LAST NAME: STUDENT NR:

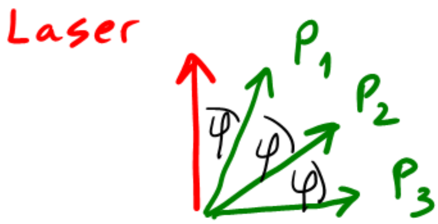
PHYS 1010 6.0: CLASS TEST 6

Time: 50 minutes; Calculators & formulae provided at the end = only aid; Total = 20 points.

1) [5] You have a laser which produces linearly polarized light of intensity I_0 in the vertical direction. It is physically tied down, you cannot rotate it. You have three ideal polarizers, meaning polarized light aligned with the polarizer axis goes through without attenuation. You know the law of Malus $I(\phi) = I_0 \cos^2 \phi$. What is your best option to produce horizontally polarized light, and how much intensity will you get? Assume that you will use your polarizers with equal relative orientation of adjacent polarizers. Which relative orientation will you use for the polarizers?

Using three polarizers with $\phi = 30^\circ = \frac{\pi}{6}$ relative orientation results in a filter which produces a 90° rotation for light aligned with the first polarizer

1



1

$$I_1 = I_0 \cos^2(30^\circ)$$

$$I_2 = I_1 \cos^2(30^\circ)$$

$$I_3 = I_2 \cos^2(30^\circ)$$

$$I_3 = I_0 \left(\cos^2(30^\circ)\right)^3 = I_0 \cos^6(30^\circ)$$ 1

Attenuation : $\cos^6(30^\circ) = 0.422$

$$I_{out} = 0.42\, I_{in}$$ 2

2) [5] A piano tuner adjusts the tension force on the A440 Hz string from 2080 N to 2100 N in order to tune it properly. He measures the vibrating portion of the string to be 64 cm long. What is the mass of this section of the string? What fundamental frequency was the string playing before it was tuned right?

standing wave, fundamental $(n=1)$

$$\lambda_1 = 2L \quad 1 \qquad v_w = \lambda f$$

$$\therefore \quad v_w = 2 \cdot 0.64 \cdot 440 \ \frac{m}{s} = 563.2 \ \frac{m}{s} \quad 1$$

$$v_w = \sqrt{\frac{F_t}{\mu}} \quad \therefore \quad v_w^2 = \frac{F_t}{\mu} \quad \therefore \quad \mu = \frac{F_t}{v_w^2}$$

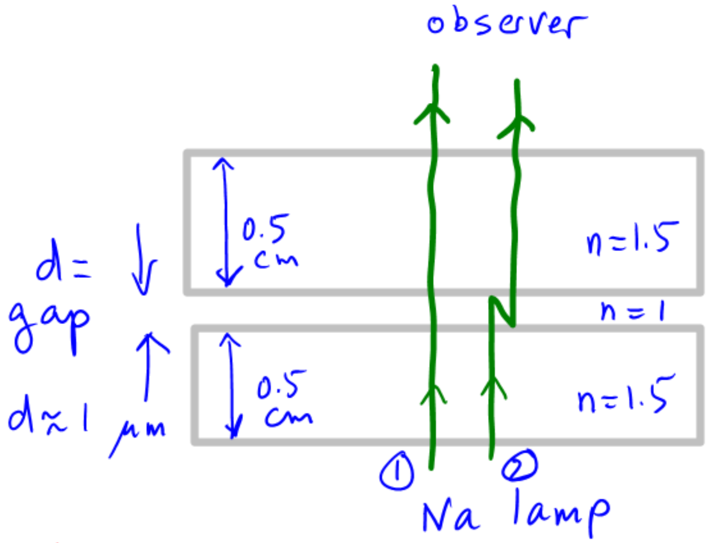
$$\left(\mu = \frac{M}{L}\right) \therefore \quad M = \frac{F_t L}{v_w^2} = \frac{2100 \cdot .64}{3.172 \cdot 10^5} \ kg = \underline{\underline{4.2(4) \ g}} \quad 1$$

$$\tilde{v}_w = \sqrt{\frac{\tilde{F}_t}{\mu}} = \sqrt{\frac{2080 \cdot .64}{4.24 \times 10^{-3}}} \ \frac{m}{s} = 560. \ \frac{m}{s} \quad 1$$

$$\tilde{f} = \frac{\tilde{v}_w}{\lambda} = \frac{560.}{1.28} \ Hz = 437.8 \ Hz = 438 \ Hz$$

1

3) [5] A sodium lamp emits light of wavelength 589 nm. Two glass plates of thickness 0.5 cm each are placed on top of each other over the lamp, and a gap forms which is typically in the [illegible] μm range. Explain using two sentences why interference fringes appear, and support them by a drawing explaining a dark fringe. Provide a calculation, i.e., make an estimate of what the local gap could be at the dark spot and state the order m for the interference. Assume normal incidence for the light. Do not forget about the phase jump under reflection when reflecting from an interface connecting lower$-n$ to higher$-n$ medium.



Two interfering paths:

Path 2 has added path length 2d with optical PL = PL (n=1 for air) ①

+① for detail (n-values, accuracy)

① for at least 2 relevant statements

The observed intensity $I \sim (E_1 + E_2)^2$ is sensitive to interference.

Destructive interference occurs when crests from ① meet with throughs from ② (or vice versa)

Accumulated phase difference (path 2) $\phi_2 - \phi_1 = \frac{2\pi}{\lambda} 2d$

When $(\phi_2 - \phi_1)$ = odd integer $\cdot \pi = (2m+1)\pi$, $m = 0, 1, 2, \ldots$ (0.5)

destructive interference.

(0.5) Phase jumps irrelevant, as there are two in ②.

(0.5) $\frac{4\pi d}{\lambda} = (2m+1)\pi \quad \therefore \quad d = \left(\frac{m+\frac{1}{2}}{2}\right)\lambda$ (0.5)

$0.589\left(\frac{m}{2} + \frac{1}{4}\right) \approx 1$ (in μm)

$m \approx 3.3$ for 1 μm

$m = 1, 2, 3, 4, 5$ works, as gap varies locally

bigger thickness → higher m

4) [5] An ambulance siren is blasting a sound with a fundamental of $f = 1.2$ kHz while parked at the front of your apartment building. You hear it from far away while driving on a straight road towards the house. You get nervous and step on the gas. What pitch (frequency) are your ears perceiving as the speedometer needle reaches 80 km/h? When you get out of the (stopped) car, what frequency do your ears record?

$$f_{obs} = f_{src}\left(1 + \frac{v_{obs}}{v_s}\right) = 1.2 \cdot 10^3 \left(1 + \frac{v_{car}}{v_s}\right)$$

1 ← only for the right formula

$$v_{car} = \frac{80}{3.6} \frac{m}{s} = 22.2 \frac{m}{s}$$

$$\frac{v_{car}}{v_s} = 0.0647 \qquad (6.5\% \text{ increase in } f)$$

$$f_{obs} = 1.28 \text{ kHz} \simeq 1.3 \text{ kHz}$$

(both ok)

3 getting this from the 'wrong' formula is OK

$$f_{stat.obs.} = f_0 = 1.2 \text{ kHz}$$

1

FORMULA SHEET

$v(t_f) = v(t_i) + \int_{t_i}^{t_f} a(t)\, dt \quad s(t_f) = s(t_i) + \int_{t_i}^{t_f} v(t)\, dt$

$v_f = v_i + a\Delta t \quad s_f = s_i + v_i\Delta t + \frac{1}{2}a\Delta t^2 \quad v_f^2 = v_i^2 + 2a\Delta s \qquad g = 9.8 \text{ m/s}^2$

$f(t) = t \quad \frac{df}{dt} = 1 \quad F(t) = \int f(t)\, dt = \frac{t^2}{2} + C$

$f(t) = a \quad \frac{df}{dt} = 0 \quad F(t) = \int f(t)\, dt = at + C \qquad F(t) =$ anti-derivative = indefinite integral

area under the curve $f(t)$ between limits t_1 and t_2: $F(t_2) - F(t_1)$

$x^2+px+q=0$ factored by: $x_{1,2}=-\frac{p}{2}\pm\sqrt{\frac{p^2}{4}-q}$

uniform circular m. $\vec{r}(t)=R(\cos\omega t\ \hat{\mathbf{i}}+\sin\omega t\ \hat{\mathbf{j}})$; $\vec{v}(t)=\frac{d\vec{r}}{dt}=...$; $\vec{a}(t)=\frac{d\vec{v}}{dt}=....$

$\exp'=\exp$; $\sin'=\cos$; $\cos'=-\sin$. $\quad \frac{d}{dx}[f(g(x))]=\frac{df}{dg}\frac{dg}{dx}$; $\quad (fg)'=f'g+fg'$

$m\vec{a}=\vec{F}_{\rm net}$; $\quad F_G=\frac{Gm_1m_2}{r^2}$; $g=\frac{GM_E}{R_E^2}$; $R_E=6370$ km; $G=6.67\times10^{-11}\frac{\rm Nm^2}{\rm kg^2}$; $M_E=6.0\times10^{24}$kg

$f_{\rm s}\le\mu_{\rm s}n$; $\quad f_{\rm k}=\mu_{\rm k}n$; $\quad f_{\rm r}=\mu_{\rm r}n$; $\quad \mu_{\rm r}<<\mu_{\rm k}<\mu_{\rm s}$. $\qquad F_H=-k\Delta x=-k(x-x_0)$.

$\vec{F}_{\rm d}\sim-\vec{v}$; linear: $F_{\rm d}=dv$; quadratic: $F_{\rm d}=0.5\rho Av^2$; $\quad A=$ cross sectional area

$W=F\Delta x=F(\Delta r)\cos\theta$. $W=$ area under $F_x(x)$. $\quad PE_{\rm H}=\frac{k}{2}(\Delta x)^2$; $PE_g=mg\Delta y$.

$\Delta\vec{p}=\vec{J}=\int\vec{F}(t)dt$; $\Delta p_x=J_x=$ area under $F_x(t)=F_x^{\rm avg}\Delta t$; $\quad \vec{p}=m\vec{v}$; $K=\frac{m}{2}v^2$

$\Delta\vec{p}_1+\Delta\vec{p}_2=0$; $K_1^{\rm in}+K_2^{\rm in}=K_1^{\rm fin}+K_2^{\rm fin}$ for elastic collisions. $\quad \vec{a}_{\rm CM}=\frac{m_1\vec{a}_1+m_2\vec{a}_2}{m_1+m_2}$

$\vec{\tau}=\vec{r}\times\vec{F}$; $\quad \tau_z=rF\sin(\alpha)$ for $\vec{r}$, $\vec{F}$ in xy plane. $\quad I=\sum_i m_ir_i^2$; $\quad I\alpha_z=\tau_z$; ($\hat{k}=$ rot. axis)

$K_{\rm rot}=\frac{I}{2}\omega^2$; $L_z=I\omega_z$; $\frac{d}{dt}L_z=\tau_z$; $\vec{L}=\vec{r}\times\vec{p}$; $\frac{d}{dt}\vec{L}=\vec{\tau}$

$x(t)=A\cos(\omega t+\phi)$; $\quad \omega=\frac{2\pi}{T}=2\pi f$; $\quad v_x(t)=...$; $\quad v_{\max}=...$

$m_{\rm e}=9.11\times10^{-31}$kg $\quad m_{\rm p}=1.67\times10^{-27}$kg $\quad e=1.60\times10^{-19}$C $\quad K=\frac{1}{4\pi\epsilon_0}=9.0\times10^9\frac{\rm Nm^2}{\rm C^2}$

$\vec{F}_{\rm C}=\frac{Kq_1q_2}{r^2}\hat{\mathbf{r}}$ $\quad \vec{F}_{\rm E}=q\vec{E}$ $\quad E_{\rm line}=\frac{2K|\lambda|}{r}=\frac{2K|Q|}{Lr}$ $\quad E_{\rm plane}=\frac{|\eta|}{2\epsilon_0}=\frac{|Q|}{2A\epsilon_0}$ $\quad \vec{E}_{\rm cap}=\left(\frac{Q}{\epsilon_0A},\text{pos}\to\text{neg}\right)$

$\frac{mv^2}{2}+U_{\rm el}(s)=\frac{mv_0^2}{2}+U_{\rm el}(s_0)$, $(U\equiv PE_{\rm el})$ $\quad U_{\rm el}=qEx$ for $\vec{E}=-E\ \hat{i}$ $\quad V_{\rm el}=U_{\rm el}/q$ $\quad E_x=-\frac{dV_{\rm el}}{dx}$

$Q=C\Delta V_C$ $\quad$ farad $=$ F $=\frac{\rm C}{\rm V}$ $\quad C=\frac{\epsilon_0A}{d}$ $\quad \epsilon_0=8.85\times10^{-12}\frac{\rm C^2}{\rm Nm^2}$

parallel C_1,C_2: $C_{\rm eq}=C_1+C_2$ $\quad$ series C_1,C_2: $C_{\rm eq}^{-1}=C_1^{-1}+C_2^{-1}$

$\Delta V_{\rm loop}=\sum_i\Delta V_i=0$ $\quad \sum I_{\rm in}=\sum I_{\rm out}$

$P=\Delta VI$ $\quad$ watt $=$ W $=$ VA $\quad P_R=\Delta V_RI=I^2R$

$\tau=RC$ $\quad Q(t)=Q_0e^{-t/\tau}$ $\quad I(t)=-\frac{dQ}{dt}=\frac{\Delta V_0}{R}e^{-t/\tau}$

$\vec{B}=\frac{\mu_0}{4\pi}\frac{q\vec{v}\times\vec{r}}{r^3}=\frac{\mu_0}{4\pi}\frac{I\Delta\vec{s}\times\vec{r}}{r^3}$ $\quad B_{\rm wire}=\frac{\mu_0}{2\pi}\frac{I}{d}$ (use RH rule) $\quad \frac{\mu_0}{4\pi}=10^{-7}\frac{\rm Tm}{\rm A}$ $\quad$ tesla $=$ T $=\frac{\rm N}{\rm Am}$

short coil, $R>>L$ (N turns): $B_{\rm coil,centre}=\frac{\mu_0NI}{2R}$ $\qquad$ solenoid, $L>>R$: $B_{\rm sol,inside}=\frac{\mu_0NI}{L}$

mag dipole: $\vec{\mu}=(AI,\text{from south to north})$ $\quad \vec{B}_{\rm dip}=\frac{\mu_0}{4\pi}\frac{2\vec{\mu}}{z^3}$ on axis, far away

$\vec{F}_{\rm onq}=q\vec{v}\times\vec{B}$ $\quad$ force on current $\perp$ to $\vec{B}$: $F_{\rm wire}=ILB$

force betw. parallel wires: $F_{\rm 2wires}=\frac{\mu_0LI_1I_2}{2\pi d}$ $\qquad$ torque on mag dipole: $\vec{\mu}$ in $\vec{B}$: $\vec{\tau}=\vec{\mu}\times\vec{B}$

bar (length L) moves w. $\vec{v}\perp\vec{B}$ gen. EMF: $\varepsilon=vLB$;

$\Phi_m=\vec{A}\cdot\vec{B}$ $\quad \Phi_m=AB\cos\theta$ $\quad \varepsilon=\left|\frac{d\Phi_m}{dt}\right|=\left|\vec{B}\cdot\frac{d\vec{A}}{dt}+\vec{A}\cdot\frac{d\vec{B}}{dt}\right|$

$L=\frac{\Phi_m}{I}$ $\quad$ henry $=$ H $=\frac{\rm Tm^2}{\rm A}$ $\quad \varepsilon_{\rm coil}=L\left|\frac{dI}{dt}\right|$ $\quad \Delta V_L=-L\frac{dI}{dt}$ $\quad {\rm PE}_L=\frac{L}{2}I^2$

series L and R: $\tau=\frac{L}{R}$ $\quad I(t)=I_0(1-e^{-t/\tau})$; parallel L and C: $\omega=\sqrt{\frac{1}{LC}}$ $\quad I(t)=\omega Q_0\sin\omega t$

$\lambda f=v_{\rm w}$ $\quad$ sinusoid +ve $x-$dir'n: $D(x,t)=A\sin\left(2\pi(\frac{x}{\lambda}-\frac{t}{T})+\phi_0\right)=A\sin\left(kx-\omega t+\phi_0\right)$

transverse wave on a string: $v_{\rm w}=\sqrt{\frac{T}{\mu}}$ where T is tension, $\mu=M/L$

$\omega=v_{\rm w}k$ $\qquad v_{\rm sound}=343$m/s in air at T $=20°$C $\quad$ in water: $v_{\rm sound}=1480$m/s

light in vac.: $v_{\rm w}=c=3.00\times10^8$m/s $\quad$ visible: $\lambda=400$nm (blue/UV); $\lambda=700$nm (red/IR)

medium: $n_{\rm glass}=1.5$; $n_{\rm water}=1.333$; speed: c/n; wavelength: $\lambda_{\rm vac}/n$; acc. phase: $\phi=\frac{2\pi n\Delta x}{\lambda_{\rm vac}}$

src speed $v_{\rm src}$: $f_+=\frac{f_0}{1-v_{\rm src}/v_{\rm w}}$; $f_-=\frac{f_0}{1+v_{\rm src}/v_{\rm w}}$; $\quad$ obs speed $v_{\rm obs}$: $f_+=f_0(1+\frac{v_{\rm obs}}{v_{\rm w}})$; $f_-=f_0(1-\frac{v_{\rm obs}}{v_{\rm w}})$

$\sin(\alpha\pm\beta)=\sin\alpha\cos\beta\pm\cos\alpha\sin\beta$ $\quad \sin\alpha+\sin\beta=2\cos\frac{\alpha-\beta}{2}\ \sin\frac{\alpha+\beta}{2}$

transverse standing wave, string length L: $\lambda_n=\frac{2L}{n}$ $\quad n=1,2,...$ $\quad f_n$ from $\lambda_nf_n=c_{\rm w}$