Lab 10: Spatial Profile of a Laser Beam

1 Introduction

Refer to Appendix D for photos of the apparatus

A laser beam can be characterized by measuring its spatial intensity profile at points perpendicular to its direction of propagation. The spatial intensity profile is the variation of intensity as a function of distance from the center of the beam, in a plane perpendicular to its direction of propagation. It is often convenient to think of a light wave as being an infinite plane electromagnetic wave. Such a wave propagating along (say) the z-axis will have its electric field uniformly distributed in the x-y plane. This implies that the spatial intensity profile of such a light source will be uniform as well. However, in reality, a light beam does not extend to infinity. Therefore, this picture of a light beam as an infinite plane wave is just an idealization.

In this experiment, we will investigate the spatial profile of a Gaussian laser beam. The shape of a laser cavity is responsible for the intensity profile. A Gaussian profile is characteristic of most lasers. We will discuss some properties of such Gaussian beams. In particular, we will concentrate on the fact that a Gaussian beam has a non-uniform intensity profile. Furthermore, by studying the profile of a laser beam, we are able to extract a characteristic radius of the Gaussian beam, known as the spot size, at a particular location along the direction of propagation of the beam.

The experiment consists of two parts. In the first part, you will verify that the laser beam has a Gaussian intensity profile. You will use this profile to extract a value for the spot size. In the second part of the experiment, you will study an alternative technique for measuring spot size. This technique is suitable for laser beams that have large diameters.

EXERCISES 3-4 PERTAIN TO THE BACKGROUND CONCEPTS AND EXERCISES 1-2 AND 5 PERTAIN TO THE EXPERIMENTAL SECTIONS.

2 Background

We will deal with a special class of Gaussian beams for which the transverse electric field can be written as,

$$E(r) = E_0 e^{-r^2/w^2}$$  \hspace{1cm} (1)

Equation 1 is expressed in plane polar coordinates where $r$ is the radial distance from the center of the beam (recall $r^2 = x^2 + y^2$) and $w$ is a parameter which is called the spot size of the Gaussian beam (it is also referred to as the Gaussian beam radius). It is assumed that the transverse direction is the x-y plane, perpendicular to the direction of propagation $(z)$ of the beam.

Since we typically measure the intensity of a beam rather than the electric field, it is more useful to recast equation 1. The intensity $I$ of the beam is related to $E$ by the following equation,

$$I = \frac{\varepsilon_0 c E^2}{2}$$  \hspace{1cm} (2)

Here, $\varepsilon_0 = 8.85 \times 10^{-12} C^2/Nm^2$ is the permittivity of free space and $c = 3.00 \times 10^8 m/s$ is the speed of light. Substituting equation 1 into equation 2 we get,

$$I(r) = I_0 e^{-2r^2/w^2}$$  \hspace{1cm} (3)

Here, $I_0 = \frac{\varepsilon_0 c E_0^2}{2}$ is the (maximum) intensity on the z-axis ($r = 0$). Equation 3 is similar to equation 1; it represents a Gaussian function centered at $r = 0$.

Figure 1 is a plot of the Gaussian beam intensity profile given by equation 3. Notice that the Gaussian is a radially symmetric distribution.

If we set $r = w$ in equation 3, we obtain $I = I_0/e^2$. This means that at a distance $w$ from the z axis, the intensity of light is reduced to $1/e^2$ of its maximum value. If we put a small screen perpendicular to the z-axis at that position, we would observe a spot of radius $\sim w$.

The spot size is a function of position along the direction of propagation $z$ of the Gaussian beam. It is given by,

$$w(z) = w_o \sqrt{1 + \left(\frac{\lambda z}{\pi w_o^2}\right)^2}$$  \hspace{1cm} (4)

where $\lambda$ is the wavelength of the light. Notice that $w_o$ is the minimum spot size which occurs at the particular position $z = 0$. The size of the beam at the
Figure 1: Gaussian intensity profile of a laser beam/determination of the spot size.

$z = 0$ position is called the **beam waist**. In practice, $z = z_o$ will often correspond to the position of a focal spot of a lens introduced in the laser beam.

Figure 2 illustrates the variation of $w$ as the beam propagates along the $z$-axis. It also shows that the radius of curvature of the wavefronts varies along the axis of propagation.

Keep in mind that although the spot size and the radius of curvature vary along the $z$-axis, the intensity profile remains Gaussian at every position along the direction of propagation. The underlying reason is that Gaussian beams have a unique property that they maintain their Gaussian form along their propagation direction. This is an important characteristic of Gaussian beams, in contrast to most ordinary sources of light which cannot maintain a specific profile along the direction of propagation. For instance, if you shine a flashlight through a circular aperture, its spatial profile is uniform close to the aperture. At distances far away from the aperture, the beam profile changes because of diffraction. Another property of Gaussian beams is that they remain Gaussian as they propagate through any combination of optical elements such as lenses and mirrors.

The unique properties of Gaussian beams that we have been discussing so far provide the motivation to investigate them. A light source which naturally produces Gaussian beams is a Laser. It is instructive to briefly mention how the laser operates, to gain insight as to how it produces Gaussian beams. A Laser consists of a “Lasing medium” which is located between two spherical mirrors, one of which can partially transmit the light produced in the cavity. Under special conditions a process known as stimulated emission can be induced inside the cavity. This causes further amplification of light. The amplified electric field has to satisfy the boundary conditions imposed by the cavity walls. This determines the number of transverse (spatial) **modes** that the cavity can support. A good Laser cavity allows only the fundamental (or TEM$_{00}$) mode to be amplified. This mode has a Gaussian spatial profile. The laser that is being used in this lab produces a number of spatial modes. Equation 3 is therefore only an approximation of the spatial profile of the laser.

Finally, notice that Gaussian beams exhibit diffraction (even without a lens!). As you can see in Figure 2, the beam is diverging, as if it is diffracting from the minimum spot size $w_o$. In fact, this imposes a limitation on our ability to focus a laser beam. It is not possible to focus laser light to a single point because it is diffraction-limited by the very fact that it is Gaussian in nature.

3 Suggested Reading

Refer to the chapters 14-17,


4 Apparatus

- Laser: N-type semiconductor diode laser (635nm wavelength)
- Translation stage and pinhole
• Photodiode
• Box for covering the photodiode
• Digital multimeter
• Two mirrors and mirror holders
• Three convex lenses with focal lengths \(\sim 8\) cm, \(\sim 15\) cm and \(\sim 30\) cm
• Three lens holders
• Large iris
• Cylindrical rod machined with variable diameters
• 90 degrees clamp
• Five 3” postholders and six 2” postholders
• Ten posts and eleven bases
• Screws, bolts, 3/16” Allen key, cardboards, transparent lens tissue
• Optical table

WARNING!!: KEEP TRACK OF YOUR LASER BEAM AT ALL TIMES. NEVER POINT THE BEAM AT PEOPLE, OR LOOK IN THE APERTURE OF THE LASER OR BE AT EYELEVEL WITH THE BEAM.

KEEP EYES AWAY FROM DIRECT OR REFLECTED LASER BEAMS. OTHERWISE SERIOUS EYE DAMAGE WILL OCCUR.

YOU SHOULD BE AWARE OF WHERE THE LASER BEAMS STRIKE OPTICAL COMPONENTS. REFLECTIONS FROM OPTICAL COMPONENTS SHOULD BE BLOCKED BY USING PIECES OF CARDBOARD THAT ARE PROVIDED. BE PARTICULARLY CAREFUL WHEN YOU INSERT OR REMOVE LENSES INTO A LASER BEAM.

DO NOT TOUCH THE OPTICAL SURFACES OF LENSES AND MIRRORS. IF THE SURFACES ARE UNCLEAN, PLEASE BRING IT TO THE ATTENTION OF THE TA IMMEDIATELY.

USE THE TRANSPARENT LENS TISSUES TO DETECT THE BEAMS.

Figure 3: Schematic diagram of the setup for experiment 1.

MAKE SURE ALL MOUNTS ARE SECURELY FASTENED ON THE OPTICAL TABLE.

5 Experiment I: Gaussian profile of a laser beam - measurement of the spot size

The purpose of this part of the experiment is to verify that the transverse beam intensity for the laser has a Gaussian profile. Once you measure the intensity profile at a particular position, you will be able to determine the spot size of the laser beam at that position.

The experimental setup is shown in Figure 3. The laser beam is reflected from two mirrors so that you can conveniently manipulate the beam by adjusting the mirrors. The beam then passes through a translation stage. In order to measure the profile of the beam, we need to measure the intensity at various points on the beam’s cross-section. This can be done by scanning a pinhole attached to a translation stage across the beam’s cross-sectional area, and measuring the light transmitted through the pinhole. Ideally, one must measure the intensity at every point in this area. But that is definitely time-consuming. For our purposes, it is enough to assume that the pinhole has radial symmetry and to scan the pinhole in the horizontal direction only. The pinhole can be moved vertically and horizontally by using the graduated micrometer screws on the translation stage.

The size of the pinhole to be used in conducting this experiment is very important. If the size of the pinhole is small compared to the spot size to be mea-
sured, the laser beam emerging from the pinhole will be strongly diffracted. This will make it difficult to measure the transmitted signal. You will be using a 100µm pinhole.

After passing through the pinhole, light will hit a photodiode which must be placed as close to the pinhole as possible. A major challenge in performing measurements with the photodiode is that it produces a weak signal. Another challenge is the noise introduced by background light hitting the photodiode. Unless the experiment is performed in the dark, the signal-to-noise ratio will be very small. For this reason, you must cover the photodiode by a box and make sure that stray light hitting the detector is minimized.

It is worth noting that photodiode does not measure light intensity. It measures light power. The voltage reading is proportional to the photocurrent it produces (V=IR). But we know that power = current*volts, therefore the voltmeter reading is proportional to the power. On the other hand, intensity (= power/area) depends on the area of the beam incident on the photodiode.

For example, consider a beam with a diameter of 2cm incident on a photodiode. Suppose the reading of the voltmeter is V. Now we expand the same beam to a diameter of 4cm. The voltmeter reading will be V again, since it is proportional to the total light power which obviously does not change by expanding the beam. It is the power per unit area, i.e. the intensity of the beam that changes by expanding the beam. (In this case it is reduced by a factor of 4). Therefore, the photodiode voltage is not necessarily proportional to the intensity of light unless you take all your data without changing the area of the incident beam. This is precisely what you need to do in this part of the experiment. The pinhole has a fixed area and thus the voltmeter readings will be proportional to the light intensity.

Setup the experiment as shown in Figure 3.

Use a clamp to fix the laser horizontally. Check that the beam is horizontal by marking the height of the beam on a piece of cardboard. Adjust the mirrors so that the beam strikes the same mark when the cardboard is placed at a large distance away. Also make sure that the beam is centered at the pinhole. Place the photodiode as close to the pinhole as possible. Verify that you see the transmitted spot with your naked eye and that the spot hits the photodiode. Use the box provided to reduce background light noise. Remember that for a BNC connector, the central pin carries the signal and the outer shield is the return (which is often connected to ground).

**Exercise 1:** Why should the photodiode be placed very close to the pinhole?

**Exercise 2a:** Adjust the micrometers to obtain the maximum signal. Then using the horizontal micrometer, scan the pinhole across the beam and record the voltmeter reading at each position. As you move the pinhole, make sure that only the part of the beam passing through the pinhole hits the photodiode. Take a number of data points across the whole width of the beam. Tabulate your results. Block the laser light and record the voltmeter reading. What is the maximum signal-to-noise ratio? Subtract this background signal from all your data points.

**Exercise 2b:** Plot the photodiode voltage as a function of micrometer position. Show error bars on your plot. This plot is the intensity profile of the laser beam. Comment on the shape of the curve. Does it convince you that the laser beam profile is Gaussian?

**Exercise 2c:** From your plot, determine the spot size of the beam at the position of the pinhole. The size of the pinhole you used is 100µm in diameter. Compare it to the value of the spot size you determined. Was the pinhole size appropriate for measuring the spot size of the laser at the particular position you chose? Discuss whether this has any impact on the accuracy of your result.

### 6 Experiment II: Power distribution within a laser beam - measurement of spot size of an expanded laser beam.

In Experiment 1, you determined the spot size of a laser beam from its intensity profile by scanning a pinhole across the beam. An alternative method for determining the spot size is to measure the power contained within a radius r of the beam. If the power is measured as a function of r, and the total power is determined, the spot size can be deduced. This technique is appropriate for determining the spot size of an expanded laser beam.
We know that the intensity distribution is given by equation 3, \( I(r) = I_o e^{-2r^2/w^2} \). Since the power \( P = I \times Area \), we can easily obtain an expression for the power \( P(r) \) contained within an arbitrary radius \( r \) of the beam, by integrating equation 3 from 0 to \( r \),

\[
P(r) = \int_0^r \int_0^{2\pi} I_o e^{-2r^2/w^2} d\theta dr
\]

(5)

where \( da = rdrd\theta \) is an infinitesimal area element in plane polar coordinates.

By evaluating equation 5, we obtain,

\[
P(r) = \frac{I_o \pi w^2}{2} [1 - e^{-2r^2/w^2}]
\]

(6)

In the limit as \( r \to \infty \), the exponential term of equation 6 goes to zero and the total power \( P(\infty) \) is given by,

\[
P(\infty) = \frac{I_o \pi w^2}{2}
\]

(7)

Note that this result makes physical sense.

Consequently, equation 6 can be written as,

\[
P(r) = P(\infty)[1 - e^{-2r^2/w^2}]
\]

(8)

Figure 4 shows a plot of the power distribution given by equation 8.

When \( r = w \), \( P(w) = P(\infty)[1 - 1/e^2] \approx 0.865P(\infty) \). It is therefore possible to infer \( w \) from the graph in Figure 4.

**Exercise 3:** Evaluate the integral in equation 5 and obtain the result in equation 6.

The purpose of this part of the experiment is to use the power distribution method to determine a spot size of the laser beam. The experimental setup is shown in Figure 5.

First, the beam has to be expanded so that we can easily measure the power within a wide range of radial distances. This can be achieved by constructing a beam expanding telescope. For constructing the telescope, use two convex lenses. The distance between the lenses should be equal to the sum of their focal lengths. Keep first convex lens fixed and adjust the position of the second convex lens until you observe that the expanded beam is properly collimated. A large iris diaphragm and a smaller diaphragm nested within it can be used to vary the radius of the beam incident on the photodiode. A cylindrical rod is machined at various accurately measured diameters. You can use it to vary the diameter of the aperture. The table placed at the end of the write up gives you the dimensions of the two cylindrical rods that will be used to define the diameters of the two apertures. A third convex lens is used to focus the beam on the photodiode. This ensures that all the light transmitted through the iris hits the photodiode.

**Note:** The beam diameter is maximized when the focal lengths of the telescope lenses are \( f_1 = 8 \) cm and \( f_3 = 30 \) cm. Use the lens with \( f_2 = 15 \) cm to focus the light on the detector.

**Exercise 4:** Using geometrical optics, show that in order for the beam emerging from the telescope to be collimated, the distance \( d \) between the concave and the convex lenses has to be equal to \( f_2-f_1 \). Here, \( f_2 \) is the focal length of the convex lens and \( f_1 \) is the absolute value of the focal length of the concave lens.

Setup the experiment as shown in Figure 5. Make sure that the laser beam is centered on the two nested irises.
Table 1: Dimensions of cylindrical rods

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<th>Large rod radii (mm)</th>
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**Exercise 5a:** Record voltmeter readings for various values of the iris radius $r$. Block the laser beam and record the voltmeter reading. Subtract this background signal from your data.

**Exercise 5b:** Plot a graph of the power distribution and show your error bars on the plot.

**Exercise 5c:** Determine the spot size of the laser beam. Does this spot size have the same value you determined in experiment 1? Explain why or why not.

**Exercise 5d:** Determine the power (in mW) within the spot size of the laser beam. Assume that the total power of the laser beam is 1mW.

**Exercise 5e:** Calculate the peak intensity $I_o$ of the laser beam. Notice from equation 7 that $I_o$ must be a function of $z$, since the spot size depends on $z$.

**Exercise 5f:** Calculate the magnitude $E_o$ of the electric field vector.

**Exercise 5g:** Calculate $I_o$ and $E_o$ again, now using the spot size you determined in the first part of the experiment.

**Note:** Once you have completed the experiment please remove all optical elements and the detector from their mounts and place them on the optical table.

**Your lab report should include:**

Answers to exercises 1-5 with relevant data tables, graphs, figures and qualitative comments.

Refer to Appendix C for Maple worksheets.