Lab 10: Capacitors and inductors in AC circuits, and electrical resonance

1 Introduction

Capacitors and inductors can be used to store energy in electrical circuits in the form of electric fields and magnetic fields respectively. This lab will introduce you to the behavior of these elements in alternating current (AC) circuits. The lab will also cover experiments associated with the phenomenon of resonance in simple circuits with a resistor, an inductor and a capacitor. Depending on the values of the inductor and capacitor, these circuits can be made to select or respond to specific resonant frequencies. Such circuits are used in radios and television sets to tune to various frequencies or channels. The principle of resonance is widely used in the design of electrical circuits.

EXERCISES 1 AND 2 PERTAIN TO THE BACKGROUND CONCEPTS AND EXERCISES 3 - 5 PERTAIN TO THE EXPERIMENTAL SECTIONS.

2 Background

When a resistor is connected to an AC voltage source, the voltage is given by,

$$V(t) = V_0 \sin(\omega t) \tag{1}$$

The current through the resistor is simply given by Ohm's law as,

$$I(t) = \frac{V_0}{R} \sin(\omega t) \tag{2}$$

The current has the same frequency ($f = \frac{\omega}{2\pi}$) and phase as the voltage source.

When the AC source is connected to a capacitor C, the charge on the plates of the capacitor is given by,

$$Q(t) = CV(t) \tag{3}$$

Differentiating equation 3, and using $I = \frac{dQ}{dt}$, we get,

$$I = C\frac{dV}{dt} \tag{4}$$

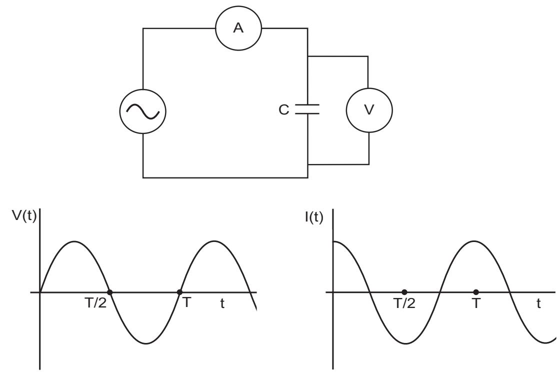

Figure 1: Voltage and current in an AC capacitor circuit. The time period $T = \frac{1}{f}$

Since $V = V_0 \sin(\omega t)$, equation 4 gives,

$$I = \omega C V_0 \cos(\omega t) = \omega C V_0 \sin(\omega t + \frac{\pi}{2}) \tag{5}$$

Equation 5 shows that the capacitor introduces a phase difference between the voltage and current since the current in the capacitor leads the voltage by 90°(figure 1). The quantity $\omega C V_0$ in equation 5 can be identified as the amplitude of the current I_0. In analogy with Ohm's law we can write,

$$I_0 = \frac{V_0}{X_C} \tag{6}$$

where the impedance X_C is the capacitive reactance (measured in ohms), $X_C = \frac{1}{\omega C}$. The impedance is clearly frequency dependent. At high frequencies it tends to zero since the current through the capacitor ($C\frac{dV}{dt}$) increases with frequency.

When an AC source is connected to an inductor L, the potential difference across the inductor is given by,

$$V_L = L\frac{dI}{dt} \tag{7}$$

Differentiating equation 2 we get $\frac{dI}{dt} = \frac{V_0}{R}\omega \cos \omega t$. Plugging this result into equation 7 we get,

$$V_L = \omega L(\frac{V_0}{R}) \cos(\omega t) = \omega L I_0 \cos(\omega t) \tag{8}$$

Equation 8 shows that the inductor introduces a phase difference between the voltage and the current since the current lags behind the applied voltage by 90°(figure 2). The quantity $\omega L I_0$ in equation 8 can be identified as the maximum amplitude of the voltage across the inductor V_0. In analogy with Ohm's law we can write,

$$I_0 = \frac{V_0}{X_L} \tag{9}$$

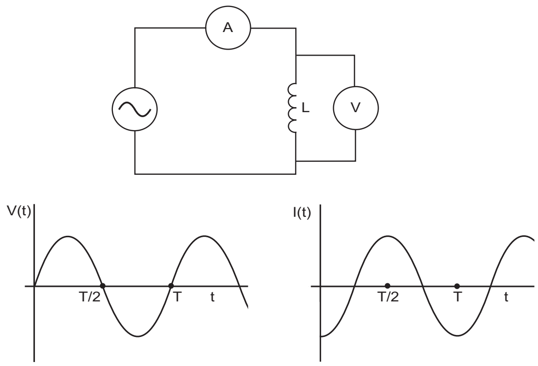

Figure 2: Voltage and current in an AC inductor circuit. The time period $T = \frac{1}{f}$

where the impedance X_L is the inductive reactance (measured in ohms), $X_L = \omega L$. The impedance of an inductor is also frequency dependent. The impedance of the inductor tends to infinity at high frequencies, since the absolute value of the induced emf in the inductor ($L\frac{dI}{dt}$) will be larger at higher frequencies and oppose the flow of current.

Now consider the series circuit shown in figure 3. Since the same current flows through the circuit, the voltage across the inductor and the capacitor should be 180° out of phase for all frequencies! At high frequency the $X_C < X_L$ and the voltage across the inductor is much larger than the voltage across the capacitor. At low frequencies, the voltage across the capacitor is higher than the voltage across the inductor. However, the phase difference between these voltages is always 180°! This suggests that there must be an intermediate frequency (called the **resonant frequency** of the circuit) at which the voltages are the same.

The value of ω for which $X_L = X_C$ is called the resonant frequency. At this frequency $\omega L = \frac{1}{\omega C}$ and f_r is the natural or resonant frequency of the circuit given by,

$$f = f_r = \frac{1}{2\pi\sqrt{LC}} \tag{10}$$

The overall resistance to the flow of current in an RLC circuit is known as impedance, Z. The impendence is found by combining the resistance, the capacitive reactance and the inductive reactance. Unlike a simple series circuit with resistors where the resistances are directly added, in an RLC circuit the resistances and reactances are added as vectors. This is due to the phase relationships.

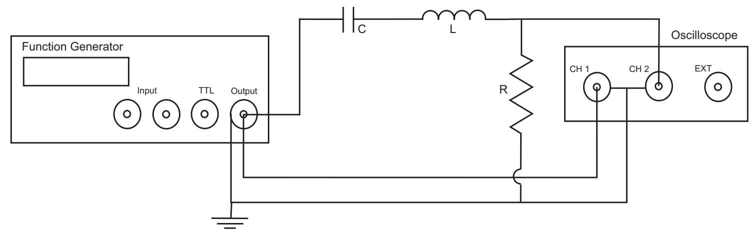

Figure 3: LCR circuit

It can be shown that the impedance Z is equal to,

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \tag{11}$$

Refer to section 37-3 of the textbook **Physics**, listed in the suggested reading section, for a derivation of the impedance Z.

The current and voltage in an RLC circuit are related by,

$$V = IZ \tag{12}$$

The phase relationship between the current and voltage can be shown to be,

$$\tan \Phi = \frac{(X_L - X_C)}{R} \tag{13}$$

If Φ is positive, the voltage leads the current by the phase angle. If the angle is negative, the voltage lags the current by Φ. When $X_L = X_C$, $\Phi = 0$ since the current and voltage are in phase.

At the resonance frequency $X_L = X_C$, Z is a minimum ($Z = R$) and hence the current through the circuit is maximum.

Exercise 1a: Using $I_0 = \frac{V_0}{Z}$, derive an expression for I_0 through the circuit in figure 3 as a function of frequency.

Exercise 1b: Use physical arguments to draw a graph of the current as a function of frequency.

Exercise 1c: What will happen to the shape of your graph when the resistance in the circuit is decreased? Draw another graph to show this effect and explain why the shape will change.

3 Suggested Reading

Refer to the chapter on AC Circuits,

R. Wolfson and J. Pasachoff, **Physics with Modern Physics** (3rd Edition, Addison-Wesley Longman, Don Mills ON, 1999)

D. Halliday, R. Resnick and K. S. Krane, **Physics** (Volume 2, 5th Edition, John Wiley, 2002)

4 Apparatus

Refer to Appendix E for photos of the apparatus

- Function generator
- Oscilloscope
- 0.002 μF capacitor
- 100 mH inductor
- Plexiglass circuit board with binding posts
- Banana cables
- Banana-BNC adapters
- Digital multi-meter
- BNC cable
- 11 kΩ resistor
- 110 Ω resistor

5 Experiment I: RLC circuit

The purpose of this part of the experiment is to determine the resonant frequency of a simple series circuit containing a resistor, an inductor and a capacitor. Connect the function generator to channel 1 of the oscilloscope. To do this, connect the positive terminal on the generator to the inner pin of the connector on the oscilloscope, and the grounded terminal of the generator to the outer shield of the connector on the oscilloscope, using a BNC cable. Set the trigger source on the oscilloscope to channel 1. Select a suitable time base on the oscilloscope so that you see a sinusoidal voltage on the scope when you vary the trigger level. Verify that the frequency that you measure on the oscilloscope corresponds to the setting of the frequency dial on the signal generator. Vary the frequency of the generator over the range of 500 Hz to 60 kHz and verify that the output of the generator is independent of frequency, i.e. the voltage should not change as the frequency is changed. Use another cable to simultaneously connect the output of the generator to the circuit as shown in figure 3. Use an 11 kΩ resistor, a 100 mH inductor and a capacitor with a value of 0.002 μF. Now connect the terminals of the resistor to channel 2 of the oscilloscope.

Exercise 2: Determine the resonance frequency by varying the generator frequency over the suggested range and measuring the peak voltage as a function of frequency. Tabulate your data. Vary the frequency in small increments near the value of the resonant frequency.

Exercise 3: Replace the 11 kΩ resistor with the 110 kΩ resistor and repeat the above measurement. Tabulate your results.

Exercise 4a: Plot the peak voltage versus frequency in SI units for both sets of data.

Exercise 4b: What are the resonance frequencies for each value of R? How do they compare with the theoretical values given by equation 10?

Exercise 4c: Are the shapes of your graphs consistent with your predictions in exercise 1?

Your lab report should include:

Answers to exercises 1-4 with relevant data tables, graphs, figures and qualitative comments.

Refer to Appendix D for Maple worksheets.