \text{ N} \quad 1$$

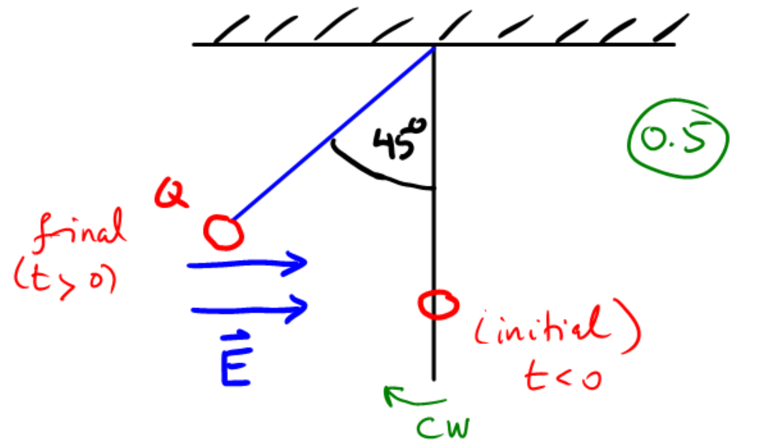
b) $$a = \frac{F_E}{m} = \frac{1.08 \times 10^{-10} \text{ N}}{9.11 \times 10^{-31} \text{ kg}} = 1.2 \times 10^{20} \frac{\text{m}}{\text{s}^2}$$

$$\vec{a} = 1.2 \times 10^{20} \hat{j} \quad \frac{\text{m}}{\text{s}^2} \quad 1$$

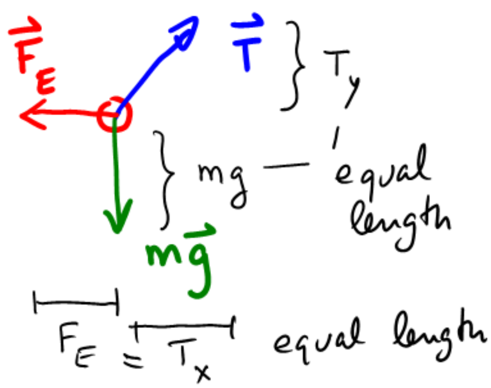
2) [5] A small plastic sphere carries an unknown charge Q. It is suspended by a string and placed in a uniform electric field $\vec{E} = 100\ \hat{\mathbf{i}}$ N/C that is turned on gently at $t = 0$. One finds that it finally makes an angle $\theta = 45°$ with respect to the vertical direction, and that it turned clockwise from the vertical as the field was turned on. Determine the charge Q (magnitude and sign). Start with a figure depicting the initial and final positions of the suspendend sphere and free-body diagrams for both positions.

For $t<0$:





For $t>0$:



0.75

$\vec{F}_E$ opposes $\vec{E}$

$\therefore$ $Q < 0$ (is negative) 1.0

For $\vartheta = 45°$ we have $T_x = T_y = mg$

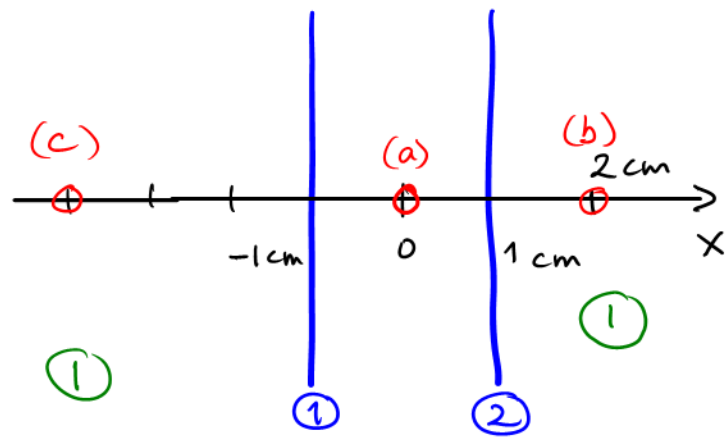
Since $F_E = T_x$ we find $F_E = mg$ 1.0

Then, $|Q|E = mg$ or $|Q| = \dfrac{mg}{E}$ 0.5

$$|Q| = \frac{0.05 \times 9.8}{100}\ \frac{C}{N}\frac{kg\ m}{s^2} = 4.9\ mC$$ 1.0

$Q = -4.9\ mC$

3) [5] Two thin, large-area insulating planes have been charged uniformly and carry a surface density of $\sigma = -10$ nC/cm^2 respectively. They are placed parallel to each other, oriented vertically, and separated by a distance of $\Delta x = 2.0$ cm. Plate 1 has location $X_1 = -1.0$ cm, and plate 2 is at $X_2 = 1.0$ cm. A proton is placed at three possible locations: (a) in the middle ($x = 0$); (b) at $x = 2.0$ cm; (c) at $x = -4.0$ cm. Determine the electric force experienced by the proton at the three locations (magnitude and direction for each case). Start with a sideview drawing. There is no need to derive the electric field from Gauss' law, you can use the expressions provided on the formula sheet.



Location (a)

the proton is attracted equally by both negatively charged plates $\Rightarrow$ $F_E = 0$ there 1

Location (b) Each plate contributes an equally strong field, (attracting the proton to the left)

The field does not depend on the distance of the proton (as long as it's closer than the plate dimension)

$$\vec{E}_1 = \vec{E}_2 = -\frac{|\sigma|}{2\varepsilon_0}\hat{\imath} \Rightarrow \vec{E}_{tot} = -\frac{|\sigma|}{\varepsilon_0}\hat{\imath} \quad 1$$

$$E_{tot} = \frac{|\sigma|}{\varepsilon_0} = \frac{10\ nC}{(10^{-2})^2\ m^2 \cdot 8.85\times10^{-12}\ \frac{Nm^2}{C^2}}$$

$$= \frac{10^{-8}\times10^{12}\times10^{4}}{8.85}\ \frac{N}{C} = 11.3\times10^{6}\ \frac{N}{C}$$

$$F_{el} = q_p E_{tot} = 1.60\times10^{-19}\,C \times 11.3\times10^{6}\ \frac{N}{C} = 1.8\times10^{-12}\,N$$

$$\vec{F}_{el} = -1.8\times10^{-12}\,\hat{\imath}\ N \quad 1$$

(c) The same field strenght, but to the right: $\vec{F}_{el} = +1.8\times10^{-12}\,\hat{\imath}\ N$ 1

4) [5] A proton moves from a location where $V = 125\text{V}$ to a place where $V = -40\text{V}$. (a) What is the change in the proton's kinetic energy? (b) Now replace the proton by an electron. What is the change in KE in this case?

Electric potential energy $U_{el} = q\, V_{el}$ 1

From energy conservation we have for locations 1, 2:

$$KE_1 + U_{el}(1) = KE_2 + U_{el}(2) \quad 1$$

$$\Delta(KE) = KE_2 - KE_1 = U_{el}(1) - U_{el}(2)$$

$$\therefore \quad \Delta(KE) = q\,(V_{el}(1) - V_{el}(2))$$

In our case: $V_{el}(1) = +125$ Volts

$V_{el}(2) = -40$ Volts

$\Delta V_{el} = 165$ Volts

$$\Delta(KE) = q \cdot (165 \text{ Volts}) \quad 1$$

(a) $q = +1.60 \times 10^{-19}\,C \Rightarrow \Delta(KE) = +2.6 \times 10^{-17}$ Nm

(it speeds up) 1

(b) $q = -1.60 \times 10^{-19}\,C \Rightarrow \Delta(KE) = -2.6 \times 10^{-17}$ Nm

(the electron slows down) 1

FORMULA SHEET

$v(t_f) = v(t_i) + \int_{t_i}^{t_f} a(t)\ dt \quad s(t_f) = s(t_i) + \int_{t_i}^{t_f} v(t)\ dt$

$v_f = v_i + a\Delta t \quad s_f = s_i + v_i\Delta t + \frac{1}{2}a\Delta t^2 \quad v_f^2 = v_i^2 + 2a\Delta s \qquad g = 9.8 \text{ m/s}^2$

$f(t) = t \quad \frac{df}{dt} = 1 \quad F(t) = \int f(t)\ dt = \frac{t^2}{2} + C$

$f(t) = a \quad \frac{df}{dt} = 0 \quad F(t) = \int f(t)\ dt = at + C \quad F(t) = \text{anti-derivative} = \text{indefinite integral}$

area under the curve $f(t)$ between limits t_1 and t_2: $F(t_2) - F(t_1)$

$x^2 + px + q = 0$ factored by: $x_{1,2} = -\frac{p}{2} \pm \sqrt{\frac{p^2}{4} - q}$

uniform circular m. $\vec{r}(t) = R(\cos\omega t\ \hat{\mathbf{i}} + \sin\omega t\ \hat{\mathbf{j}})$; $\vec{v}(t) = \frac{d\vec{r}}{dt} = ...$; $\vec{a}(t) = \frac{d\vec{v}}{dt} = ...$

$\exp' = \exp$; $\sin' = \cos$; $\cos' = -\sin$. $\quad \frac{d}{dx}[f(g(x))] = \frac{df}{dg}\frac{dg}{dx}$; $\quad (fg)' = f'g + fg'$

$m\vec{a} = \vec{F}_{\text{net}}$; $\quad F_G = \frac{Gm_1m_2}{r^2}$; $g = \frac{GM_E}{R_E^2}$; $R_E = 6370$ km; $G = 6.67 \times 10^{-11}\frac{\text{Nm}^2}{\text{kg}^2}$; $M_E = 6.0 \times 10^{24}$kg

$f_s \leq \mu_s n$; $\quad f_k = \mu_k n$; $\quad f_r = \mu_r n$; $\quad \mu_r << \mu_k < \mu_s$. $\qquad F_H = -k\Delta x = -k(x - x_0)$.

$\vec{F}_d \sim -\vec{v}$; linear: $F_d = dv$; quadratic: $F_d = 0.5\rho A v^2$; $\quad A$ = cross sectional area

$W = F\Delta x = F(\Delta r)\cos\theta$. W = area under $F_x(x)$. $\quad PE_H = \frac{k}{2}(\Delta x)^2$; $PE_g = mg\Delta y$.

$\Delta\vec{p} = \vec{J} = \int \vec{F}(t)dt$; $\Delta p_x = J_x$ = area under $F_x(t) = F_x^{\text{avg}}\Delta t$; $\quad \vec{p} = m\vec{v}$; $K = \frac{m}{2}v^2$

$\Delta\vec{p}_1 + \Delta\vec{p}_2 = 0$; $K_1^{\text{in}} + K_2^{\text{in}} = K_1^{\text{fin}} + K_2^{\text{fin}}$ for elastic collisions. $\quad \vec{a}_{\text{CM}} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2}{m_1 + m_2}$

$\vec{\tau} = \vec{r} \times \vec{F}$; $\quad \tau_z = rF\sin(\alpha)$ for $\vec{r}$, $\vec{F}$ in xy plane. $\quad I = \sum_i m_i r_i^2$; $\quad I\alpha_z = \tau_z$; ($\hat{k}$ = rot. axis)

$K_{\text{rot}} = \frac{I}{2}\omega^2$; $L_z = I\omega_z$; $\frac{d}{dt}L_z = \tau_z$; $\vec{L} = \vec{r} \times \vec{p}$; $\frac{d}{dt}\vec{L} = \vec{\tau}$

$x(t) = A\cos(\omega t + \phi)$; $\quad \omega = \frac{2\pi}{T} = 2\pi f$; $\quad v_x(t) = ...$; $\quad v_{\max} = ...$

$m_e = 9.11 \times 10^{-31}$kg $\quad m_p = 1.67 \times 10^{-27}$kg $\quad e = 1.60 \times 10^{-19}$C $\quad K = \frac{1}{4\pi\epsilon_0} = 9.0 \times 10^9\frac{\text{Nm}^2}{\text{C}^2}$

$\vec{F}_C = \frac{Kq_1q_2}{r^2}\hat{\mathbf{r}} \quad \vec{F}_E = q\vec{E} \quad E_{\text{line}} = \frac{2K|\lambda|}{r} = \frac{2K|Q|}{Lr} \quad E_{\text{plane}} = \frac{|\eta|}{2\epsilon_0} = \frac{|Q|}{2A\epsilon_0} \quad \vec{E}_{\text{cap}} = \left(\frac{Q}{\epsilon_0 A}, \text{pos} \to \text{neg}\right)$

$\frac{mv^2}{2} + U_{\text{el}}(s) = \frac{mv_0^2}{2} + U_{\text{el}}(s_0)$, $(U \equiv PE_{\text{el}}) \quad U_{\text{el}} = qEx$ for $\vec{E} = -E\ \hat{i} \quad V_{\text{el}} = U_{\text{el}}/q \quad E_x = -\frac{dV_{\text{el}}}{dx}$

$Q = C\Delta V_C \quad$ farad = F = $\frac{\text{C}}{\text{V}} \quad C = \frac{\epsilon_0 A}{d} \quad \epsilon_0 = 8.85 \times 10^{-12}\frac{\text{C}^2}{\text{Nm}^2}$

parallel C_1, C_2: $C_{\text{eq}} = C_1 + C_2 \quad$ series C_1, C_2: $C_{\text{eq}}^{-1} = C_1^{-1} + C_2^{-1}$