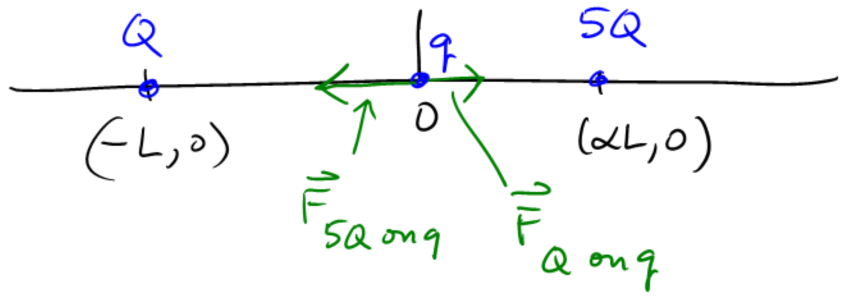
LAST NAME: STUDENT NR:

PHYS 1010 6.0: CLASS TEST 4

Time: 50 minutes; Calculators & formulae provided at the end = only aid; Total = 20 points.

1) [5] A charge Q is placed at $(-L, 0)$ and a second charge $5Q$ is placed at $(\alpha L, 0)$, where α is unknown. Determine α such that when a third charge is placed at $(0, 0)$ the force on it is zero. Start with a figure depicting the situation and indicate the forces acting on the third charge. Explain whether it matters whether the third charge is positive or negative.



1

we want

$$F_{5Q \text{ on } q} = F_{Q \text{ on } q} \qquad (F_{net} = 0)$$ 1

$$\left(\text{or } \vec{F}_{5Q \text{ on } q} + \vec{F}_{Q \text{ on } q} = \vec{0}\right)$$

$$F_{5Q \text{ on } q} = \frac{K\,Qq}{L^2} \quad , \quad F_{Q \text{ on } q} = \frac{5KQq}{\alpha^2 L^2}$$

0.5 0.5

$$\therefore \quad \frac{\cancel{K}\ \cancel{Q}\cancel{q}}{\cancel{L^2}} = \frac{5\cancel{K}\cancel{Q}\cancel{q}}{\alpha^2 \cancel{L^2}}$$

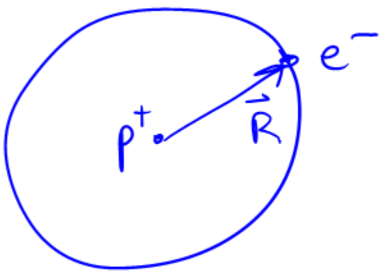
$$\therefore \quad 1 = \frac{5}{\alpha^2} \quad \therefore \quad \alpha^2 = 5$$ 0.5

$$\alpha = \sqrt{5} \quad (= 2.2..)$$ 0.5

q (as well as Q) dropped out of the calculation, the sign of q doesn't matter (q > 0 or q < 0 work the same) 1

2) [5] A classical model for the hydrogen atom can be imagined as the (heavy) proton being at rest, while the electron executes a circular orbit of radius $R = 0.53 \times 10^{-10}$ m. Find the speed of the electron from the assumption that the static Coulomb force provides the centripetal acceleration for the electron. Compare the electron's kinetic energy to the electric potential energy (calculate both, and ignore gravity; the electrostatic potential energy is $U_{\text{el}}(r)) = Kq_1q_2/r$).

e^- motion:



$$m a_{cp} = \frac{m v^2}{R} = \frac{Ke^2}{R^2} \quad 1$$

$$v^2 = \frac{Ke^2}{mR} = \frac{9.0 \times 10^9 (1.6 \times 10^{-19})^2}{9.11 \times 10^{-31} \cdot 0.53 \times 10^{-10}}$$

$$v^2 = 4.77 \times 10^{12} \ \frac{m^2}{s^2}$$

$$v = 2.18 \times 10^6 \ \frac{m}{s} \quad 1$$

$$KE = \frac{1}{2} m v^2 = 2.17 \times 10^{-18} \ J \quad (\text{or } Nm) \quad 1$$

$$PE = \frac{-Ke^2}{R} = \frac{-9.0 \times 10^9 (1.60 \times 10^{-19})^2}{0.53 \times 10^{-10}}$$

$$= -4.35 \times 10^{-18} \ J \quad 1$$

$\therefore \quad KE = -\frac{1}{2} PE \quad 1$ or $KE = \frac{1}{2} |PE|$ is also OK.

(or $PE = -2\ KE$)

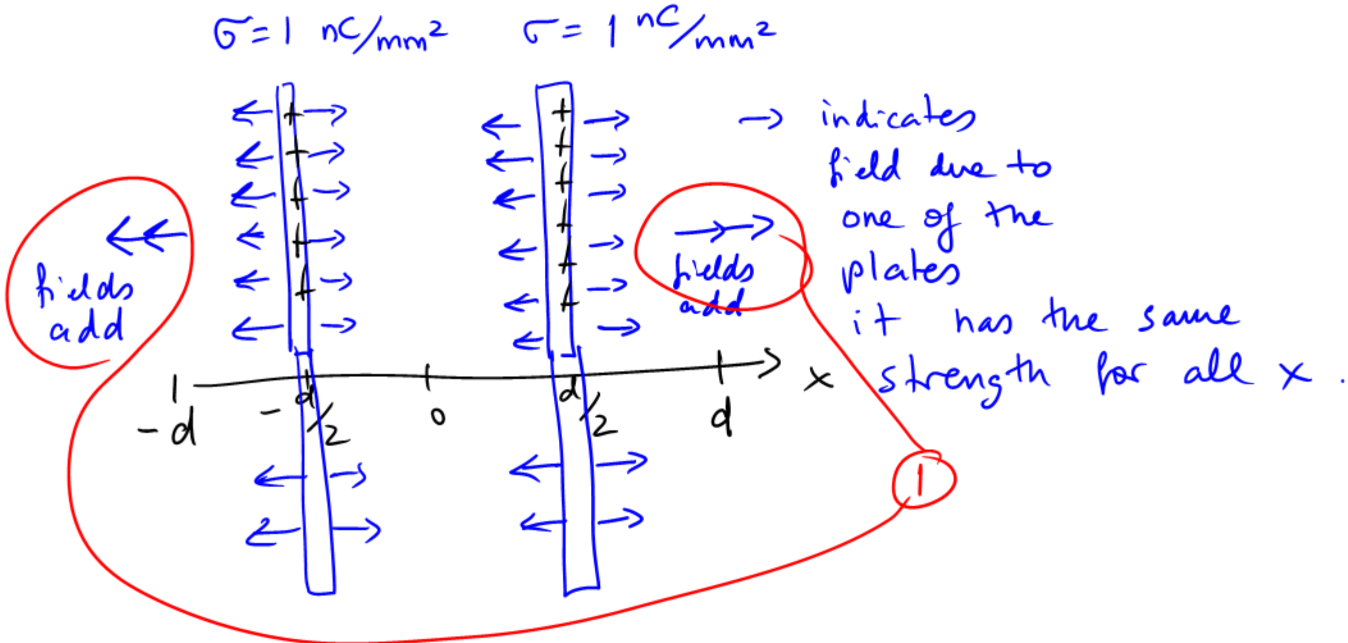
or: $KE + PE = E$

$E = -KE = -2.17 \times 10^{-18}$ J

expressed in eV:

$$E = -13.6 \ eV$$

3) [5] Consider two parallel (infinitely) large plates separated by distance d, both charged to the same surface density of $+1\ \mathrm{nC/mm^2}$. The plates are oriented vertically, one is placed at $x = -d/2$, the other at $x = +d/2$. Start with a figure and indicate the direction of the electric field vector at the two locations: $x = -d$ and $x = d$. Calculate the field strength at these locations, and also at $x = 0$, and $x = 2d$.



a) $x = d$: $\vec{E} \sim \hat{\imath}$, $E = \dfrac{\sigma}{2\varepsilon_0} + \dfrac{\sigma}{2\varepsilon_0} = \dfrac{\sigma}{\varepsilon_0}$

$$E = \frac{1\,\mathrm{nC}}{\mathrm{mm}^2} \cdot \left(8.85 \times 10^{-12} \frac{\mathrm{C}^2}{\mathrm{N\,m}^2}\right)^{-1}$$

(NB: $1\ \mathrm{mm}^2 = 10^{-6}\ \mathrm{m}^2$)

$$= \frac{10^{-9}}{8.85 \times 10^{-12} \times 10^{-6}} \frac{\mathrm{N}}{\mathrm{C}}$$

$$= \frac{10^{9}}{8.85} \frac{\mathrm{N}}{\mathrm{C}} = 1.1 \times 10^{8} \frac{\mathrm{N}}{\mathrm{C}} \quad \therefore \vec{E} = 1.1 \times 10^{8}\, \hat{\imath}\ \frac{\mathrm{N}}{\mathrm{C}} \quad 1$$

b) $\vec{E} = -1.1 \times 10^{8}\, \hat{\imath}\ \dfrac{\mathrm{N}}{\mathrm{C}}$ 1

c) $E = 0$ 1

d) $x = 2d$: Same result as at $x = d$: 1

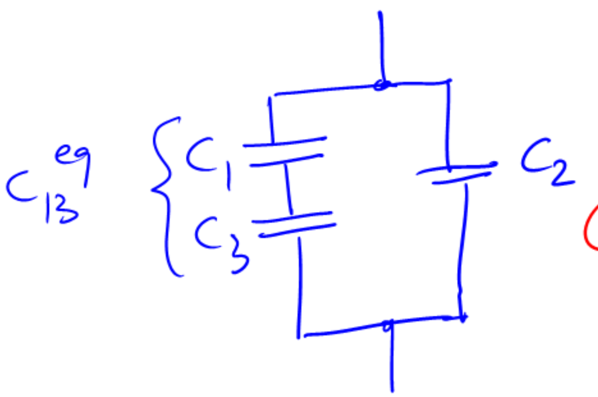
$$\vec{E} = 1.1 \times 10^{8}\, \hat{\imath}\ \frac{\mathrm{N}}{\mathrm{C}}$$

4) [5] You are given three capacitors. $C_1 = C_2 = 5.0\mu$F, and $C_3 = 10\mu$F. You are connecting C_1 and C_3 in series. What is the equivalent capacitance? Now you are adding to this circuit C_2 in parallel. Draw the circuit diagram and calculate the equivalent capacitance of the circuit. Now you apply a battery voltage $\Delta V = 1.5$ V. How much charge will be displaced in C_2? How much in C_1? How much in C_3?

C_1 and C_3 in series : $\frac{1}{C_{13}} = \frac{1}{C_1} + \frac{1}{C_3}$

$$C_{13}^{eq} = \frac{C_1 C_3}{C_1 + C_3} = \frac{5 \cdot 10}{15} \mu F$$

$$C_{13}^{eq} = \frac{10}{3} = 3.3 \mu F \quad 1$$



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$$C_{eq} = C_{13}^{eq} + C_2 = 3.3 + 5.0$$
$$= 8.3 \mu F \quad 1$$

$C \Delta V = Q$ For C_2 the full voltage applies

$$Q_2 = 5 \mu F \cdot 1.5 V = 7.5 \mu C \quad 1$$

The amount of charge displaced in C_1 and C_3 is the same, since the same current passes through

$\therefore$ Use C_{eq}^{13} : $Q_{13} = 3.3 \mu F \cdot 1.5 V = 5.0 \mu C$

C_1 has a charge displaced of $5.0 \mu C$ 0.5

C_3 " " " " " $5.0 \mu C$ 0.5

FORMULA SHEET

$v(t_\mathrm{f}) = v(t_\mathrm{i}) + \int_{t_\mathrm{i}}^{t_\mathrm{f}} a(t)\ dt \quad s(t_\mathrm{f}) = s(t_\mathrm{i}) + \int_{t_\mathrm{i}}^{t_\mathrm{f}} v(t)\ dt$

$v_\mathrm{f} = v_\mathrm{i} + a\Delta t \quad s_\mathrm{f} = s_\mathrm{i} + v_\mathrm{i}\Delta t + \frac{1}{2}a\Delta t^2 \quad v_\mathrm{f}^2 = v_\mathrm{i}^2 + 2a\Delta s \qquad g = 9.8\ \mathrm{m/s^2}$

$f(t) = t \quad \frac{df}{dt} = 1 \quad F(t) = \int f(t)\ dt = \frac{t^2}{2} + C$

$f(t) = a \quad \frac{df}{dt} = 0 \quad F(t) = \int f(t)\ dt = at + C \quad F(t) =$ anti-derivative = indefinite integral

area under the curve $f(t)$ between limits t_1 and t_2: $F(t_2) - F(t_1)$

$x^2 + px + q = 0$ factored by: $x_{1,2} = -\frac{p}{2} \pm \sqrt{\frac{p^2}{4} - q}$

uniform circular m. $\vec{r}(t) = R(\cos\omega t\ \hat{\mathbf{i}} + \sin\omega t\ \hat{\mathbf{j}})$; $\vec{v}(t) = \frac{d\vec{r}}{dt} = ...$; $\vec{a}(t) = \frac{d\vec{v}}{dt} = ...$

$\exp' = \exp$; $\sin' = \cos$; $\cos' = -\sin$. $\quad \frac{d}{dx}[f(g(x))] = \frac{df}{dg}\frac{dg}{dx}$; $\quad (fg)' = f'g + fg'$

$m\vec{a} = \vec{F}_\mathrm{net}$; $\quad F_G = \frac{Gm_1m_2}{r^2}$; $g = \frac{GM_E}{R_E^2}$; $R_E = 6370$ km; $G = 6.67\times 10^{-11}\frac{\mathrm{Nm^2}}{\mathrm{kg^2}}$; $M_E = 6.0\times 10^{24}$kg

$f_\mathrm{s} \le \mu_\mathrm{s} n$; $\quad f_\mathrm{k} = \mu_\mathrm{k} n$; $\quad f_\mathrm{r} = \mu_\mathrm{r} n$; $\quad \mu_\mathrm{r} << \mu_\mathrm{k} < \mu_\mathrm{s}$. $\qquad F_H = -k\Delta x = -k(x - x_0)$.

$\vec{F}_\mathrm{d} \sim -\vec{v}$; linear: $F_\mathrm{d} = dv$; quadratic: $F_\mathrm{d} = 0.5\rho A v^2$; $\quad A =$ cross sectional area

$W = F\Delta x = F(\Delta r)\cos\theta$. $W =$ area under $F_x(x)$. $\quad PE_\mathrm{H} = \frac{k}{2}(\Delta x)^2$; $PE_g = mg\Delta y$.

$\Delta\vec{p} = \vec{J} = \int \vec{F}(t)dt$; $\Delta p_x = J_x =$ area under $F_x(t) = F_x^\mathrm{avg}\Delta t$; $\quad \vec{p} = m\vec{v}$; $K = \frac{m}{2}v^2$

$\Delta\vec{p}_1 + \Delta\vec{p}_2 = 0$; $K_1^\mathrm{in} + K_2^\mathrm{in} = K_1^\mathrm{fin} + K_2^\mathrm{fin}$ for elastic collisions. $\quad \vec{a}_\mathrm{CM} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2}{m_1 + m_2}$

$\vec{\tau} = \vec{r}\times\vec{F}$; $\quad \tau_z = rF\sin(\alpha)$ for $\vec{r}$, $\vec{F}$ in xy plane. $\quad I = \sum_i m_i r_i^2$; $\quad I\alpha_z = \tau_z$; ($\hat{k} =$ rot. axis)

$K_\mathrm{rot} = \frac{I}{2}\omega^2$; $L_z = I\omega_z$; $\frac{d}{dt}L_z = \tau_z$; $\vec{L} = \vec{r}\times\vec{p}$; $\frac{d}{dt}\vec{L} = \vec{\tau}$

$x(t) = A\cos(\omega t + \phi)$; $\quad \omega = \frac{2\pi}{T} = 2\pi f$; $\quad v_x(t) = ...$; $\quad v_\mathrm{max} = ...$

$m_\mathrm{e} = 9.11\times 10^{-31}$kg $\quad m_\mathrm{p} = 1.67\times 10^{-27}$kg $\quad e = 1.60\times 10^{-19}$C $\quad K = \frac{1}{4\pi\epsilon_0} = 9.0\times 10^9\frac{\mathrm{Nm^2}}{\mathrm{C^2}}$

$\vec{F}_\mathrm{C} = \frac{Kq_1q_2}{r^2}\hat{\mathbf{r}} \quad \vec{F}_\mathrm{E} = q\vec{E} \quad E_\mathrm{line} = \frac{2K|\lambda|}{r} = \frac{2K|Q|}{Lr} \quad E_\mathrm{plane} = \frac{|\eta|}{2\epsilon_0} = \frac{|Q|}{2A\epsilon_0} \quad \vec{E}_\mathrm{cap} = \left(\frac{Q}{\epsilon_0 A}, \text{pos} \to \text{neg}\right)$

$\frac{mv^2}{2} + U_\mathrm{el}(s) = \frac{mv_0^2}{2} + U_\mathrm{el}(s_0)$, $(U \equiv PE_\mathrm{el}) \quad U_\mathrm{el} = qEx$ for $\vec{E} = -E\,\hat{i} \quad V_\mathrm{el} = U_\mathrm{el}/q \quad E_x = -\frac{dV_\mathrm{el}}{dx}$

$Q = C\Delta V_C \quad$ farad = F = $\frac{\mathrm{C}}{\mathrm{V}} \quad C = \frac{\epsilon_0 A}{d} \quad \epsilon_0 = 8.85\times 10^{-12}\frac{\mathrm{C^2}}{\mathrm{Nm^2}}$

parallel C_1, C_2: $C_\mathrm{eq} = C_1 + C_2 \quad$ series C_1, C_2: $C_\mathrm{eq}^{-1} = C_1^{-1} + C_2^{-1}$