Lab 6: The Earth's Magnetic Field

1 Introduction

The earth just like other planetary bodies has a magnetic field. The purpose of this experiment is to measure the horizontal component of the earth's magnetic field B_H using a very simple apparatus. The measurement involves combining the results of two separate experiments to obtain B_H. The first experiment will involve studies of how a freely suspended bar magnet interacts with the earth's magnetic field. The second experiment will involve an investigation of the combined effect of the magnetic field of a bar magnet and the earth's magnetic field on a compass needle.

A **magnetic dipole** is the fundamental entity in magnetostatics just like a point charge is the simplest configuration in electrostatics. A bar magnet is an example of a magnetic dipole. The magnetic field lines of the magnet are closed loops that are directed from north to south outside the magnet, and from south to north inside the magnet, as illustrated in figure 1. The **magnetic dipole moment** (by convention) is a vector drawn pointing from the south pole toward the north pole. At any point in space, the direction of the magnetic field is along the tangent to the field line at that point. The earth is another example of a magnetic dipole, with the north magnetic pole located at an inclination of $\sim 11.5°$ with respect to the rotational axis passing through the north and south geographic poles. In fact, the north magnetic pole should be labeled as the south pole of the earth's magnetic dipole, and the south magnetic pole should be labeled as the north pole of the dipole (figure 2). This can be inferred by considering the behavior of a freely suspended bar magnet or compass needle. The north pole of the magnetic needle is attracted toward the south pole of the earth's magnetic dipole.

Studies of the evolution of the earth's magnetic field have revealed that magnetic poles migrate relative to the geographic pole, and that the direction and the magnitude of the field changes. Most interestingly, the magnetic poles reverse periodically, once every several hundred thousand years!

EXERCISES 1, 2, 5 AND 9 PERTAIN TO

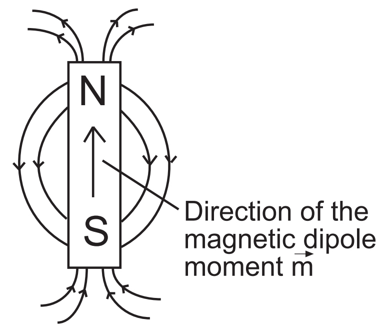

Figure 1: Magnetic field lines of a bar magnet

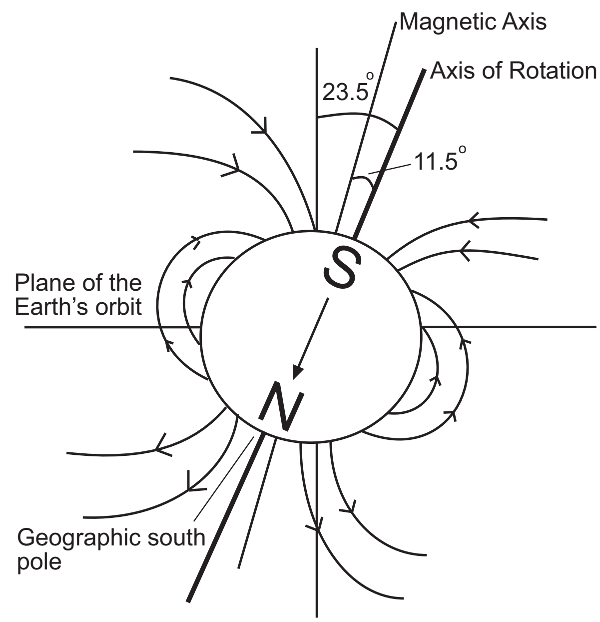

Figure 2: Earth's magnetic field

THE BACKGROUND CONCEPTS AND EXERCISES 3-4, 6-8 AND 10-13 PERTAIN TO THE EXPERIMENTAL SECTIONS.

2 Background

Gauss's Method: The procedure to be used in this experiment was developed by Gauss more than a century ago. At that time it was known that the earth had a magnetic field, but the field had not been measured. Gauss used principles of classical mechanics and very simple tools. In fact, with the exception of a digital stop watch (completely unnecessary, but nevertheless compact) the apparatus you will use is perhaps no different from what Gauss may have used!

The first part of the experiment involves measuring the period of oscillation of a bar magnet suspended in the earth's magnetic field, in order to obtain a value for the product of m and B_H. Here, m is the magnetic dipole moment of the bar magnet and B_H is the horizontal component of the Earth's magnetic field. The second part of the experiment involves a determination of the ratio $\frac{m}{B_H}$ by measuring the deflection of a compass needle due to the combined effects of the magnetic fields of the bar magnet and the horizontal component of the earth's magnetic field.

B_H can be measured by combining the results of both experiments. The result can be compared to current (accepted) values of B_H. To appreciate that some things have indeed become easier, you will be allowed to measure the earth's magnetic field using a conventional method involving a Hall probe.

3 Suggested Reading

Refer to the chapters on the Magnetic Field,

R. Wolfson and J. Pasachoff, **Physics with Modern Physics** (3rd Edition, Addison-Wesley Longman, Don Mills ON, 1999)

D. Halliday, R. Resnick and K. S. Krane, **Physics** (Volume 2, 5th Edition, John Wiley, 2002)

4 Apparatus

Refer to Appendix E for photos of the apparatus

- Bar magnet
- Sensitive compass needle mounted on a protractor
- Stopwatch for timing the oscillations
- Balance to measure the mass of the magnet
- Aluminum post and clamp for suspending the magnet
- Torsionless string for suspending the magnet
- Meter stick (without magnetized metal ends)
- Two aluminum posts and two clamps for supporting the meter stick on 3" post holders

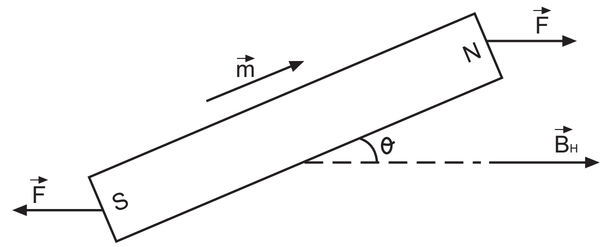

Figure 3: Torque on a bar magnet

- Two aluminum bases attached to a wooden board to support post holders
- Magnetic field sensor, controller and shield
- 5V DC power supply for sensor
- Digital voltmeter
- BNC to banana cable to connect sensor to voltmeter

5 Experiment I: Oscillation period of a bar magnet suspended in the earth's magnetic field

Consider a bar magnet suspended in the earth's (uniform) magnetic field. If the magnet is displaced from the position of equilibrium, a torque due to the earth's field acts on the magnet and tends to align it with the direction of the field, as shown in figure 3. The torque is given by,

$$\overrightarrow{\tau} = \overrightarrow{m} \times \overrightarrow{B_H} \tag{1}$$

Here, $\overrightarrow{m}$ is the magnetic moment of the magnet, and $\overrightarrow{B_H}$ is the horizontal component of the earth's magnetic field.

Exercise 1: If the magnet is displaced from its position of equilibrium by a small angle Θ, show that it will oscillate with a period T given by,

$$T = 2\pi\sqrt{\frac{I_M}{mB_H}} \tag{2}$$

Here, I_M is the moment of inertia of the bar magnet. It is given by,

$$I_M = \frac{M(L^2 + a^2)}{12} \tag{3}$$

where, L is the length of the magnet, a is its width, and M is its mass (which can be measured using a

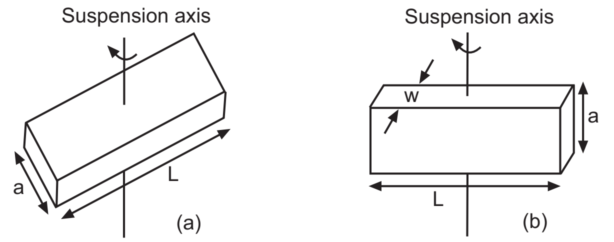

Figure 4: Moments of Inertia for the bar magnet, (a) suspension axis perpendicular to the surface of area La and (b) suspension axis perpendicular to the surface of area Lw

balance). This is the moment of inertia with the 'suspension axis' of the bar magnet perpendicular to the surface of area La, figure 4a.

In figure 4b the moment of inertia with suspension axis perpendicular to the surface of area Lw is $I_M = \frac{M(L^2+w^2)}{12}$. It is important to realize that the torque on the magnet restores the magnet to its equilibrium position. Hence $\tau = -mB_H \sin\Theta$. To solve for the period of oscillation, use the rotational analog of Newton's second law.

Exercise 2: Solve equation 2 to obtain an expression for mB_H. What are the SI units of m and B_H?

Suspend the magnet from its midpoint by a torsionless string, so that only the horizontal component of the Earth's magnetic field will cause a torque. Determine the period of small amplitude oscillations. Repeat the experiment several times and obtain the average value of the oscillation period. During each trial measure the period of oscillation by timing ∼10 oscillations. **Make sure that there are no other magnets or magnetized materials in the vicinity of the magnet. Also make sure that you shield the experiment from air currents.** If a wooden bar is used instead of a magnet there would be no restoring torque. Why?

Exercise 3a: Organize your experimental data in a table. Next determine the moment of inertia of the bar magnet, using equation 3. Estimate the experimental errors associated with each measured quantity.

Exercise 3b: Determine the value of mB_H and the associated uncertainty.

Exercise 4: How would your results differ if you used a copper wire to suspend the magnet?

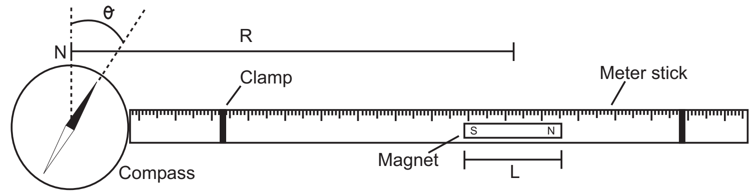

Figure 5: Magnetometer setup, top view

6 Experiment II: Determination of $\frac{m}{B_H}$

This part of the experiment requires the use of a compass needle that is extremely sensitive to magnetic fields. So make sure that apart from the bar magnet, there are no other magnetized materials in the vicinity of the compass needle. **This is crucial for obtaining accurate results**

The experimental setup consists of a magnetometer as shown in figure 5. To set up the magnetometer, find the direction of the Earth's magnetic field (i.e. the N-S direction) using the compass needle. Then place the axis of the bar magnet perpendicular to the N/S direction, with the magnet's center at a distance R from the center of the compass needle. It is assumed that the distance R is very much larger than L, the length of the magnet.

If $R \gg L$, the magnitude of the magnetic field B_b produced by the bar magnet at the compass is given by,

$$B_b = \frac{2k_m m}{R^3} \tag{4}$$

where, $k_m = \frac{\mu_0}{4\pi} = 10^{-7}\frac{N}{A^2}$ is a constant, m is the magnetic dipole moment of the bar magnet and R is the distance from the center of the magnet to the center of the compass.

Note that equation 4 can represent the magnetic field due to a single current carrying loop of wire for $R \gg r$, where r is the radius of the loop (in this case m is the magnetic moment of the loop).

Equation 4 is also analogous to the expression for the electric field perpendicular to the axis of an electric dipole. For R much larger than the distance separating the charges, the magnitude of the electric field E is given by,

$$E = \frac{2k_e p}{R^3} \tag{5}$$

Here, $k_e = \frac{1}{4\pi\varepsilon 0}$ is Coulomb's constant and p is the electric dipole moment.

Exercise 5a: Show that in the presence of the bar magnet and the earth's field, the compass needle will deflect from its initial direction by an angle Θ where,

$$\tan\Theta = \frac{2k_m m}{R^3 B_H} \tag{6}$$

Exercise 5b: If Θ is measured as a function of R, what is the simplest method of plotting your data to extract $\frac{m}{B_H}$?

Set up the magnetometer as shown in figure 5. The compass should be in the horizontal plane. When you want to use the compass to indicate the direction of the earth's magnetic field, place the bar magnet very far away from the compass. Find the direction of the Earth's magnetic field and rotate the protractor base of the compass until the tip of the needle coincides with the zero reading. Fix the compass in this position. Set up the meter stick in the plane of the compass needle. Carefully adjust the height of the meter stick to be at the same level as the plane of rotation of the compass needle.

Place the magnet at a distance R from the compass on the meter stick and measure the deflection angle of the compass needle. For the same value of R, reverse the magnet and measure the angle of deflection again. Averaging the two deflections will give you a more accurate value for the deflection angle.

Exercise 6: Estimate the errors associated with these measurements. Find the deflection angle for a number of values of R for which $R \gg L$. Choose values of R that are appropriate for your graph (exercise 5b). Organize your experimental data in the form of a table.

NOTE: Since the compass needle is very sensitive to magnetic fields, it oscillates for a long time after it is perturbed. You may dampen its motion by using a plastic ruler or an appropriate non-magnetized object.

Exercise 7: Why is it important to place the magnet at distances from the compass that are much larger than the length of the bar magnet?

Exercise 8: Plot a graph of $\tan\Theta$ as a function of $\frac{1}{R^3}$. Plot error bars associated with each data point. Do not use a computer for plotting your results. Do as Gauss probably did. From the slope of your graph, find $\frac{m}{B_H}$. Consult Appendix A. Find the error in the slope, and hence the error associated with $\frac{m}{B_H}$.

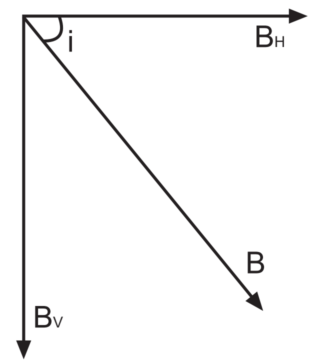

Figure 6: Dip angle

Exercise 9: Starting from the expressions for mB_H and $\frac{m}{B_H}$, eliminate m to derive the general expression,

$$B_H = 2\pi\sqrt{\frac{2I_M k_m}{T^2 R^3 \tan\theta}} \tag{7}$$

Show that the unit Tesla is equivalent to $\frac{N}{A.m}$.

Exercise 10: Combining the values of mB_H and $\frac{m}{B_H}$, calculate B_H. Propagate the errors in all your measurements and estimate the error in B_H.

The magnetic inclination angle i, (also called the dip angle) is defined as the angle measured from the horizontal plane down to the direction of the magnetic field vector (due to the earth's field) as shown in figure 6. In Toronto, the dip angle is about 71°.

The total magnetic field in Toronto is about 0.56 Gauss ($1G = 10^{-4}T$). Verify this by using the magnetic field sensor, which is a conventional method of measuring magnetic fields. The sensor is a probe based on the classical Hall effect. When a current passes through the sensor, the magnetic field causes a force on the charge carriers. This results in the accumulation of excess charge on one side of the sensor. This effect gives rise to a potential difference across the sensor. This voltage is proportional to the magnetic field.

Make sure that the power supply for the sensor is turned on. You must position the sensor perpendicular to the magnetic field to be measured. Measure the sensor voltage on a digital voltmeter. Then shield the sensor with the end cap made of mu-metal and measure the offset voltage on the voltmeter. Subtract the value of the offset from your measurement. The calibration of the sensor is marked on the sensor's controller. Use this information to convert your measurement into magnetic field units (make sure you use the correct calibration for the amplification you are using).

Exercise 11a: What is the value of the total magnetic field in Toronto measured by the sensor?

Exercise 11b: How does the measured value compare with the accepted value of 0.56G?

Exercise 12a: Calculate the value for the horizontal component of the earth's magnetic field in Toronto using the (accepted) total magnetic field and the dip angle.

Exercise 12b: How does this value compare with your measurement of B_H? Does the accepted value fall within the limits specified by the experimental uncertainty?

Exercise 13: The earth's magnetic field can be described as the magnetic field of a dipole. However, throughout the experiment, it was assumed to be uniform. How can you justify this assumption?

Your lab report should include:

Answers to exercises 1-13 with relevant data tables, graphs, figures and qualitative comments.

Do not use the Maple worksheets for your graphs.