LAST NAME: STUDENT NR:

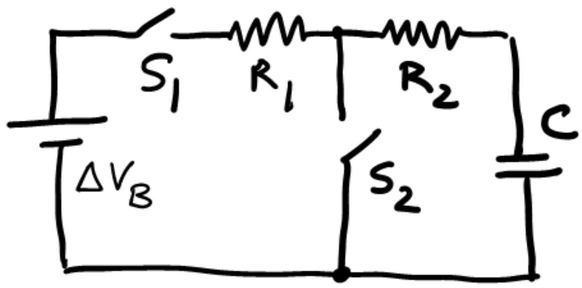
PHYS 1010 6.0: CLASS TEST 5

Time: 50 minutes; Calculators & formulae provided at the end = only aid; Total = 20 points.

1) [5] The capacitor in the circuit shown is initially uncharged. a) What is the battery current right after switch S_1 is closed? b) S_1 was closed now for a long time. At time t_1 simultaneously S_1 is opened and S_2 is closed. What is the current through S_2 at this instant. c) A long time later $t_2 >> t_1$ (while S_2 is closed) what is the the current through S_2?

$C = 90\ \mu F$, $R_1 = 2.2\ k\Omega$,

$\Delta V_B = 3.0\ V$, $R_2 = 4.9\ k\Omega$



a) $R_{eq} = R_1 + R_2 = 7.1\ k\Omega$, 1

C has $Q = 0$, i.e., $\Delta V_C = \frac{Q}{C} = 0$ ∴ $I(0) = \frac{\Delta V_B}{R_{eq}}$

$I(0) = \frac{3.0}{7.1 \times 10^3} = 0.42\ mA$ 1.5 ← includes reasoning (intermediate steps)

b) discharge current. $\Delta V_C = \Delta V_B$, since $I(\infty) = 0$ no voltage drop across R_1 or R_2

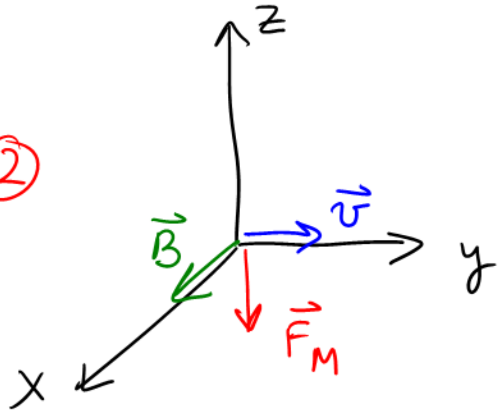
$I(t_1) = \frac{\Delta V_C}{R_2} = \frac{3.0\ V}{4.9\ k\Omega} = 0.61\ mA$ 1.5 direction is

c) $I(\infty) = 0$, since $I(t) = I_0\, e^{-t/\tau}$, where 1

$\tau = R_2 C$

2) [5] An alpha particle is a helium nucleus (two protons and two neutrons bound together). If it moves along the positive $y-$axis and experiences a magnetic force along the negative $z-$axis, what can you conclude about the direction of $\vec{B}$? Begin with a sketch indicating all vectors involved, and explain your reasoning.

2



$F = q\,\vec{v} \times \vec{B}$ 1

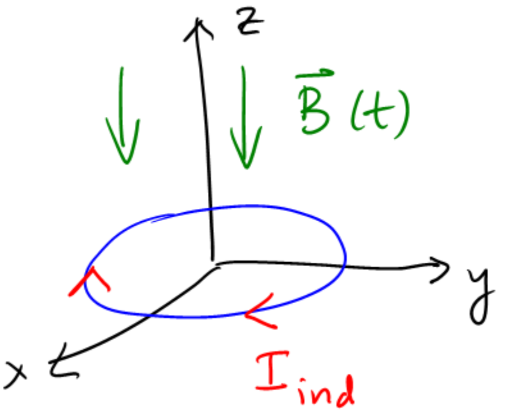
$q = 2e > 0$

The magnetic field is along the positive x-axis

$\vec{B} = (B, 0, 0) = B\,\hat{i}$ 2

By the RH rule

3) [5] A circular loop of wire lies in the $x - y$ plane. A time-varying magnetic field is given as $\vec{B} = (0, 0, -B(t))$, with the field strength $B(t)$ decreasing over time. a) Explain the direction of the induced current. (b) The wire loop has a resistance of $R = 1\Omega$, and a diameter of 1 cm. The constant rate at which the field decreases is -0.1 T/s. Calculate the current in the loop and the power dissipated.



a) The strength of $\vec{B}$ decreasing. By the Lenz rule $\vec{B}_{ind}$ will support $\vec{B}$, i.e.,

$$\vec{B}_{ind} = (0, 0, -B_{ind}) = -B_{ind}\hat{k} \quad 1$$

By the simple RH rule the induced current is CW when viewed from above 1

b) $d = 1\text{ cm} = 10^{-2}\text{ m}$ $\therefore$ loop area $= A = \pi\frac{d^2}{4}$

$$A = 7.85 \times 10^{-5}\ \text{m}^2 \quad 0.5$$

$\phi_M = B(t)\,A$ since $\vec{B} \perp$ to loop area

$$\frac{d\phi_M}{dt} = \frac{dB}{dt}A \quad 0.5 \qquad \frac{dB}{dt} = -0.1\,\frac{T}{s}$$

$$\mathcal{E} = -\frac{d\phi_M}{dt} = -A\frac{dB}{dt} = 7.85 \times 10^{-5} \times 10^{-1}\ \frac{Tm^2}{s} \quad 0.5$$

$$= 7.85\ \mu V$$

$$I = \frac{\mathcal{E}}{R} = \frac{7.85 \times 10^{-6}\ V}{1\ \Omega} = 7.85\ \mu A \quad 2$$

includes partial steps

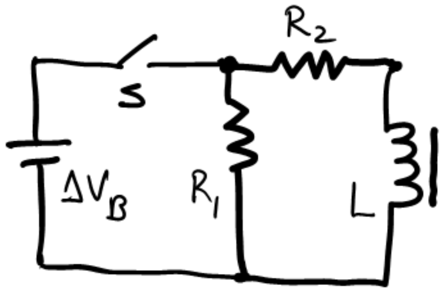
$$P = \mathcal{E}\,I = 7.85^2 \times 10^{-12}\ W = 6.2 \times 10^{-11}\ W \quad 1$$

(0.5)

4) [5] In the diagram shown switch S was open a long time, and it is closed at $t = 0$. a) What is the current I_1 and I_2 through R_1 and R_2 respectively at $t = 0$, just when S is closed? b) What is the voltage across inductor L at $t = 0$ (S closed), i.e., $\Delta V_L(0)$? c) S was closed now for a long time. What are the currents I_1, I_2 (through R_1, R_2 respectively), and the inductor voltage ΔV_L? d) A long time later S is re-opened at t_o. What is the magnitude and direction of $I_1(t_o)$ (after re-opening)? e) What is the inductor voltage $\Delta V_L(t_o)$ at this time?

$\Delta V_B = 9.0\ V$, $R_1 = 0.5\ k\Omega$,

$R_2 = 1.2\ k\Omega$, $L = 3.0\ mH$



a) $I_1 = \dfrac{\Delta V_B}{R_1} = 18\ mA$ 0.5

$I_2 = 0$ 0.5

b) $\Delta V_L(0) = -\Delta V_B$ 0.5

c) $I_1 = 18\ mA$ 0.5, $I_2 = \dfrac{\Delta V_B}{R_2} = 7.5\ mA$ 0.5

d) I_2 continues, $|I_1(t_o)| = I_2 = 7.5\ mA$ ↑ 0.5

The direction is opposite to the original I_1 0.5

e) To pass $I_2 = 7.5\ mA$ through the circuit with $R_{eq} = R_1 + R_2 = 1.7\ K\Omega$ we need

$$\mathcal{E} = R_{eq} I_2 = 1.7 \times 10^3 \cdot 7.5 \times 10^{-3}\ V$$

$$= 12.75\ V = 13\ V$$ 1.5

FORMULA SHEET

$v(t_\mathrm{f}) = v(t_\mathrm{i}) + \int_{t_\mathrm{i}}^{t_\mathrm{f}} a(t)\ dt \quad s(t_\mathrm{f}) = s(t_\mathrm{i}) + \int_{t_\mathrm{i}}^{t_\mathrm{f}} v(t)\ dt$

$v_\mathrm{f} = v_\mathrm{i} + a\Delta t \quad s_\mathrm{f} = s_\mathrm{i} + v_\mathrm{i}\Delta t + \frac{1}{2}a\Delta t^2 \quad v_\mathrm{f}^2 = v_\mathrm{i}^2 + 2a\Delta s \qquad g = 9.8\ \mathrm{m/s^2}$

$f(t) = t \quad \frac{df}{dt} = 1 \quad F(t) = \int f(t)\ dt = \frac{t^2}{2} + C$

$f(t) = a \quad \frac{df}{dt} = 0 \quad F(t) = \int f(t)\ dt = at + C \quad F(t) = \text{anti-derivative} = \text{indefinite integral}$

area under the curve $f(t)$ between limits t_1 and t_2: $F(t_2) - F(t_1)$

$x^2 + px + q = 0$ factored by: $x_{1,2} = -\frac{p}{2} \pm \sqrt{\frac{p^2}{4} - q}$

uniform circular m. $\vec{r}(t) = R(\cos\omega t\ \hat{\mathbf{i}} + \sin\omega t\ \hat{\mathbf{j}})$; $\vec{v}(t) = \frac{d\vec{r}}{dt} = \ldots$; $\vec{a}(t) = \frac{d\vec{v}}{dt} = \ldots.$

$\exp' = \exp$; $\sin' = \cos$; $\cos' = -\sin$. $\quad \frac{d}{dx}[f(g(x))] = \frac{df}{dg}\frac{dg}{dx}$; $\quad (fg)' = f'g + fg'$

$m\vec{a} = \vec{F}_\mathrm{net}$; $\quad F_G = \frac{Gm_1m_2}{r^2}$; $g = \frac{GM_E}{R_E^2}$; $R_E = 6370$ km; $G = 6.67\times10^{-11}\frac{\mathrm{Nm^2}}{\mathrm{kg^2}}$; $M_E = 6.0\times10^{24}$kg

$f_\mathrm{s} \le \mu_\mathrm{s}n$; $\quad f_\mathrm{k} = \mu_\mathrm{k}n$; $\quad f_\mathrm{r} = \mu_\mathrm{r}n$; $\quad \mu_\mathrm{r} << \mu_\mathrm{k} < \mu_\mathrm{s}$. $\qquad F_H = -k\Delta x = -k(x - x_0)$.

$\vec{F}_\mathrm{d} \sim -\vec{v}$; linear: $F_\mathrm{d} = dv$; quadratic: $F_\mathrm{d} = 0.5\rho Av^2$; $\quad A$ = cross sectional area

$W = F\Delta x = F(\Delta r)\cos\theta$. $\ W$ = area under $F_x(x)$. $\quad PE_\mathrm{H} = \frac{k}{2}(\Delta x)^2$; $PE_g = mg\Delta y$.

$\Delta\vec{p} = \vec{J} = \int \vec{F}(t)dt$; $\Delta p_x = J_x$ = area under $F_x(t) = F_x^\mathrm{avg}\Delta t$; $\quad \vec{p} = m\vec{v}$; $K = \frac{m}{2}v^2$

$\Delta\vec{p}_1 + \Delta\vec{p}_2 = 0$; $K_1^\mathrm{in} + K_2^\mathrm{in} = K_1^\mathrm{fin} + K_2^\mathrm{fin}$ for elastic collisions. $\quad \vec{a}_\mathrm{CM} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2}{m_1 + m_2}$

$\vec{\tau} = \vec{r}\times\vec{F}$; $\quad \tau_z = rF\sin(\alpha)$ for $\vec{r}$, $\vec{F}$ in xy plane. $\quad I = \sum_i m_ir_i^2$; $\quad I\alpha_z = \tau_z$; ($\hat{k}$ = rot. axis)

$K_\mathrm{rot} = \frac{I}{2}\omega^2$; $L_z = I\omega_z$; $\frac{d}{dt}L_z = \tau_z$; $\vec{L} = \vec{r}\times\vec{p}$; $\frac{d}{dt}\vec{L} = \vec{\tau}$

$x(t) = A\cos(\omega t + \phi)$; $\quad \omega = \frac{2\pi}{T} = 2\pi f$; $\quad v_x(t) = \ldots$; $\quad v_\mathrm{max} = \ldots$

$m_\mathrm{e} = 9.11\times10^{-31}$kg $\quad m_\mathrm{p} = 1.67\times10^{-27}$kg $\quad e = 1.60\times10^{-19}$C $\quad k = \frac{1}{4\pi\epsilon_0} = 9.0\times10^9\frac{\mathrm{Nm^2}}{\mathrm{C^2}}$

$\vec{F}_\mathrm{C} = \frac{kq_1q_2}{r^2}\hat{\mathbf{r}} \quad \vec{F}_\mathrm{E} = q\vec{E} \quad E_\mathrm{line} = \frac{2k|\lambda|}{r} = \frac{2k|Q|}{Lr} \quad E_\mathrm{plane} = \frac{|\eta|}{2\epsilon_0} = \frac{|Q|}{2A\epsilon_0} \quad \vec{E}_\mathrm{cap} = \left(\frac{Q}{\epsilon_0 A}, \text{pos} \to \text{neg}\right)$

$\frac{mv^2}{2} + U_\mathrm{el}(s) = \frac{mv_0^2}{2} + U_\mathrm{el}(s_0)$, $(U \equiv PE_\mathrm{el}) \quad U_\mathrm{el} = qEx$ for $\vec{E} = -E\,\hat{i} \quad V_\mathrm{el} = U_\mathrm{el}/q \quad E_x = -\frac{dV_\mathrm{el}}{dx}$

$Q = C\Delta V_C \quad$ farad = F = $\frac{\mathrm{C}}{\mathrm{V}} \quad C = \frac{\epsilon_0 A}{d} \quad \epsilon_0 = 8.85\times10^{-12}\frac{\mathrm{C^2}}{\mathrm{Nm^2}}$

parallel C_1, C_2: $C_\mathrm{eq} = C_1 + C_2 \quad$ series C_1, C_2: $C_\mathrm{eq}^{-1} = C_1^{-1} + C_2^{-1}$

$\Delta V_\mathrm{loop} = \sum_i \Delta V_i = 0 \quad \sum I_\mathrm{in} = \sum I_\mathrm{out}$

$P = \Delta VI \quad$ watt = W = VA $\quad P_R = \Delta V_RI = I^2R$

$\tau = RC \quad Q(t) = Q_0e^{-t/\tau} \quad I(t) = -\frac{dQ}{dt} = \frac{\Delta V_0}{R}e^{-t/\tau}$

$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\vec{r}}{r^3} = \frac{\mu_0}{4\pi}\frac{I\Delta\vec{s}\times\vec{r}}{r^3} \quad B_\mathrm{wire} = \frac{\mu_0}{2\pi}\frac{I}{d}$ (use RH rule) $\quad \frac{\mu_0}{4\pi} = 10^{-7}\frac{\mathrm{Tm}}{\mathrm{A}} \quad$ tesla = T = $\frac{\mathrm{N}}{\mathrm{Am}}$

short coil, $R >> L$ (N turns): $B_\mathrm{coil,centre} = \frac{\mu_0 NI}{2R} \qquad$ solenoid, $L >> R$: $B_\mathrm{sol,inside} = \frac{\mu_0 NI}{L}$

mag dipole: $\vec{\mu} = (AI$, from south to north$) \quad \vec{B}_\mathrm{dip} = \frac{\mu_0}{4\pi}\frac{2\vec{\mu}}{z^3}$ on axis, far away

$\vec{F}_{\mathrm{on}q} = q\vec{v}\times\vec{B} \quad$ force on current $\perp$ to $\vec{B}$: $F_\mathrm{wire} = ILB$

force betw. parallel wires: $F_\mathrm{2wires} = \frac{\mu_0 LI_1I_2}{2\pi d} \qquad$ torque on mag dipole: $\vec{\mu}$ in $\vec{B}$: $\vec{\tau} = \vec{\mu}\times\vec{B}$

bar (length L) moves w. $\vec{v} \perp \vec{B}$ gen. EMF: $\varepsilon = vLB$;

$\Phi_m = \vec{A}\cdot\vec{B} \quad \Phi_m = AB\cos\theta \quad \varepsilon = \left|\frac{d\Phi_m}{dt}\right| = \left|\vec{B}\cdot\frac{d\vec{A}}{dt} + \vec{A}\cdot\frac{d\vec{B}}{dt}\right|$

$L = \frac{\Phi_m}{I} \quad$ henry = H = $\frac{\mathrm{Tm^2}}{\mathrm{A}} \quad \varepsilon_\mathrm{coil} = L\left|\frac{dI}{dt}\right| \quad \Delta V_L = -L\frac{dI}{dt} \quad \mathrm{PE}_L = \frac{L}{2}I^2$

series L and R: $\tau = \frac{L}{R} \quad I(t) = I_0(1 - e^{-t/\tau})$