Lab 9: Capacitors and Inductors - behavior of RC circuits and RL circuits

1 Introduction

An RC circuit contains a resistor and a capacitor. Similarly, a circuit with a resistor and an inductor is an RL circuit. When these circuits are connected to a DC power supply (such as a battery), the current through the circuits and the potential difference between the terminals of the circuit elements will vary with time. During this experiment, you will study the basic behavior of an RC circuit and an RL circuit. In particular, you will study how the voltage across the plates of a capacitor (also known as a condenser) changes with time, first when the capacitor is being charged, and then when it is being discharged. You will measure the time constant associated with this change in voltage, and use it to understand the rules for how capacitors combine in series and parallel circuits. Finally, you will study how the potential difference across the terminals of an inductor changes with time when an RL circuit is connected to or disconnected from a power supply.

EXERCISES 1-12 PERTAIN TO THE BACKGROUND CONCEPTS AND EXERCISES 13-18 PERTAIN TO THE EXPERIMENTAL SECTIONS.

2 Background

Consider the circuit shown in figure 1 in which a power supply, with a potential difference V between its terminals, is connected across a resistor R. Assume that the switch has been open for a long time.

Exercise 1a: What is the electric potential at points A, B, C, D, E and F?

Exercise 1b: What is the potential across the resistor (between points D and E)?

Exercise 1c: Immediately after the switch is closed, what is the electric potential at points A, B, C, D, E and F?

Exercise 1d: What is the electric potential difference across the resistor?

Exercise 1e: A long time after the switch has been closed, what is the electric potential at points A, B, C, D, E and F? What is the electric potential difference across the resistor?

Now check your answers with your TA before proceeding further.

A parallel plate capacitor is easy to understand. It is made up of two metal plates separated by a small air gap. When the plates are connected to a battery, opposite charges build up on the plates. The amount of charge that can be stored is directly proportional to the potential difference between the plates of the capacitor. The capacitance C (farads) is the proportionality constant relating the charge Q (coulombs) stored on the plates, and voltage V_0 (volts) between the plates. This relationship is given by,

$$Q = CV_0 \tag{1}$$

After the capacitor is "charged up" (a long time after the switch is closed) no current flows through the circuit. The voltage drop across R (recall $V = IR$) is zero and the potential difference between the plates of the capacitor is equal to the potential difference between the plates of the battery. The capacitor is now fully charged.

Now consider the circuit shown in figure 2 in which the battery (voltage V_0 between terminals) is connected in series with a resistor R and a capacitor C through a switch. Assume that that there is initially no potential difference between the plates of the capacitor. Assume that the switch has been open for a long time.

Exercise 2a: What is the voltage across the plates of the capacitor when the switch is open?

Exercise 2b: Same as 2a, but immediately after the switch is closed?

Exercise 2c: Same as 2a, but a long time after the switch is closed?

Exercise 2d: Based on your answers to parts 2a, 2b, and 2c, sketch a graph for the voltage across the plates of the capacitor as a function of time.

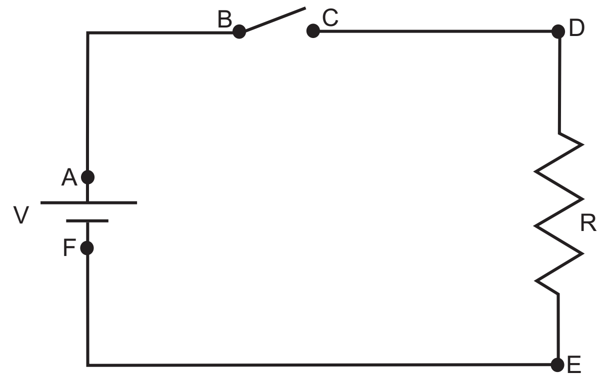

Figure 1: A simple resistor circuit

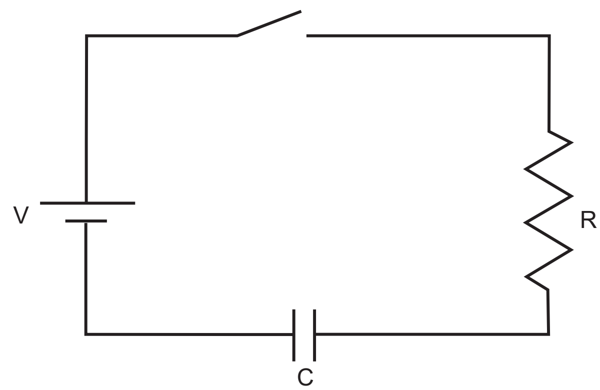

Figure 2: An RC circuit

Exercise 2e: Sketch a graph for the voltage across the resistor as a function of time.

Have your TA check your graphs before you proceed with the experiment.

Now consider the situation when the capacitor is fully charged with the battery V_0. Assume that the capacitor is disconnected from the battery and connected across a resistor through a switch. At $t = 0$, the switch is closed and the capacitor drives a current through the resistor.

Exercise 3a: What is voltage across the capacitor: i) When the switch is open? ii) Immediately after the switch is closed? iii) A long time after the switch is closed?

Exercise 3b: Based on your answers to exercise 3a, sketch a graph of the voltage across the capacitor as a function of time. Also, sketch a graph showing the voltage across the resistor as a function of time.

Have your TA check your graphs before you proceed.

We shall now discuss how the voltage across the plates of the capacitor varies with time.

After the switch is closed (figure 2), Kirchoff's voltage rule can be applied to the circuit. Assume that the instantaneous value of the current in the circuit is $I(t)$. Hence, we get,

$$V_0 - I(t)R - V_C(t) = 0 \tag{2}$$

Now recall that $V_C(t) = \frac{Q(t)}{C}$ and that $I = \frac{dQ}{dt}$. Hence, equation 2 becomes,

$$V_0 = R\frac{dQ}{dt} + \frac{Q(t)}{C} \tag{3}$$

Rearranging equation 3,

$$\frac{dQ}{dt} = -\frac{1}{RC}(Q(t) - CV_0) \tag{4}$$

Differentiating the following substitution $\beta(t) = (Q(t) - CV_0)$, we get $\frac{d\beta(t)}{dt} = \frac{dQ(t)}{dt}$. Therefore the solution to the differential equation becomes,

$$\beta(t) = \beta_0 e^{\frac{-t}{RC}} \tag{5}$$

Assume that the initial condition for the charge on the plates of the capacitor, is $Q(t = 0) = 0$. The solution to equation 5 can be shown to be,

$$Q(t) = CV_0(1 - e^{-\frac{t}{RC}}) \tag{6}$$

Equation 6 describes the time-dependent charge buildup on the plates of the capacitor. Using equation 1, we now write the time-dependent voltage across the plates of the capacitor as,

$$V(t) = V_0(1 - e^{-\frac{t}{RC}}) \tag{7}$$

Exercise 4a: Verify that equation 7 makes sense by applying limits $t = 0$ and $t = \infty$. How does the graph of $V(t)$ predicted by equation 7 compare with your graph in question 2?

Exercise 4b: What are the values of V for t = RC, 2RC, 3RC and 5RC?

Exercise 4c: Verify that the quantity RC (which is called the time constant for the circuit) has the units of time.

Now again consider the situation when the capacitor is fully charged with the battery V_0. Assume that the capacitor is disconnected from the battery and connected across a resistor through a switch. At $t = 0$, the switch is closed and the capacitor drives a current through the resistor.

Applying Kirchoff's rule to this circuit we get,

$$\frac{Q(t)}{C} - IR = 0 \tag{8}$$

Knowing that the current is the rate at which charge decreases on the plates of the capacitor, $I = -\frac{dQ}{dt}$, we get,

$$\frac{dQ}{dt} = -\frac{Q}{RC} \tag{9}$$

The solution to equation 9 can be found by applying the initial condition $Q(t = 0) = Q_0$. The solution can be shown to be,

$$Q(t) = Q_0 e^{-\frac{t}{RC}} \tag{10}$$

Since $Q_0 = CV_0$, we can describe the potential difference across the plates of the capacitor by the following equation,

$$V(t) = V_0 e^{-\frac{t}{RC}} \tag{11}$$

Exercise 5a: Verify that equation 11 makes sense by applying limits $t = 0$ and $t = \infty$. How does the graph of $V(t)$ predicted by equation 11 compare with your graph in exercise 3?

Exercise 5b: What are the values of V for t = RC, 2RC, 3RC and 5RC?

Exercise 6a: Consider the RL circuit shown in figure 3. Assume that the switch is open, and that there is no current flow through the circuit. Immediately after the switch is connected to position A: i) What is the current through the circuit? ii) What is the potential difference across the inductor? iii) What is the potential difference across the resistor?

Exercise 6b: A long time after the switch is connected to A (figure 3): i) What is the current through the circuit? ii) What is the potential difference across the indutor? iii) What is the potential difference across the resistor?

Exercise 6c: Give reasons for your answers. Sketch a graph of the potential difference across the inductor as a function of time.

A long time after the current in the circuit has stabilized (with the switch in position A), the switch is thrown to position B.

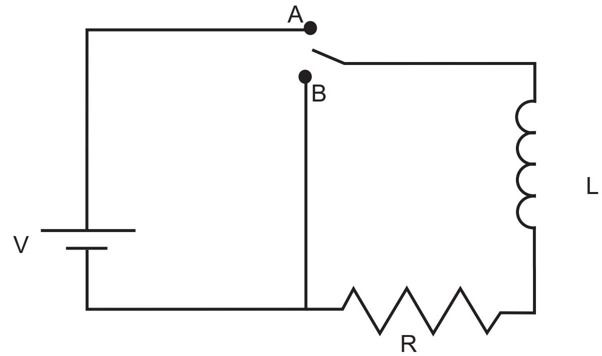

Figure 3: An RL circuit

Exercise 7a: Consider again the circuit shown in figure 3. Immediately after the switch is closed: i) What is the current through the circuit? ii) What is the potential difference across the inductor? Give reasons for your answers.

Exercise 7b: A long time after the switch is in position B (figure 3): i) What is the current through the circuit? ii) What is the potential difference across the inductor? Give reasons for your answers.

Consult the TA and verify that your predictions are valid before proceeding further.

We shall now discuss how the voltage across the inductor varies with time.

After the switch is connected to position A, Kirchoff's voltage rule can be applied to the circuit. Assume that the instantaneous value of current in the circuit is $I(t)$ keeping in mind that $L(\frac{dI}{dt})$ is the absolute value of the induced emf in an inductor. Hence we get,

$$V_0 - I(t)R - L(\frac{dI}{dt}) = 0 \tag{12}$$

Exercise 8: Draw the circuit and show which terminal of the inductor is at a higher potential.

Rearranging equation 12 we get,

$$\frac{dI}{dt} = -\frac{R}{L}(I(t) - \frac{V_0}{R}) \tag{13}$$

Using the substitution,

$$\beta(t) = I(t) - \frac{V_0}{R} \tag{14}$$

differentiating equation 14 and plugging it into equation 13 we get,

$$\frac{d\beta}{dt} = -\frac{R}{L}\beta(t) \tag{15}$$

After solving the differential equation, the solution can be found by applying the initial condition $\beta(t = 0) = \beta_0$ and plugging back equation 14,

$$I(t) = \frac{V_0}{R}(1 - e^{-\frac{t}{\tau}}) \tag{16}$$

where the time constant $\tau = \frac{L}{R}$.

Exercise 9: Draw $I(t)$ versus t, showing the initial $(t = 0)$ and $(t = \infty)$ current values.

Multiplying equation 16 by L and differentiating it we get,

$$L\frac{dI}{dt} = V_0 e^{-\frac{t}{\tau}} \tag{17}$$

Which can be written as,

$$V_L = V_0 e^{-\frac{t}{\tau}} \tag{18}$$

Equation 18 describes the potential difference across the terminals of the inductor as a function of time when the inductor is connected to a power supply. In this case the inductor prevents the discontinuous change of current, due to the induced emf.

Exercise 10: Draw a graph of V_L versus t, showing initial and final values.

Now consider the situation when the circuit is connected to position B. Using Kirchoff's rule and since $\frac{dI}{dt} < 0$ we get,

$$-L\frac{dI}{dt} - I(t)R = 0 \tag{19}$$

Exercise 11: Draw the circuit and show which one of the inductor's terminals is at a higher potential, immediately after the switch is connected to B.

Rearranging equation 19 we get,

$$L\frac{dI}{dt} = -I(t)R \tag{20}$$

Using initial conditions, this results in,

$$I(t) = \frac{V_0}{R} e^{-\frac{t}{\tau}} \tag{21}$$

Differentiating equation 21 and multiplying it by L we get,

$$L\frac{dI}{dt} = -V_0 e^{-\frac{t}{\tau}} \tag{22}$$

where the time constant $\tau = \frac{L}{R}$.

In this case the inductor prevents the discontinuous change of current, by reducing it gradually after the power supply is abruptly disconnected.

Exercise 12a: Graph the potential drop across the inductor as a function of time, marking the time constant on your sketch.

Exercise 12b: Does your theoretical prediction agree with your expectations (exercise 6 and exercise 7)?

3 Suggested Reading

Refer to the chapters on DC Circuits, Inductance and AC Circuits,

R. Wolfson and J. Pasachoff, **Physics with Modern Physics** (3rd Edition, Addison-Wesley Longman, Don Mills ON, 1999)

D. Halliday, R. Resnick and K. S. Krane, **Physics** (Volume 2, 5th Edition, John Wiley, 2002)

4 Apparatus

Refer to Appendix E for photos of the apparatus

- 100 mH inductor
- 1 MΩ resistor trimpot
- Function generator
- 100 kΩ resistor
- 1 kΩ resistor
- 0.002 μF capacitors
- Oscilloscope
- Digital multimeter
- Banana cables
- Alligator clips
- Coaxial cables with BNC connectors on one end and banana connectors on other end
- Plexiglass circuit board with binding posts

5 Experiment I: Potential difference between the terminals of a resistor

You can use a function generator and an oscilloscope to test your answers. The terminals of the resistor can be connected to an oscilloscope to understand how the potential difference changes with time. Oscilloscopes are preferred to voltmeters because voltages in typical circuits often change on very short time scales ($\sim 10^{-6}$ seconds). Therefore, the measuring instrument should be sufficiently fast to measure, record, and analyze the change in voltage. The function generator can output a voltage that is a particularly simple function of time. In the “square pulse mode” the output is either zero or non-zero. The amplitude and frequency of the output can be adjusted easily.

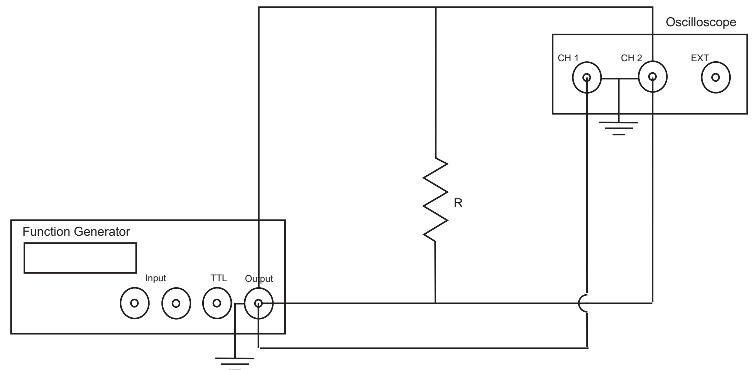

Figure 4: Experimental setup for resistor circuit

R (Ω)	C/L (F)/(H)	$\tau_{expr.}$	$\tau_{calc.}$	$\tau_{diffr.}$

Table 1: Table template for experiments 2 & 4 with τ experimental, calculated and difference.

Now construct the circuit shown in figure 4 using an oscilloscope connected across the resistor, and the function generator serving as the power supply. Assemble the circuit on the circuit board. Choose a resistor and before connecting it to the circuit, check its resistance with a digital ohmmeter. Use a tee connector and a BNC coaxial cable to connect the output of the waveform generator directly to channel 1 of the oscilloscope. The other arm of the tee connector can be connected to the resistor using a suitable cable.

The central pin of the generator’s output connector is connected to the central pin of the connector on the oscilloscope via the inner connector of the coaxial cable. The outer connector (shield) of the cable provides the return pathway to complete the circuit. It connects the grounded outer connector on the generator to the grounded outer connector on the oscilloscope.

Set the frequency of the generator to 100 Hz. Set the trigger source on the scope to channel 1. Trigger the scope by adjusting the trigger level knob until you see the waveform. If you have trouble triggering the scope, consult your TA. Measure the frequency of the waveform using the oscilloscope and check how this compares with the setting on the generator.

Now connect the terminals of the resistor to channel 2 of the oscilloscope and observe how the potential difference across the terminals of the resistor changes with time. Observe the waveform on a suitable time base of the oscilloscope.

Exercise 13: Carefully, sketch the two waveforms that you see on the scope. Mark the period on the sketch and explain your results. Are your expectations for the answers to exercise 1 verified? Explain why or why not.

6 Experiment II: Time constants for RC circuits

Verify that you are using a resistor with R = 100 kΩ using the digital multi-meter. The capacitor should have a value C = 0.002 μF. Connect the series circuit as shown in figure 5 on the circuit board. Set the generator to the “square pulse mode”. Connect channel 2 of the oscilloscope to the terminals of the capacitor. Connect the output of the waveform generator (using a tee connector) directly to channel 1 of the scope.

Ensure that the ground of the oscilloscope and the ground of the function generator are connected to THE GROUND TERMINAL OF THE CAPACITOR. Trigger the oscilloscope, using channel 1 as the trigger source. Adjust the frequency of the generator so that the waveform has a period that is much longer than the expected value of the time constant.

For experiments 2 and 4 use Table 6 to record your results for the time constants and their uncertainties.

Exercise 14: Measure the time constant of the signal (displayed on channel 2) when the capacitor is charged and discharged. Consult your TA and discuss

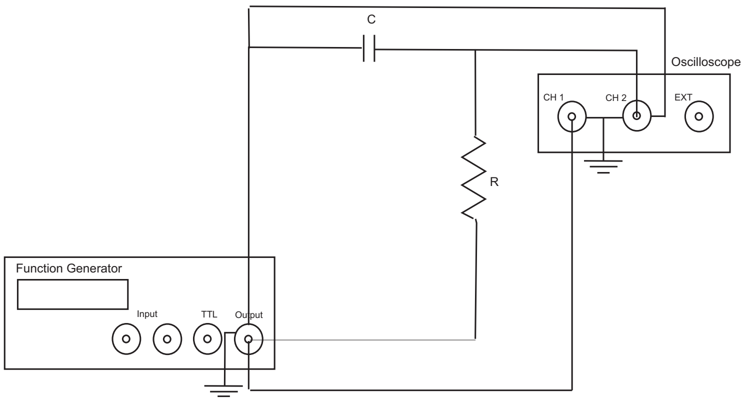

Figure 5: Experimental setup for an RC circuit

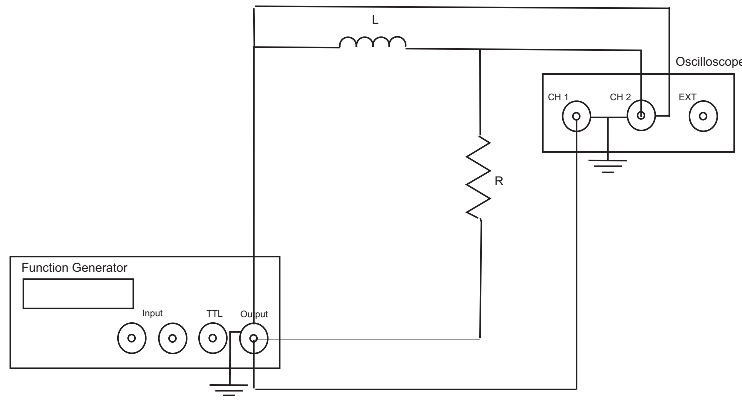

Figure 6: Experimental setup for an RL circuit

your measurement. Record your results in the form of a table. Compare the time constants associated with the charging and discharging of the capacitor. Compare the measured time constants with the expected values. Find out the uncertainties in the values of R and C used in the experiment. Does the measured value fall within this uncertainty?

Exercise 15a: Vary the resistance or the capacitance and check if the time constant changes in the expected manner. Tabulate your data.

Exercise 15b: Draw a sketch of the signal. Mark the time constant on the figure. Does the shape of the signal correspond to the theoretical predictions of equation 7 and equation 11?

7 Experiment III: Combination of capacitors

When two capacitors C_1 and C_2 are combined in series as in figure 7a, it is possible to show that the effective capacitance C_{eff} is given by,

$$\frac{1}{C_{eff}} = \frac{1}{C_1} + \frac{1}{C_2} \tag{23}$$

When the capacitors are combined in parallel as in figure 7b, the effective capacitance of the circuit is,

$$C_{eff} = C_1 + C_2 \tag{24}$$

NOTE: Make sure that you use identical capacitors for this experiment.

Exercise 16: Use one of the capacitors in the RC circuit shown in figure 5 and record the time constant.

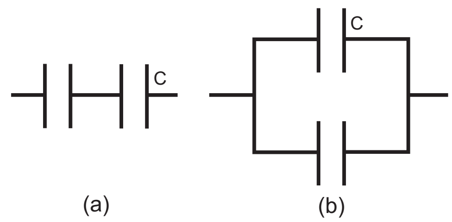

Figure 7: Capacitors in series (a) and in parallel (b)

Replace the single capacitor by the capacitor combinations shown in figure 7a and figure 7b respectively. Record the time constant in each case. Tabulate your results.

Exercise 17: Compare the measured time constants to the corresponding expected values. Do your measurements agree with the predictions of equation 23 and equation 24?

8 Experiment IV: Time constants for RL circuits

Connect the circuit shown in figure 6.

Exercise 18: Test your predictions for the potential difference across the terminals of the inductor. Use a 100 mH inductor and a suitable resistor (1 kΩ). Sketch a graph of your observations. Measure the time constant. Compare the experimental value of the time constant with the calculated value.

Your lab report should include:

Answers to exercises 1-18 with relevant data tables, graphs, figures and qualitative comments.

Refer to Appendix D for Maple worksheets.