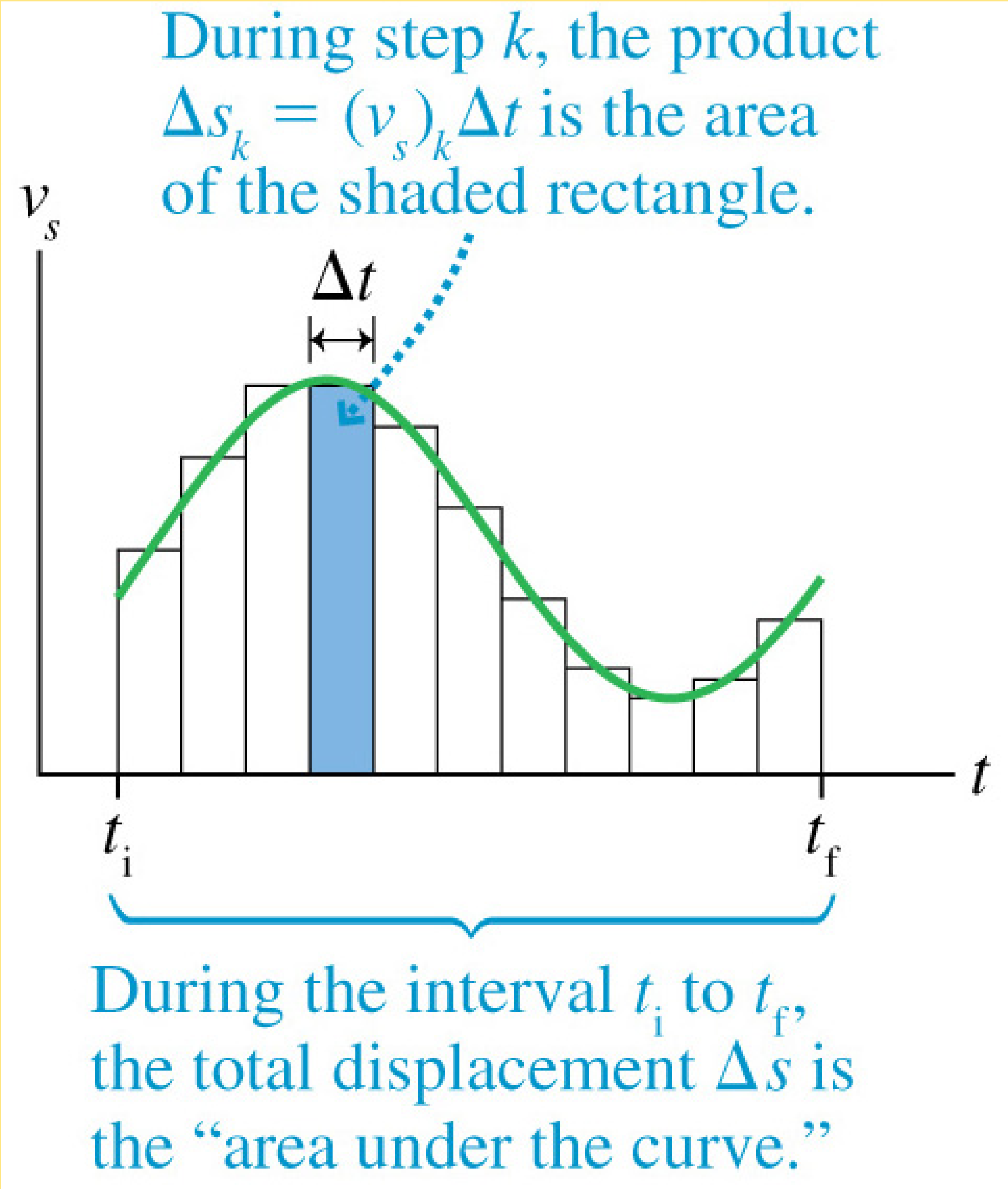
# Avoiding the Limit




Integral Calculus uses a limit to define the area under the curve

Use the increased frame rate idea: $v_s(t_k)$ becomes slowly varying

Area is estimated well assuming piecewise constant velocity

Sum the area from all rectangles:

$$s_f \approx s_i + \sum_{k=1}^{N} v_s(t_k)\Delta t$$

A partition of the $t-$axis consisting of $N$ points (frames) was used

A computational strategy that is practical and used often

In math define proper lower and upper sums and send $N \to \infty$

Lower and upper sums converge to the same number $\to$ integral is defined

# Example

When we release a slinky the velocity looks like a sine curve: it starts from zero, reaches a maximum, decreases to zero, becomes negative, reaches a minimum (most negative value), increases again, etc. (see previous graph with baseline moved)

Task: Create a partition with $N = 3$ to reach the first maximum at $t \approx 1.57$ s, evaluate the displacement estimate, and compare with the Calculus result.

Solution: The time frames are spaced by $\Delta t = 1.57/3 = 0.523$ s.
The time slices are then $t_k = (k - 0.5)\Delta t$, i.e., [0.262,0 .784, 1.31] s.
The velocities $v(t_k) = \sin(\omega t_k)$, where $\omega = 2\pi f$ 1/s, and $\omega = 1$/s assumed, are calculated as [0.259,0 .706,0 .966] m/s.
Displacement estimate: $\Delta s = 0.523(0.259 + 0.706 + 0.966)$ m $= 1.011$ m.

Calculus result: Anti-derivative to sin is -cos, but we want $s(0) = 0$!
Thus, $s(t) = -\cos t + 1$, where SI units are assumed ($\omega = 1$/s).
Evaluate: $s(1.57) = 0.999$ m.
Calculus saves time, and is more accurate (exact).
In fact, Calculus gave us the answer for ALL times, not just $t = 1.57$ s!

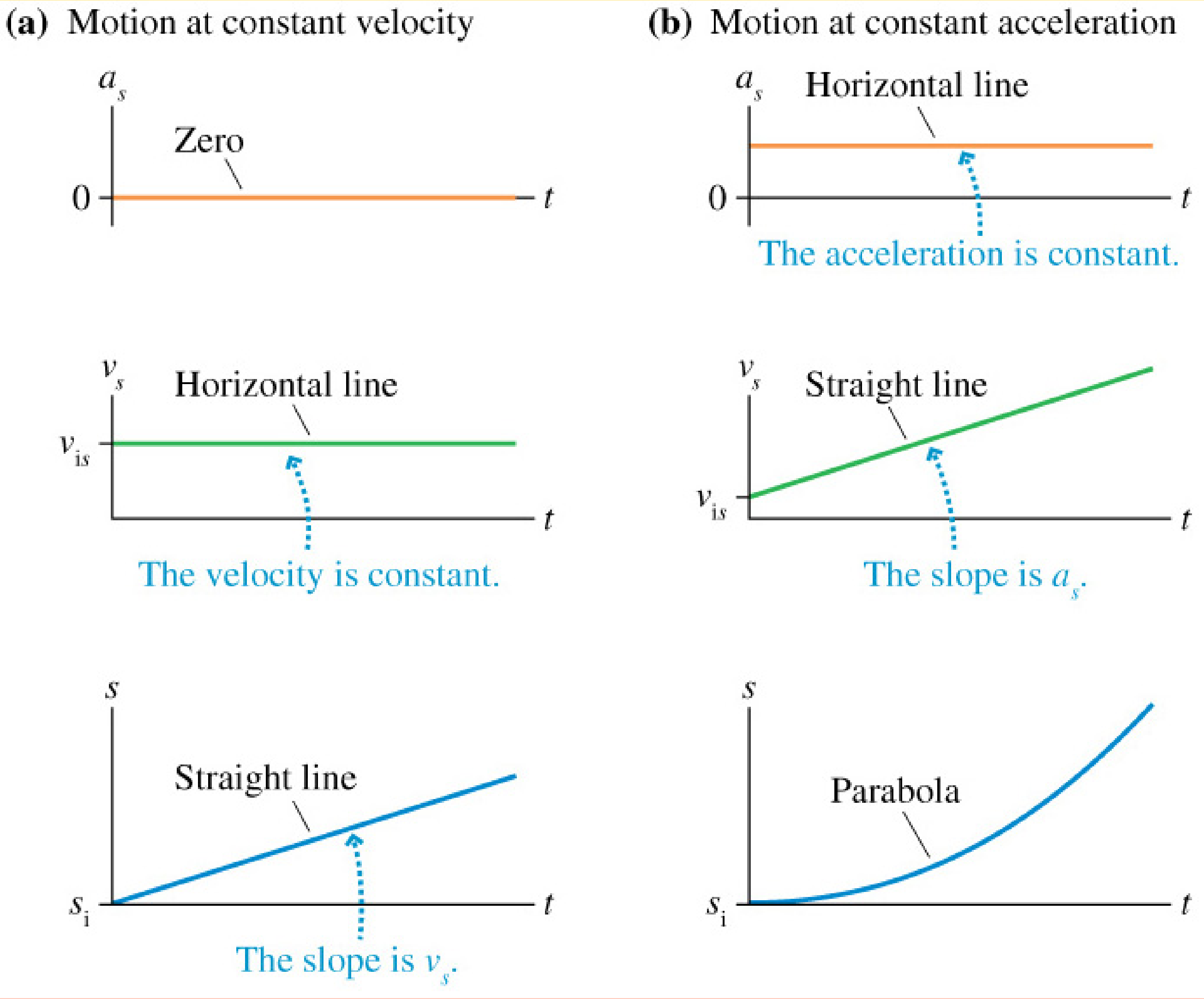
# Velocity from Acceleration

Position from velocity is solved (Calculus - the better way)

In a similar way we get velocity from acceleration

By definition the instantaneous acceleration

$$a_s = \lim_{\Delta t \to 0} \frac{\Delta v_s}{\Delta t} = \lim_{\Delta t \to 0} \frac{v_s(t+\Delta t) - v_s(t)}{\Delta t} = \frac{\mathrm{d}v_s}{\mathrm{d}t}$$




Velocity change from acceleration:

$$v_s(t_f) - v_s(t_i) = \int_{t_i}^{t_f} a_s(t')\mathrm{d}t'$$

Definite integral is a number.

Anti-derivative is a function.

# Free Fall

Objects near the surface of the earth experience a downward acceleration $g \approx 9.8\ \mathrm{m/s^2}$.
Describe the motion of a ball with initial height $y_0 = 1$ m, thrown vertically upwards with initial velocity $v_y(0) = 1.5$ m/s.

Solution: First calculate the velocity as a function fo time.

$$v_y(t) = v_y(0) + \int_0^t a_y(t')\mathrm{d}t' = v_y(0) + \int_0^t g\mathrm{d}t' = v_y(0) + gt$$

Thus, we know: $v_y(t) = (1.5 - 9.8t)$ m/s.

Now calculate the height as a function of time:

$$y(t) = y(0) + \int_0^t v_y(t')\mathrm{d}t' = y(0) + \int_0^t (v_y(0) + gt')\mathrm{d}t'$$

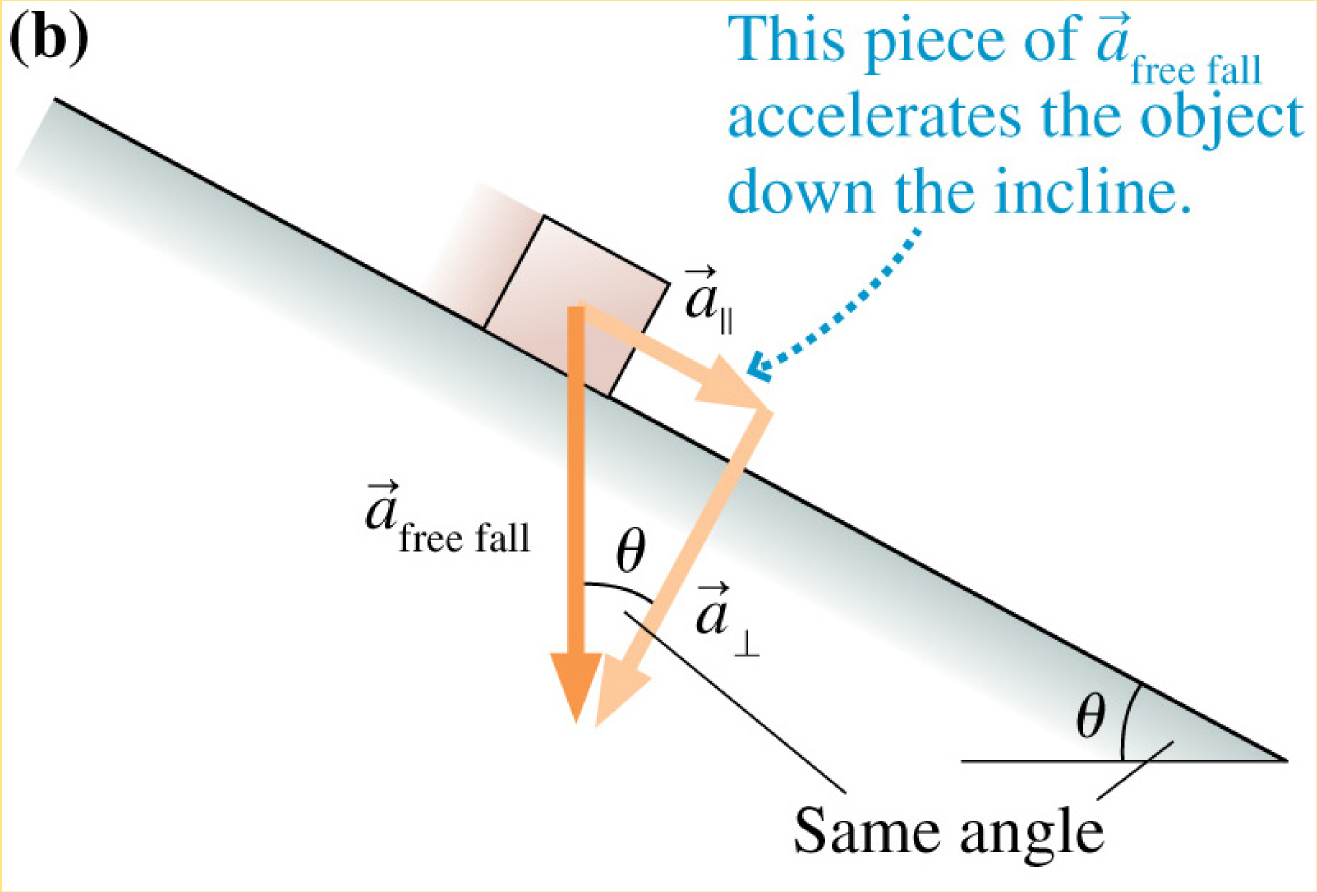
$$= y_0 + v_y(0)t + \frac{1}{2}gt^2$$

Understand the 3 terms!

Insert the numbers: $y(t) = (1 + 1.5t - 4.9t^2)$ m.

Now graph the velocity and height as a function of time and compare with the graphs from Fig. 2.29(b).
Why do they look different ?

# Inclined Plane




On an incline we define $(x, y)$ coordinates parallel and perpendicular to the surface;

$\vec{a}_{\perp}$ is compensated by the surface;
$\vec{a}_{\parallel}$ determines the motion;

air track ≈ frictionless surface;

geometry and trigonometry
→ $a_{\parallel}$ (the magnitude!)

Why does the angle $\theta$ appear in the vector diagram?

How do we get $a_{\parallel}$ in terms of $a$ and $\theta$ ?

$$\sin\theta = \frac{a_{\parallel}}{a} \quad \Longrightarrow \quad a_{\parallel} = a\sin\theta$$

Makes sense?