Lab 4: The Classical Hall Effect

1 Introduction

Understanding the motion of charge carriers in magnetic fields has led to several interesting practical applications. The deflection of an electron beam in a magnetic field can be used to measure the charge to mass ratio of electrons and ions. Mass spectrometers that are used to measure the masses of isotopes find widespread applications in archaeology, geology, and planetary science. The period of orbital motion of a charged particle in a uniform magnetic field is independent of the velocity of the particle. This is the basis for particle accelerators such as cyclotrons. High-energy collisions in particle accelerators and cloud chambers (used for studying cosmic rays) are analyzed by photographing the curved trajectories of charged particles (produced during these collisions) in magnetic fields.

The classical Hall effect (discovered in 1879 by E. H. Hall) is also related to the motion of charge carriers in magnetic fields. It led to some of the earliest experiments that established the sign of charge carriers in conductors. It has been used to measure the number density of charge carriers in conductors, as well as the drift velocity of charge carriers in the presence of a uniform electric field in the conductor. Such an electric field can be established by connecting the terminals of a battery to the conductor. The classical Hall effect is commonly used in magnetic field sensors that are reliable, accurate, and relatively inexpensive.

During this lab, you will carry out some simple experiments that will allow you to verify the Hall effect. You will determine the sign of the charge carriers in two kinds of semiconductors. The drift velocity of the charge carriers in the sample and the number of charge carriers per unit volume can also be determined from your data.

EXERCISES 1 AND 7 PERTAIN TO THE BACKGROUND CONCEPTS AND EXERCISES 2-6 AND 8-10 PERTAIN TO THE EXPERIMENTAL SECTIONS.

2 Background

A particle with charge q moving with a velocity $\overrightarrow{v}$ in a uniform magnetic field $\overrightarrow{B}$ will experience a force $\overrightarrow{F}$,

$$\overrightarrow{F} = q(\overrightarrow{v} \times \overrightarrow{B}) \tag{1}$$

Consider the motion of free electrons in a conducting wire at room temperature. In the absence of an electric field, the electrons will move in random directions with a characteristic velocity distribution. The average velocity of the electrons is zero, and their most probable speed is related to the thermal energy of the electrons. This thermal energy is proportional to the temperature of the conductor. If the conductor is connected to a battery, the electrons will experience a force due to the electric field $\overrightarrow{E_B}$ inside the conductor. The electrons will be accelerated in a direction opposite the field. However, collisions with the lattice of ions that make up the wire will dissipate the increase in the kinetic energy of the electrons due to the electric field. As a result, the electrons acquire a **drift velocity** $\overrightarrow{v_d}$ in addition to their random thermal velocity. The magnitude of the drift velocity is usually small in comparison with the most probable speed associated with the random motion.

The total number of charge carriers in a conductor of infinitesimal length $d\overrightarrow{L}$ and cross-sectional area A is $nA(d\overrightarrow{L})$, where n is the number of charge carriers per unit volume.

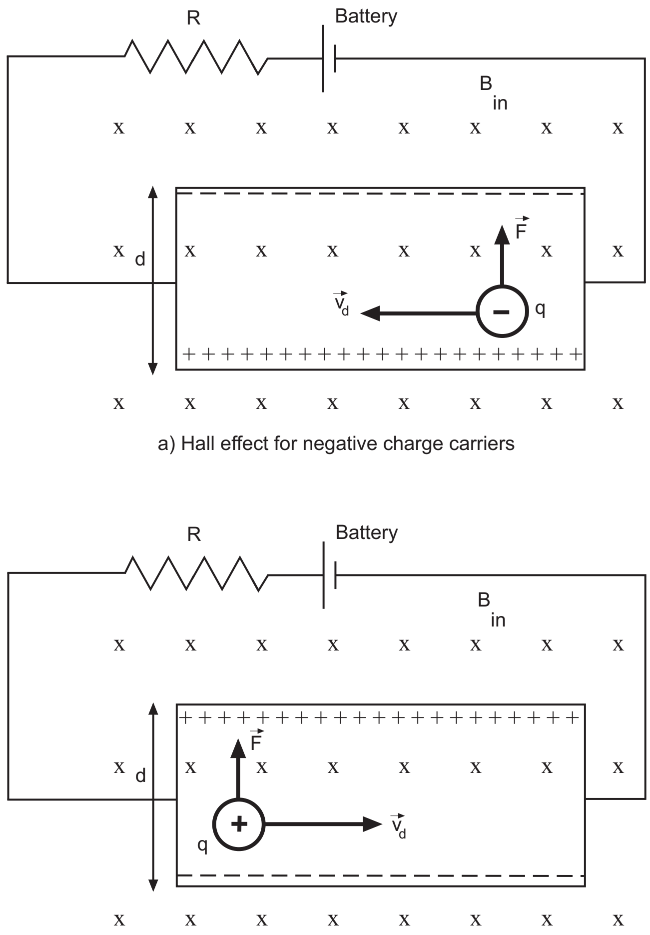
Since a total charge $Q = nA(d\overrightarrow{L})q$ flows through the length $d\overrightarrow{L}$ in time $\Delta t = \frac{d\overrightarrow{L}}{v_d}$, the current flowing through the wire ($I = \frac{Q}{\Delta t}$) is,

$$I = nAqv_d \tag{2}$$

If the conductor is placed in a uniform magnetic field perpendicular to the drift velocity, the electrons will feel a force transverse to their motion as specified by equation 1. The electrons will therefore pile up on one side of the conductor as shown in figure 1. The other side of the conductor will have an excess positive charge. The separation of charges creates a uniform electrostatic field $\overrightarrow{E}$ inside the conductor that opposes the magnetic field. During the charge pileup, the magnitude of the electrostatic field increases until the force due to this field balances the force due to the magnetic field. This condition is expressed as,

$$qE = qv_dB \tag{3}$$

When this condition is satisfied, electrons will no longer experience a deflection due to the magnetic



a) Hall effect for negative charge carriers

b) Hall effect for positive charge carriers

Figure 1: Hall effect

field. The electric potential difference between the two sides of the conductor is known as the Hall voltage V_H. If the width of the conductor is d, the relationship between the electrostatic field and the Hall voltage is given by,

$$E = \frac{V_H}{d} \tag{4}$$

Using equations 3 and 4, we can express the drift velocity as,

$$v_d = \frac{V_H}{Bd} \tag{5}$$

It is therefore possible to infer the drift velocity by measuring the Hall voltage and the magnetic field.

Rearranging equation 2, the number of charge carriers per unit volume can be expressed as,

$$n = \frac{I}{qAv_d} \tag{6}$$

It should be evident that a measurement of the current flowing through the conductor can be combined with the inferred drift velocity to find the value of n.

Exercise 1: A rectangular slab of silver (an excellent electrical conductor) has a width of 2.5 cm and thickness 2.0 mm. The slab is placed in a magnetic field of magnitude 1.0 T directed perpendicular to the slab. A Hall voltage of 0.107 μV is measured when a current of 2.0 A flows through the slab.

a) Assuming that the electron charge is $q = -1.6 \times 10^{-19}$ C, calculate the number of charge carriers per unit volume.

b) Compare the number of charge carriers n obtained in exercise 1a to the number of atoms per unit volume in silver obtained from the density of silver (10.5 g/cm^3) and its molar mass (108 g/mol).

In a semi-conductor, the number of charge carriers per unit volume is much smaller than in an excellent electrical conductor like silver. From equation 6, it can be expected that the drift velocity must be higher in a semiconductor than in a good conductor assuming that both materials have the same area of cross section, and that the current flowing through each material is the same. Since the Hall voltage is proportional to the drift velocity (see equation 5), it can be expected that the Hall voltage will be larger, and therefore easier to measure in a semiconductor.

Semi-conductors have another interesting property. The charge carriers in a semi-conductor are not always **electrons**. In some cases, the charge carriers are positively charged and are referred to as **holes**. From figure 1, it should be clear that the sign of the Hall voltage should depend on the type of charge carrier. **If the charge carriers are electrons, the material is referred to as an n-type semiconductor. If the charge carriers are holes, the material is called a p-type semiconductor.**

3 Suggested Reading

Refer to the chapters on Current and Resistance and the Magnetic Field,

R. Wolfson and J. Pasachoff, **Physics with Modern Physics** (3rd Edition, Addison-Wesley Longman, Don Mills ON, 1999)

D. Halliday, R. Resnick and K. S. Krane, **Physics** (Volume 2, 5th Edition, John Wiley, 2002)

4 Apparatus

Refer to Appendix E for photos of the apparatus

- Digital multimeter
- n and p type semiconductors
- Circuit board with Potentiometer
- 4.5V power supply (3 batteries)
- Ammeter
- Permanent magnet
- Magnetic field sensor
- Banana wires
- Vernier calipers

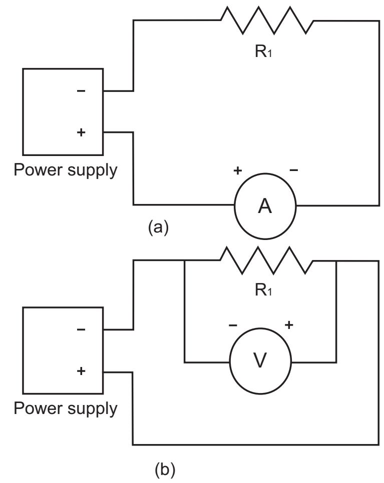


Figure 2: Ammeter (a) and Voltmeter (b) setup

5 Experiment I: Determining the sign of the charge carriers

In order to interpret the results one needs to determine the direction of the current I, the direction of the magnetic field B and the sign of the Hall voltage. The following discussion describes how the signs of the quantities can be determined.

If an ammeter is connected to a power source as shown in figure 2a, it is configured to show a positive deflection or reading.

If a voltmeter is connected between two points on a circuit, as shown in figure 2b, the voltmeter will read a positive voltage. In other words, the voltmeter will show a positive deflection or reading if the positive terminal is at a higher potential than the negative terminal.

The direction of the magnetic field can be deduced on the basis of magnetic field lines. The direction of the field between the poles of the magnet is indicated on the magnet.

To determine the sign of the charge carriers we will use the apparatus shown below. Figure 4a shows the circuit board that holds resistors and connection plugs. The reversing switch can be used to change the direction of current through the semiconductor wafer. The current can be set to a convenient value by adjusting the variable resistor, or potentiometer.

Figure 4b is the circuit diagram for the experiment. In the circuit diagram, when the switch is in position

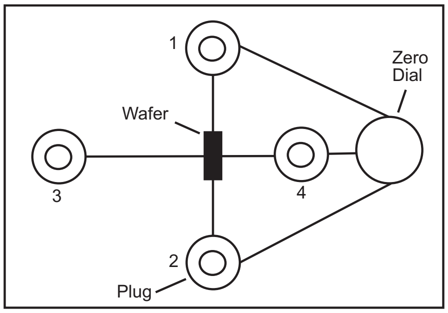


Figure 3: Diagram of the wafer board

I the current flows from X to Y, on the other hand, when the switch is in position II the current flows from Y to X.

Figure 3 is a sketch of a wafer board. Note the knob labeled zero and the four terminals surrounding the wafer. The contacts of the wafer are labeled 1 and 2. If the wafer is placed in a magnetic field, the Hall voltage can be measured between points 3 and 4.

Before beginning the experiment, the voltage across the wafer needs to be reset to zero. In general, it will not read zero since the wafer is in the earth's magnetic field. Using two banana wires connect the power supply to the terminals labeled 7 and 8 on the circuit board. Next, connect the ammeter to the two terminals labeled 5 and 6 in figure 4. Now connect terminals 1 and 2 on the wafer board in figure 3 to terminals 1 and 2 in figure 4. Finally, you need to connect the wafer to the voltmeter. This can be done in three different ways,

1. Plug two banana cables from the voltmeter to the circuit board in figure 4 at the terminals labeled 3 and 4. Next, connect the wafer board's wires labeled 3 and 4 to the voltmeter's banana cables on the circuit board.
2. Plug two banana wires directly from the voltmeter to the terminals labeled 3 and 4 on the front panel of the wafer board in figure 3.
3. Plug in the wafer board's wires labeled 3 and 4 directly to the voltmeter.

It is recommended that you use the first method to measure the voltage, since it is the most convenient setup. After the circuit has been setup, hold down the power supply's press button and turn the wafer's zero knob until the voltmeter reads zero. Make sure you do this for both samples. For these measurements,

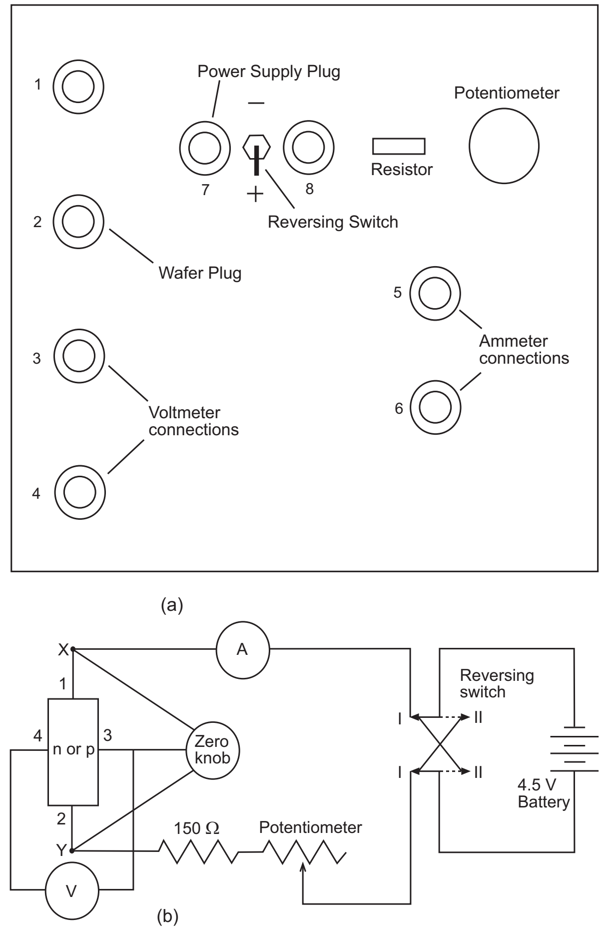


Figure 4: Circuit Board (a) and schematic of circuit (b).

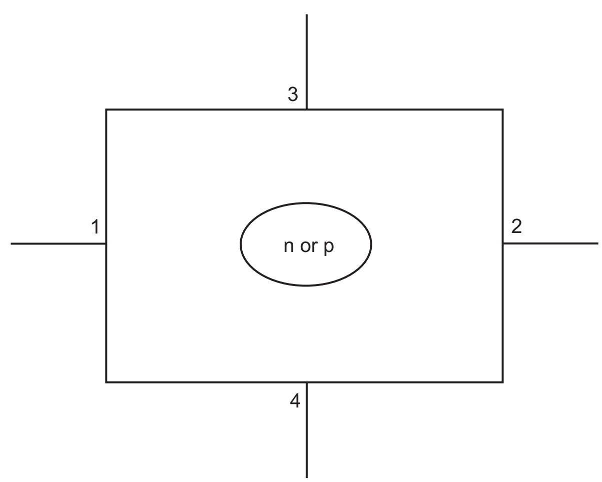


Figure 5: Semiconductor wafer, exercise 3

ensure that the magnet is moved away from the wafer. Consult the TA and make sure you understand why it is necessary to do this.

Now you can place the wafer in a magnetic field. You can do so by sliding the thick wooden block into the magnet and placing the thin wooden block with a slit on top of it. The wafer board can be positioned firmly between the magnet's poles using the wooden block.

Exercise 2: Measure the potential difference between points 3 and 4.

Exercise 3: Based on your knowledge of which terminal is at a higher potential, the direction of the magnetic field and the direction of the current, draw a diagram like the one shown in figure 5 (diagram of a semiconductor wafer). Indicate the type of sample (n or p type semiconductor wafer), direction of current (for example, from 1 to 2), direction of magnetic field, and charge buildup (as shown in figure 1. Sketch the direction of forces responsible for achieving the charge separation.

Exercise 4: Reverse the direction of the current. Draw a figure based on the measurement of the potential difference as in the previous exercise.

Exercise 5: Reverse the direction of the magnetic field by rotating the magnet. Based on your measurement, draw a figure as in the previous exercise.

Exercise 6: Repeat exercises 2, 3, 4 and 5 for the second sample. Draw figures consistent with your measurements of the potential difference.

From exercises 2, 3, 4 and 5 determine the sign on the charge carriers in each sample.

6 Experiment II: Determining the drift velocity and the number of charge carriers per unit volume

As discussed in the previous section, the drift velocity v_d can be determined using equation 5 if the Hall voltage V_H, the magnetic field $\overrightarrow{B}$, and the width of the semi-conductor d are known. From equation 2, we can see that the drift velocity should be proportional to the current through the sample. This is because the current is proportional to the potential difference across the sample (Ohm's Law). The potential difference across the sample is proportional to the electric field $\overrightarrow{E_B}$ due to the battery that drives the current through the circuit. It should also be obvious that there is no drift velocity without current flow.

It is therefore possible to make an accurate measurement of the drift velocity if the Hall voltage is measured as a function of the current at a fixed magnetic field.

Exercise 7: Combine equation 2 and equation 5, and express the Hall voltage as a function of current.

The dimensions of the two samples can be measured using a vernier caliper. Refer to Lab 3's background section for instructions on how to use a vernier caliper.

To measure the magnetic field, use the magnetic field sensor. This is a calibrated probe with a small semiconductor wafer like the one used in this lab. This device actually relies on the Hall effect to produce a voltage proportional to the magnetic field (at a fixed current)! To measure the magnetic field, connect the digital multi-meter to the field sensor. Use the dial labeled zero on the sensor controller to zero the voltage recorded by the multi-meter.

The sensor can then be placed between the pole pieces of the permanent magnet used in these experiments. **The conversion factor for the field sensor is 1V = 8.03×10^{-3} T.**

The Hall voltage is measured by connecting the multimeter across terminals 3 and 4 of the sample. The current can be measured using the ammeter. It is possible to vary the current by adjusting the variable resistor.

Exercise 8: Measure the magnetic field and the dimensions of the wafer. Record V_H as a function of current for each sample. Tabulate your data.

Exercise 9a: Plot a graph of Hall voltage versus current for each sample. Is the shape of the graph consistent with expectations?

Exercise 9b: Use your measurements of B and d to find the drift velocity of the charge carriers in each sample for a particular value of current (refer to equation 5).

Exercise 10: The drift velocity of charge carriers in the copper conductors connecting the local power station to this room is $\sim 10^{-4}$ m/s. Why do the room lights seem to turn on instantly when a wall switch is turned on even though the lights are several miles from the power station?

Exercise 11: Use the results of your graph and your measurements of the dimensions of the sample to find the number of charge carriers per unit volume n for the two samples used in these experiments. Assume $q = 1.6 \times 10^{-19}$ C. How do these values compare to the value of n for silver, from exercise 1?

Your lab report should include:

Answers to exercises 1-11 with relevant data tables, graphs, figures and qualitative comments.

Refer to Appendix D for Maple worksheets.