

EXERCISES AND PROBLEMS

Exercises

Section 3.2 Properties of Vectors

1. a. Can a vector have nonzero magnitude if a component is zero? If no, why not? If yes, give an example.
 b. Can a vector have zero magnitude and a nonzero component? If no, why not? If yes, give an example.
2. Suppose $\vec{C} = \vec{A} + \vec{B}$.
 a. Under what circumstances does $C = A + B$?
 b. Could $C = A - B$? If so, how? If not, why not?
3. Suppose $\vec{C} = \vec{A} - \vec{B}$.
 a. Under what circumstances does $C = A - B$?
 b. Could $C = A + B$? If so, how? If not, why not?
4. Trace the vectors in Figure Ex3.4 onto your paper. Then find (a) $\vec{A} + \vec{B}$ and (b) $\vec{A} - \vec{B}$.

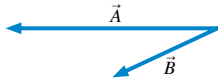


FIGURE EX3.4

5. Trace the vectors in Figure Ex3.5 onto your paper. Then find (a) $\vec{A} + \vec{B}$ and (b) $\vec{A} - \vec{B}$.

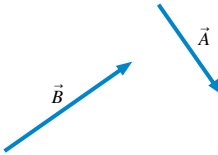


FIGURE EX3.5

Section 3.3 Coordinate Systems and Vector Components

6. A position vector in the first quadrant has an x -component of 6 m and a magnitude of 10 m. What is the value of its y -component?
7. A velocity vector 40° below the positive x -axis has a y -component of 10 m/s. What is the value of its x -component?
8. a. What are the x - and y -components of vector \vec{E} in terms of the angle θ and the magnitude E shown in Figure Ex3.8?
 b. For the same vector, what are the x - and y -components in terms of the angle ϕ and the magnitude E ?

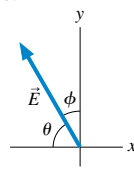


FIGURE EX3.8

9. Draw each of the following vectors, then find its x - and y -components.
 - a. $\vec{r} = (100 \text{ m}, 45^\circ \text{ below } +x\text{-axis})$
 - b. $\vec{v} = (300 \text{ m/s}, 20^\circ \text{ above } +x\text{-axis})$
 - c. $\vec{a} = (5.0 \text{ m/s}^2, -y\text{-direction})$
 - d. $\vec{F} = (50 \text{ N}, 36.9^\circ \text{ above } -x\text{-axis})$
10. Draw each of the following vectors, then find its x - and y -components.
 - a. $\vec{r} = (2 \text{ km}, 30^\circ \text{ left of } +y\text{-axis})$
 - b. $\vec{v} = (5 \text{ cm/s}, -x\text{-direction})$
 - c. $\vec{a} = (10 \text{ m/s}^2, 40^\circ \text{ left of } -y\text{-axis})$
 - d. $\vec{F} = (50 \text{ N}, 36.9^\circ \text{ right of } +y\text{-axis})$
11. Let $\vec{C} = (3.15 \text{ m}, 15^\circ \text{ above the negative } x\text{-axis})$ and $\vec{D} = (25.67, 30^\circ \text{ to the right of the negative } y\text{-axis})$. Find the magnitude, the x -component, and the y -component of each vector.

12. The quantity called the *electric field* is a vector. The electric field inside a scientific instrument is $\vec{E} = (125\hat{i} - 250\hat{j}) \text{ V/m}$, where V/m stands for volts per meter. What are the magnitude and direction of the electric field?

Section 3.4 Vector Algebra

13. Draw each of the following vectors, label an angle that specifies the vector's direction, then find the vector's magnitude and direction.
 - a. $\vec{A} = 4\hat{i} - 6\hat{j}$
 - b. $\vec{r} = (50\hat{i} + 80\hat{j}) \text{ m}$
 - c. $\vec{v} = (-20\hat{i} + 40\hat{j}) \text{ m/s}$
 - d. $\vec{a} = (2\hat{i} - 6\hat{j}) \text{ m/s}^2$
14. Draw each of the following vectors, label an angle that specifies the vector's direction, then find its magnitude and direction.
 - a. $\vec{B} = -4\hat{i} + 4\hat{j}$
 - b. $\vec{r} = (-2\hat{i} - \hat{j}) \text{ cm}$
 - c. $\vec{v} = (-10\hat{i} - 100\hat{j}) \text{ mph}$
 - d. $\vec{a} = (20\hat{i} + 10\hat{j}) \text{ m/s}^2$
15. Let $\vec{A} = 2\hat{i} + 3\hat{j}$ and $\vec{B} = 4\hat{i} - 2\hat{j}$.
 - a. Draw a coordinate system and on it show vectors \vec{A} and \vec{B} .
 - b. Use graphical vector subtraction to find $\vec{C} = \vec{A} - \vec{B}$.
16. Let $\vec{A} = 5\hat{i} + 2\hat{j}$, $\vec{B} = -3\hat{i} - 5\hat{j}$, and $\vec{C} = \vec{A} + \vec{B}$.
 - a. Write vector \vec{C} in component form.
 - b. Draw a coordinate system and on it show vectors \vec{A} , \vec{B} , and \vec{C} .
 - c. What are the magnitude and direction of vector \vec{C} ?
17. Let $\vec{A} = 5\hat{i} + 2\hat{j}$, $\vec{B} = -3\hat{i} - 5\hat{j}$, and $\vec{D} = \vec{A} - \vec{B}$.
 - a. Write vector \vec{D} in component form.
 - b. Draw a coordinate system and on it show vectors \vec{A} , \vec{B} , and \vec{D} .
 - c. What are the magnitude and direction of vector \vec{D} ?
18. Let $\vec{A} = 5\hat{i} + 2\hat{j}$, $\vec{B} = -3\hat{i} - 5\hat{j}$, and $\vec{E} = 2\vec{A} + 3\vec{B}$.
 - a. Write vector \vec{E} in component form.
 - b. Draw a coordinate system and on it show vectors \vec{A} , \vec{B} , and \vec{E} .
 - c. What are the magnitude and direction of vector \vec{E} ?
19. Let $\vec{A} = 5\hat{i} + 2\hat{j}$, $\vec{B} = -3\hat{i} - 5\hat{j}$, and $\vec{F} = \vec{A} - 4\vec{B}$.
 - a. Write vector \vec{F} in component form.
 - b. Draw a coordinate system and on it show vectors \vec{A} , \vec{B} , and \vec{F} .
 - c. What are the magnitude and direction of vector \vec{F} ?
20. Are the following statements true or false? Explain your answer.
 - a. The magnitude of a vector can be different in different coordinate systems.
 - b. The direction of a vector can be different in different coordinate systems.
 - c. The components of a vector can be different in different coordinate systems.
21. Let $\vec{A} = (4.0 \text{ m}, \text{vertically downward})$ and $\vec{B} = (5.0 \text{ m}, 120^\circ \text{ clockwise from } \vec{A})$. Find the x - and y -components of \vec{A} and \vec{B} in each of the two coordinate systems shown in Figure Ex3.21.

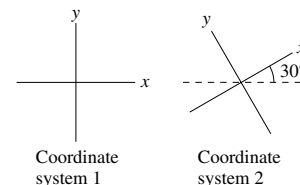


FIGURE EX3.21

22. What are the x - and y -components of the velocity vector shown in Figure Ex3.22?

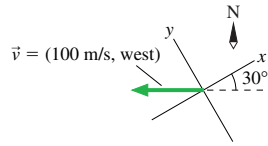


FIGURE EX3.22

Problems

23. Figure P3.23 shows vectors \vec{A} and \vec{B} . Let $\vec{C} = \vec{A} + \vec{B}$.
- Reproduce the figure on your page as accurately as possible, using a ruler and protractor. Draw vector \vec{C} on your figure, using the graphical addition of \vec{A} and \vec{B} . Then determine the magnitude and direction of \vec{C} by measuring it with a ruler and protractor.
 - Based on your figure of part a, use geometry and trigonometry to calculate the magnitude and direction of \vec{C} .
 - Decompose vectors \vec{A} and \vec{B} into components, then use these to calculate algebraically the magnitude and direction of \vec{C} .
24. a. What is the angle ϕ between vectors \vec{E} and \vec{F} in Figure P3.24?
 b. Use geometry and trigonometry to determine the magnitude and direction of $\vec{G} = \vec{E} + \vec{F}$.
 c. Use components to determine the magnitude and direction of $\vec{G} = \vec{E} + \vec{F}$.

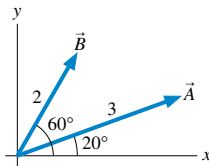


FIGURE P3.23

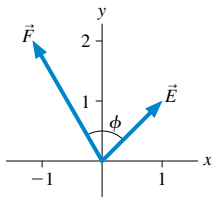


FIGURE P3.24

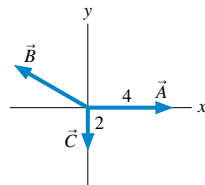


FIGURE P3.25

25. For the three vectors shown above in Figure P3.25, $\vec{A} + \vec{B} + \vec{C} = -2\hat{i}$. What is vector \vec{B} ?
- Write \vec{B} in component form.
 - Write \vec{B} as a magnitude and a direction.
26. Figure P3.26 shows vectors \vec{A} and \vec{B} . Find vector \vec{C} such that $\vec{A} + \vec{B} + \vec{C} = \vec{0}$. Write your answer in component form.

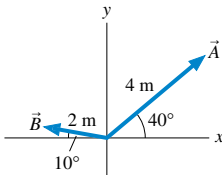


FIGURE P3.26

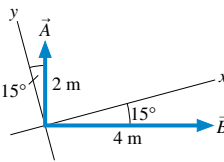


FIGURE P3.27

27. Figure P3.27 shows vectors \vec{A} and \vec{B} . Find $\vec{D} = 2\vec{A} + \vec{B}$. Write your answer in component form.

28. Let $\vec{A} = (3.0 \text{ m}, 20^\circ \text{ south of east})$, $\vec{B} = (2.0 \text{ m}, \text{north})$, and $\vec{C} = (5.0 \text{ m}, 70^\circ \text{ south of west})$.
- Draw and label \vec{A} , \vec{B} , and \vec{C} with their tails at the origin. Use a coordinate system with the x -axis to the east.
 - Write \vec{A} , \vec{B} , and \vec{C} in component form, using unit vectors.
 - Find the magnitude and the direction of $\vec{D} = \vec{A} + \vec{B} + \vec{C}$.
29. Trace the vectors in Figure P3.29 onto your paper. Use the graphical method of vector addition and subtraction to find the following.
- $\vec{D} + \vec{E} + \vec{F}$
 - $\vec{D} + 2\vec{E}$
 - $\vec{D} - 2\vec{E} + \vec{F}$

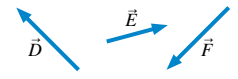


FIGURE P3.29

30. Let $\vec{E} = 2\hat{i} + 3\hat{j}$ and $\vec{F} = 2\hat{i} - 2\hat{j}$. Find the magnitude of
- \vec{E} and \vec{F}
 - $\vec{E} + \vec{F}$
 - $-\vec{E} - 2\vec{F}$
31. Find a vector that points in the same direction as the vector $(\hat{i} + \hat{j})$ and whose magnitude is 1.
32. The position of a particle as a function of time is given by $\vec{r} = (5\hat{i} + 4\hat{j})t^2 \text{ m}$, where t is in seconds.
- What is the particle's distance from the origin at $t = 0, 2,$ and 5 s ?
 - Find an expression for the particle's velocity \vec{v} as a function of time.
 - What is the particle's speed at $t = 0, 2,$ and 5 s ?
33. While vacationing in the mountains you do some hiking. In the morning, your displacement is $\vec{S}_{\text{morning}} = (2000 \text{ m}, \text{east}) + (3000 \text{ m}, \text{north}) + (200 \text{ m}, \text{vertical})$. After lunch, your displacement is $\vec{S}_{\text{afternoon}} = (1500 \text{ m}, \text{west}) + (2000 \text{ m}, \text{north}) - (300 \text{ m}, \text{vertical})$.
- At the end of the hike, how much higher or lower are you compared to your starting point?
 - What is your total displacement?
34. The minute hand on a watch is 2.0 cm in length. What is the displacement vector of the tip of the minute hand
- From 8:00 to 8:20 A.M.?
 - From 8:00 to 9:00 A.M.?
35. Bob walks 200 m south, then jogs 400 m southwest, then walks 200 m in a direction 30° east of north.
- Draw an accurate graphical representation of Bob's motion. Use a ruler and a protractor!
 - Use either trigonometry or components to find the displacement that will return Bob to his starting point by the most direct route. Give your answer as a distance and a direction.
 - Does your answer to part b agree with what you can measure on your diagram of part a?
36. Jim's dog Sparky runs 50 m northeast to a tree, then 70 m west to a second tree, and finally 20 m south to a third tree.
- Draw a picture and establish a coordinate system.
 - Calculate Sparky's net displacement in component form.
 - Calculate Sparky's net displacement as a magnitude and an angle.
37. A field mouse trying to escape a hawk runs east for 5.0 m, darts southeast for 3.0 m, then drops 1.0 m down a hole into its burrow. What is the magnitude of the net displacement of the mouse?
38. Carlos runs with velocity $\vec{v} = (5 \text{ m/s}, 25^\circ \text{ north of east})$ for 10 minutes. How far to the north of his starting position does Carlos end up?
39. A cannon tilted upward at 30° fires a cannonball with a speed of 100 m/s. What is the component of the cannonball's velocity parallel to the ground?

40. Jack and Jill ran up the hill at 3.0 m/s. The horizontal component of Jill's velocity vector was 2.5 m/s.
- What was the angle of the hill?
 - What was the vertical component of Jill's velocity?
41. The treasure map in Figure P3.41 gives the following directions to the buried treasure: "Start at the old oak tree, walk due north for 500 paces, then due east for 100 paces. Dig." But when you arrive, you find an angry dragon just north of the tree. To avoid the dragon, you set off along the yellow brick road at an angle 60° east of north. After walking 300 paces you see an opening through the woods. Which direction should you go, and how far, to reach the treasure?
42. Mary needs to row her boat across a 100-m-wide river that is flowing to the east at a speed of 1.0 m/s. Mary can row her boat with a speed of 2.0 m/s relative to the water.
- If Mary rows straight north, how far downstream will she land?
 - Draw a picture showing Mary's displacement due to rowing, her displacement due to the river's motion, and her net displacement.
43. A jet plane is flying horizontally with a speed of 500 m/s over a hill that slopes upward with a 3% grade (i.e., the "rise" is 3% of the "run"). What is the component of the plane's velocity perpendicular to the ground?
44. A flock of ducks is trying to migrate south for the winter, but they keep being blown off course by a wind blowing from the west at 6.0 m/s. A wise elder duck finally realizes that the solution is to fly at an angle to the wind. If the ducks can fly at 8.0 m/s relative to the air, what direction should they head in order to move directly south?
45. A pine cone falls straight down from a pine tree growing on a 20° slope. The pine cone hits the ground with a speed of 10 m/s. What is the component of the pine cone's impact velocity

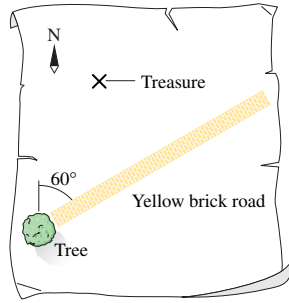


FIGURE P3.41

46. The car in Figure P3.46 speeds up as it turns a quarter-circle curve from north to east. When exactly halfway around the curve, the car's acceleration is $\vec{a} = (2 \text{ m/s}^2, 15^\circ \text{ south of east})$. At this point, what is the component of \vec{a} (a) tangent to the circle and (b) perpendicular to the circle?

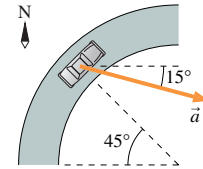


FIGURE P3.46

47. Figure P3.47 shows three ropes tied together in a knot. One of your friends pulls on a rope with 3 units of force and another pulls on a second rope with 5 units of force. How hard and in what direction must you pull on the third rope to keep the knot from moving?

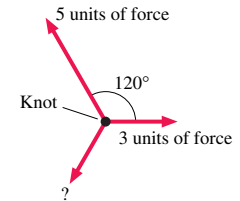


FIGURE P3.47

48. Three forces are exerted on an object placed on a tilted floor in Figure P3.48. The forces are measured in newtons (N). Assuming that forces are vectors,
- What is the component of the net force $\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$ parallel to the floor?
 - What is the component of \vec{F}_{net} perpendicular to the floor?
 - What are the magnitude and direction of \vec{F}_{net} ?

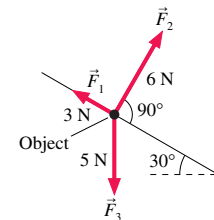


FIGURE P3.48

49. Figure P3.49 shows four electrical charges located at the corners of a rectangle. Like charges, you will recall, repel each other while opposite charges attract. Charge B exerts a repulsive force (directly away from B) on charge A of 3 N. Charge C exerts an attractive force (directly toward C) on charge A of 6 N. Finally, charge D exerts an attractive force of 2 N on charge A. Assuming that forces are vectors, what is the magnitude and direction of the net force \vec{F}_{net} exerted on charge A?

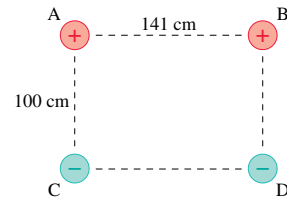
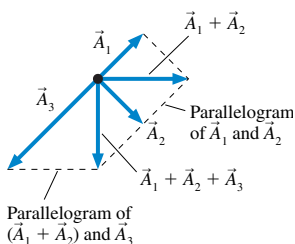


FIGURE P3.49

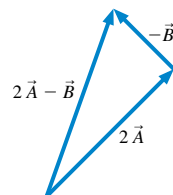
STOP TO THINK ANSWERS

Stop to Think 3.1: c. The graphical construction of $\vec{A}_1 + \vec{A}_2 + \vec{A}_3$ is shown below.

Stop to Think 3.2: a. The graphical construction of $2\vec{A} - \vec{B}$ is shown below.



STOP TO THINK 3.1



STOP TO THINK 3.2

Stop to Think 3.3: $C_x = -4 \text{ cm}, C_y = 2 \text{ cm}$.

Stop to Think 3.4: c. Vector \vec{C} points to the left and down, so both C_x and C_y are negative. C_x is in the numerator because it is the side opposite ϕ .