Lab 5: The Biot-Savart law - magnetic fields due to current carrying coils

1 Introduction

Coulomb's law describes the electric field of a point charge q. It predicts that the magnitude of the electric field $\overrightarrow{E}$ at a point located at a distance r from the charge is proportional to the charge, and inversely proportional to r^2. The direction of the electric field is along the radial direction from the point charge, and can be found by considering the force acting on (a unit positive) test charge. In an analogous manner, the Biot-Savart law describes the magnetic field due to a point charge moving with a velocity $\overrightarrow{v}$. Since a moving charge represents a current, this law can also be used to specify the magnetic field of a current carrying element of infinitesimal length $d\overrightarrow{l}$. It is most commonly written in this form and is widely used to calculate the magnetic field generated by a system of current carrying coils.

The purpose of this experiment is to verify the predictions of the Biot-Savart law by measuring the magnetic field produced by some simple configurations of coils. You will measure the magnetic fields associated with the following configurations.

1. One of the simplest derivations of the Biot-Savart law is a calculation using the field along the axis of symmetry of a current carrying loop of wire. This derivation can also give you the field at the center of the coil. You can measure the field as a function of current and test the derivation. This is useful because using a single coil is often the easiest method of producing a magnetic field.
2. The magnetic field produced by a single current carrying coil is not uniform - it varies as a function of position. Consider a pair of identical coils (radius R) that are separated by a distance equal to R. If the current flowing through each coil is in the same direction, then the magnetic field along the axis of symmetry is uniform at the point midway between the coils. This is one of the most convenient methods of producing a uniform magnetic field at a desired spatial location. This configuration is known as a pair of **Helmholtz coils** (The magnetic field inside a long solenoid is also uniform. Such a field is often more difficult to implement because of spatial constraints).
3. If the currents in the two coils (separated by R) are in opposite directions, the arrangement may be referred to as a pair of **anti-Helmholtz coils**. The field along the axis of symmetry is zero at the point midway between the coils. The magnetic field varies linearly as a function of position in the vicinity of the midpoint. This spatial variation of the field is called a field gradient, and using anti-Helmholtz coils is the most convenient method of generating field gradients. Such field gradients are widely used in magnetic resonance imaging, which is a powerful, noninvasive technique used for obtaining highly resolved spatial images. When a sample (such as the human body) is placed in a field gradient, the precession frequency of protons in the sample (which depends on the magnetic field) will vary linearly as a function of position. This precession frequency can be detected and converted into a spatial image.

EXERCISES 5, 6, 7, 8a-b AND 14 PERTAIN TO THE BACKGROUND CONCEPTS AND EXERCISES 1-4, 8c-13 AND 15 PERTAIN TO THE EXPERIMENTAL SECTIONS.

2 Background

The Biot-Savart law defines the magnetic field $\overrightarrow{B}$ due a point charge q moving with a velocity $\overrightarrow{v}$ as,

$$\overrightarrow{B} = \frac{\mu_0}{4\pi}\left(\frac{q\overrightarrow{v} \times \hat{r}}{r^2}\right) \tag{1}$$

Here, $\hat{r}$ is a unit vector that points from the position of the charge to the point at which the field is evaluated, r is the distance between the charge and the point at which the field is evaluated and $\mu_0 = 4\pi \times 10^{-7}$ Tm/A is the permeability of free space. The direction of $\overrightarrow{B}$ is perpendicular to the direction of both $\overrightarrow{v}$ and $\hat{r}$.

Equation 1 can be modified to express the magnetic field ($\overrightarrow{dB}$) due to an infinitesimal current element $\overrightarrow{dl}$, which can be written as,

$$\overrightarrow{dB} = \frac{\mu_0}{4\pi}\left(\frac{I\overrightarrow{dl} \times \hat{r}}{r^2}\right) \tag{2}$$

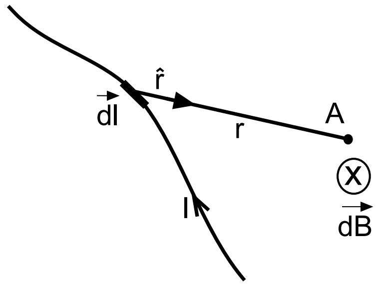

Figure 1: Illustration of the Biot-Savart Law

The quantities in equation 2 are illustrated in figure 1. The direction of the magnetic field due to the current element at the point A can be inferred using the right hand rule. $\overrightarrow{dB}$ is directed into the plane of the figure at A. The field due to the entire wire can be evaluated at A by adding up the contributions of all the current elements in the wire.

3 Suggested Reading

Refer to the chapter on Ampere's Law,

R. Wolfson and J. Pasachoff, **Physics with Modern Physics** (3rd Edition, Addison-Wesley Longman, Don Mills ON, 1999)

D. Halliday, R. Resnick and K. S. Krane, **Physics** (Volume 2, 5th Edition, John Wiley, 2002)

4 Apparatus

Refer to Appendix E for photos of the apparatus

- DC Power Supply (Model PS-303)
- Two, 110 turn magnetic field coils attached to wooden stand
- Meter stick attached along the axis of symmetry
- Banana wires to connect coils
- Alligator clips
- Magnetic field sensor with power supply
- Digital voltmeter
- Banana connectors to connect magnetic field sensor to voltmeter
- 3" post holder attached to rectangular base that is glued onto wooden base
- Three aluminum posts and two aluminum clamps to mount magnetic field sensor along axis of symmetry of coils

5 Some remarks concerning the magnetic field sensor

The magnetic field probe is based on the classical Hall effect. As current passes through the sensor (i.e. there is a flow of charge), the applied magnetic field exerts a force on the charges, resulting in the accumulation of charge carriers on one side of the sensor. This results in a potential difference (called the Hall voltage) between two sides of the sensor. The Hall voltage is proportional to the applied magnetic field.

Make sure that the sensor is connected to its power supply.

The sensor used in this experiment can be set to detect magnetic fields along two directions perpendicular to the sensor. A switch with two settings marked radial and transverse respectively is provided for this purpose on the sensor controller. Make sure that the switch is set to **THE RADIAL POSITION FOR ALL EXPERIMENTS** that will be carried out during this experiment.

The Earth's magnetic field is $\sim$1G (=10^{-4} T). To eliminate its effect on the experiment, adjust the set zero knob on the sensor controller so that the voltmeter connected to the sensor reads zero (with no current through the coils). **Check if the earth's field has been zeroed at the start of each of the three main experiments that you will carry out during this lab.**

The calibration of the sensor (when the switch is set to the radial position) is marked on the sensor controller [1V (voltmeter reading) = 8.03×10^{-3} Tesla]. Consult the TA, and make sure that you understand the calibration. The calibration is necessary to convert the voltmeter readings into magnetic field units.

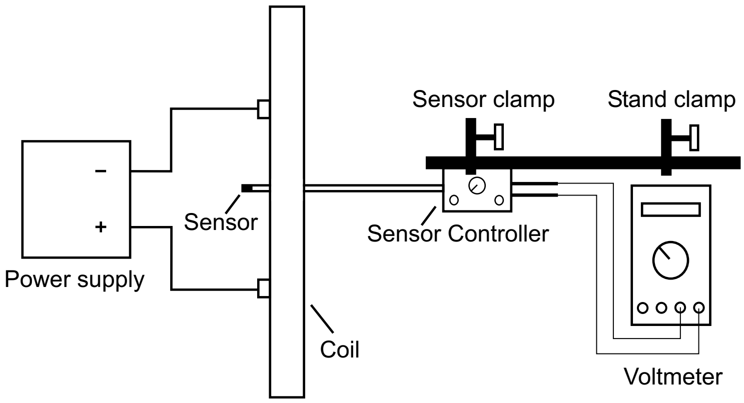

Figure 2: Experiment 1, single coil setup

6 Experiment I: Magnetic field at the center of a circular coil carrying a current

The general expression for the magnetic field along the axis of symmetry of the coil can be calculated using the Biot-Savart law. The magnitude of the field is given by,

$$B = \frac{\mu_0 N I R^2}{2(R^2 + z^2)^{3/2}} \tag{3}$$

where, N is the number of turns, I is the current, R is the radius and z is the distance from the center of the coil along its axis of symmetry to the point at which the field is evaluated.

For this part of the experiment, you will be measuring the field at the center of the coil, where $z = 0$. Hence equation 3 becomes,

$$B = \frac{\mu_0 N I}{2R} \tag{4}$$

The experimental setup is as shown in figure 2. You will be using a coil with 110 turns.

Measure the average radius of the coil. Connect the voltmeter to the magnetic field probe and measure the magnetic field at the center of the coil before the coil is connected to a power supply. **Zero the earth's magnetic field**. Connect one of the coils to a DC power supply, as shown in figure 2. Adjust the current to 2 Amperes and measure the magnetic field at the center of the coil.

Exercise 1: What is the direction of the magnetic field? Reverse the current through the coil and measure B again. What is its magnitude and direction?

Exercise 2: Calculate the magnetic field at the center of the coil using equation 4, and evaluate the associated uncertainty. Does your calculation agree with the experimentally determined value?

Exercise 3: Vary the current in a range between 0.1A and 3.0A and measure the magnetic field at the center of the coil in each case. Organize your data in a table.

Exercise 4: Plot a graph of B versus I. Explain your results. Calculate the slope and the uncertainty in the slope. Does the result agree with the prediction of equation 4?

Exercise 5: Make a rough sketch of $B(z)$ given by equation 3.

Exercise 6: Consider equation 3. If $z \gg R$ then the expression for B reduces to,

$$B = \frac{\mu_0 N I R^2}{2z^3} \tag{5}$$

What does the magnetic field described by equation 5 represent? Explain the result in physical terms.

7 Experiment II: Magnetic field of Helmholtz coils

Consider a pair of identical coils connected in series. Think of the coils in a coordinate system such that $z = 0$ and $z = R$ represent the centers of the two coils. For the case where the current through both coils is flowing in the same direction, the general expression for the total magnetic field along the axis of symmetry can be derived using the Biot-Savart law and the superposition principle. This magnetic field is given by,

$$B = \frac{\mu_0 N I R^2}{2}\left(\frac{1}{(R^2 + z^2)^{3/2}} + \frac{1}{(R^2 + (z - R)^2)^{3/2}}\right) \tag{6}$$

When the current is flowing in opposite directions through the two coils, the magnetic field is given by,

$$B = \frac{\mu_0 N I R^2}{2}\left(\frac{1}{(R^2 + z^2)^{3/2}} - \frac{1}{(R^2 + (z - R)^2)^{3/2}}\right) \tag{7}$$

Exercise 7: Show how equations 6 and 7 can be derived starting from equation 3. Use the coordinate system specified earlier. Sketch magnetic fields given

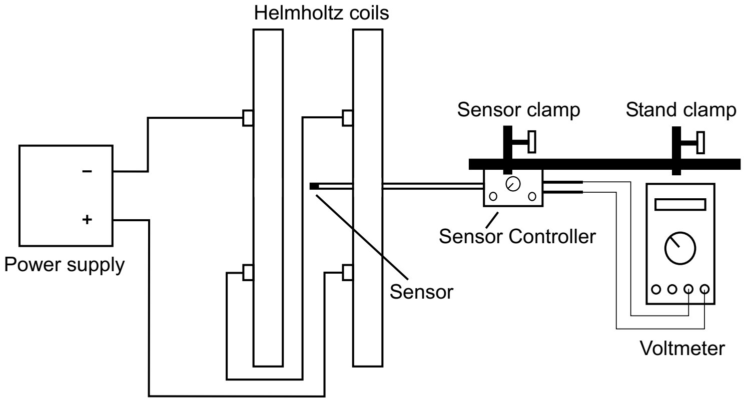

Figure 3: Experiment 2, Helmholtz coils

by equation 6 and equation 7 using the superposition principle.

Exercise 8a: Assume that the current through each coil is flowing in the same direction. Starting from equation 6, show that the magnetic field at $z = \frac{R}{2}$ is given by,

$$B = (\frac{4}{5})^{3/2}\frac{\mu_0 NI}{R} \tag{8}$$

Exercise 8b: What is $B(z = \frac{R}{2})$ if the current is flowing in opposite directions through the two coils?

Exercise 8c: How can you experimentally determine whether the current through the coils is flowing in the same direction or in opposite directions?

The setup for this part of the experiment is shown in figure 3. The distance between the two coils should be set equal to the average radius of the coils.

For the first part of this experiment, the current should flow in the **same direction** through the coils. Connect each coil to the DC power supply and connect the coils to each other as shown in figure 3. Verify that the currents are in the same direction by making a few measurements with the probe. Another way to do this would be to trace the direction of windings in the coil. Adjust the power supply voltage so that the current through the coils is $\sim$3 A.

NOTE: The current should not be changed during the experiment.

Exercise 9: Measure the magnetic field at the midpoint ($z = \frac{R}{2}$) between the coils. Then calculate the magnetic field at this point using equation 8. Does your measurement agree with theory?

Exercise 10a: Vary the position of the magnetic field sensor in steps of $\sim$1 cm and record the magnetic field as a function of position. The wooden base of the sensor mount can be guided along the wooden stand on which the coils are mounted. This will ensure that the sensor moves along the axis of symmetry. Be sure to cover a range of $\sim 2R$ along the axis on either side of the coils. Tabulate your data.

Exercise 10b: Plot B as a function of the distance z. Denote the positions of the coils on your graph. What conclusions can you draw from the graph? How does the plot compare to the sketch based on your expectations (exercise 7)?

Exercise 10c: Calculate $\frac{dB}{dz}$ at $z = \frac{R}{2}$ from equation 6. Is this result obvious on your graph?

8 Experiment III: Magnetic field of anti-Helmholtz coils

This part of the experiment requires the current to flow in **opposite directions** through the coils. Make the appropriate connections and set the current to $\sim$3A.

Verify that the current is flowing in opposite directions through the coils. Measure the magnetic field on the axis of symmetry at midpoint between the coils ($z = \frac{R}{2}$).

Exercise 11: Does the result agree with your calculation (exercise 8b)?

Again, vary the position of the sensor in small steps ($\sim$1 cm) and measure the magnetic field at each position. Cover a total distance of $2R$ outside each coil. Tabulate your data.

Exercise 12: Plot B as a function of the distance z. Denote the positions of the coils on your graph. Interpret your graph. How does your result compare with the sketch based on your expectations (exercise 7)?

Exercise 13: Calculate the slope at $z = \frac{R}{2}$.

Exercise 14: Starting from equation 7 show that,

$$\frac{dB}{dz}(z = \frac{R}{2}) = 1.1 \times 10^{-6}\frac{NI}{R^2} \tag{9}$$

The units of $\frac{dB}{dz}$ in equation 9 are T/m.

Exercise 15: Compare the slope of your graph at $z = \frac{R}{2}$ to the prediction of equation 9. What is the percentage difference between the experimental value and the theoretical value? Identify the errors that can lead to this discrepancy.

Your lab report should include:

Answers to exercises 1-15 with relevant data tables, graphs, figures and qualitative comments.

Refer to Appendix D for Maple worksheets.