Lab 3: Lenses

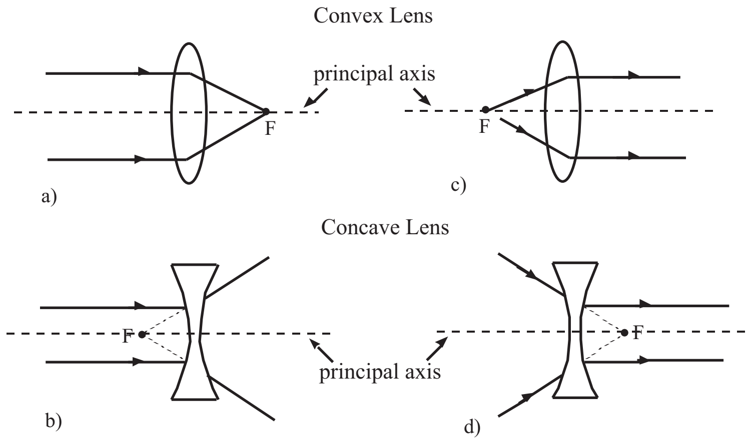

Figure 1: Properties of convex and concave lenses.

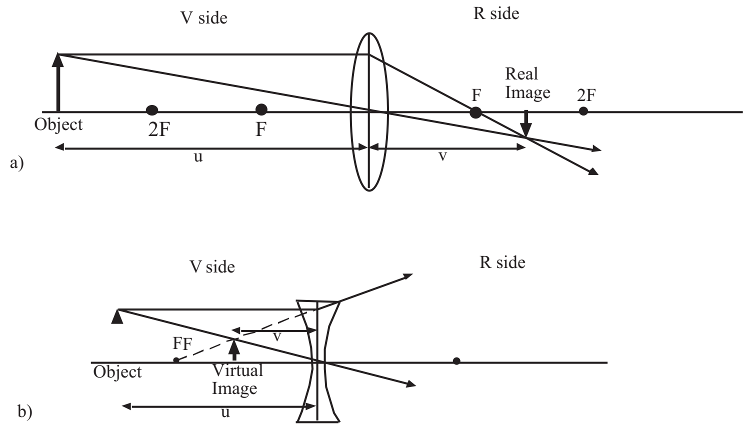

Figure 2: Image formation with a convex lens (a) and a concave lens (b).

1 Introduction

Refer to Appendix D for photos of the apparatus

In this lab, you will investigate different methods of finding the focal length of a convex lens. You will verify the predictions of the lens equation (for a thin lens) and understand the transverse magnification of a convex lens. You will then find the focal length of a concave lens based on your understanding of the properties of the convex lens. Finally, you will use a collimated light source to understand the properties of two types of beam expanding telescopes.

EXERCISE 1 PERTAINS TO THE BACKGROUND CONCEPTS AND EXERCISES 2-7 PERTAIN TO THE EXPERIMENTAL SECTIONS.

2 Background

The surfaces of the lenses that you will use are spherical. The lenses are made of glass (index of refraction ∼1.5). If the thickness of a lens is small compared to the object and image distances, you can derive a simple expression based on geometric considerations. This equation (called the lens equation) relates the **focal length** of the lens to the object and image distances.

The lenses that you will use are convex or concave. The behavior of these lenses is illustrated in figure 1.

In figure 1a, a collimated beam of light, which is parallel to the **principal axis** of a convex lens, is brought to a focus at a point F that lies on this axis. The distance between the center of the lens and the point F is known as the **focal length** of the lens (f). In figure 1b, a collimated beam of light (parallel to the principal axis) is incident on a concave lens. After passing through the lens it appears to diverge from the focal point F. The property of time reversal invariance is illustrated in figures 1c and 1d. If a point source of light is placed at the focus of a convex lens the rays that strike the lens will be rendered parallel. Similarly, if rays of light converge on the focal point of a concave lens, they will be rendered parallel (figure 1d). It should be clear from figure 1 that lenses have two focal points one on each side of the lens.

We can illustrate how lenses form images by using ray-tracing diagrams. In these diagrams we will use the property that a ray of light going through the center of the lens is not deviated. Figures 2a and 2b show how images are formed by a convex lens and a concave lens respectively. To construct these figures, it is enough to consider the behavior of two rays, one parallel to the principal axis of the lens, and the other passing through the center of the lens.

Figure 3 shows how the lens equation can be derived. Notice that triangles ABY and X'Y'Y in figure 3 are similar triangles. We can therefore define the magnification, M, which is the ratio of the transverse dimensions of the image and the object.

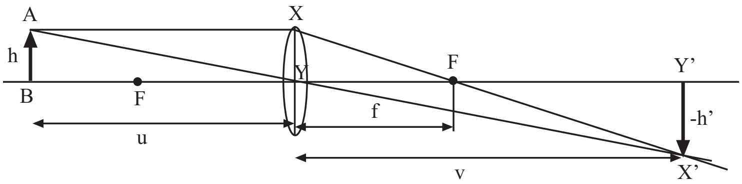

Figure 3: Deriving the magnification.

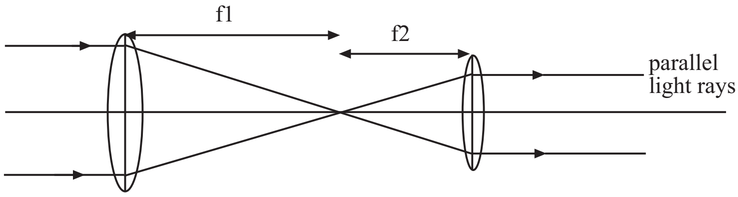

Figure 4: Ray diagram for a refracting telescope.

$$| M |= \frac{h'}{h} = \frac{v}{u} \tag{1}$$

Here, u and v are the object and image distances respectively, and h and h' represent the heights of the object and image respectively. Now consider similar triangles XYF and X'Y'F in figure 3. We can write,

$$\frac{h'}{v-f} = \frac{h}{f} \tag{2}$$

Rearranging equation 2 and using equation 1, we get the **lens equation** given below.

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \tag{3}$$

Here $\frac{1}{f} = (n-1)(\frac{1}{R_1} + \frac{1}{R_2})$ and R_1 and R_2 are the radii of curvature of the two lens surfaces as explained below. It is important to apply the correct sign conventions when we use equation 3.

Sign Conventions: We now list the sign conventions that should be followed.

1. Radii of curvature R_1, R_2 of the two surfaces of the lens are positive if the respective centers of curvature c_1 and c_2 are on the R side of the lens. If c_1 and c_2 are on the V side R_1 and R_2 are negative. Here R_1 is the radius of curvature of the surface that is first encountered by the light beam and R_2 is the radius of curvature of the second lens surface. For example in figure 2a and 2b, the lenses are symmetric and so the centers of curvature lie on either side of the lens. In figure 2a, R_1 is positive and R_2 is negative. Similarly in figure 2b, R_1 is negative and R_2 is positive.

2. The object distance u is positive for real objects and negative for virtual objects. For example the object distance is positive in both figure 2a and 2b.

3. The image distance v is positive for real images and negative for virtual images. For example the image is formed on the R side in figure 2a and v is positive. The image is formed on the V side in figure 2b and v is negative.

4. The magnification M is defined as $-\frac{v}{u}$. For example, in figure 3, $u = 2f$, $v = 2f$ and $M = -1$ (inverted image).

Exercise 1: Show the derivation of equation 3 using equations 1 and 2.

Lenses can also be combined in several ways to construct useful optical instruments. Figure 4 shows how two convex lenses of different focal lengths can be combined to form a telescope.

If the lenses are placed such that they are separated by a distance equal to the sum of their focal lengths, it is possible expand or contract the size of an incident beam. It can be shown that magnification, m, of the telescope is given by the following equation,

$$m = \frac{f2}{f1} \tag{4}$$

Here $f1$ and $f2$ are the focal lengths of the first and second lenses respectively that are encountered by the laser beam. For example figure 4 shows a beam-contracting telescope for which $m < 1$.

3 Suggested Reading

Refer to the chapter on Lenses,

D. Halliday, R. Resnick and K. S. Krane, **Physics** (Volume 2, 5th Edition, John Wiley, 2002)

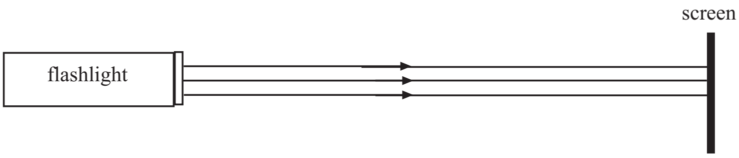

Figure 5: Expected flashlight beam with no mask placed in front.

4 Apparatus

- He-Ne laser
- Light Source (flashlight)
- 3-convex lenses ($f_A = +5\text{cm}$, $f_B = +10\text{cm}$, $f_C = +30\text{cm}$)
- 1-concave lens ($f_D = -10\text{cm}$)
- 12V, 3A Power Supply
- Screen
- 4-post holders
- 1-meter stick
- '+' shaped mask
- Optical bench
- 5/32" Allen key

Collimating the Light Source (flashlight)

The flashlight consists of a light bulb (point source) placed inside a shielded enclosure. A concave lens is in the enclosure and collimates the light source.

In the lenses experiments, a mask (shaped like +) is placed over the aperture of the flashlight. Light from the flashlight is scattered by the mask and the scattered light is imaged by a system of one of more lenses. Without an object being placed in front of the beam the light emerging from the flashlight would be collimated as shown in figure 5.

5 Experiment I: Determining the focal length of a convex lens

We will henceforth refer to the lenses listed in the apparatus as lenses A, B, C and D.

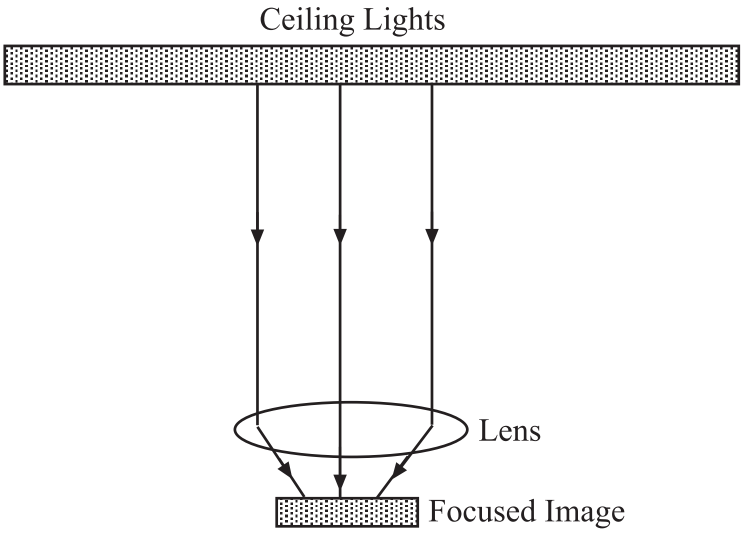

Figure 6: Finding the focal length.

A fluorescent panel of ceiling lights is an example of a two-dimensional light source. You may be aware that the intensity of light from such sources is uniform. To a good approximation we can assume that light from such a distant source is a plane wave. Hold lens A close to the floor and slowly vary its position until you can observe a focused image of the ceiling lights on the floor (see figure 6). If the collimated light is incident on the lens, it is clear that the image distance will be equal to the focal length. You can therefore measure the image distance from the location of the lens and infer its focal length. Repeat this procedure for lenses B and C.

Exercise 2: Compare the measured values of the focal lengths with the values of the focal lengths on the lenses. Tabulate your results. Estimate the errors associated with each technique.

6 Experiment II: Testing the lens equation (equation 3)

Setup the optical components as shown in figure 7. The '+' shaped mask, is attached to the front of the flashlight. It will serve as the object for the lens. Measure the height, h, of the object attached to the flashlight.

Place lens B more than one focal length away from the flashlight. Next, place the screen close to the lens and slide it away until you can observe the image of the mask. It is hard to observe where the image is best fo-

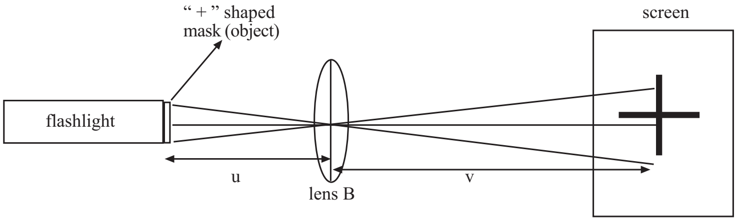

Figure 7: Verifying the focal length.

cussed. We suggest that you move the screen past the point of focus and then slowly move the screen backward until the image is clearly resolved. It is easier to find the correct position of the image by observing the edges of the image. Next, measure the object and image distances u and v respectively. Measure the height of the image, h'. Repeat these measurements for various object distances up to u 6f. Tabulate your results along with the uncertainties

Exercise 3a: You can plot your results in many ways to infer the value of the focal length. Plot a graph of $\frac{1}{v}$ vs. $\frac{1}{u}$. Interpret your graph and explain how you can extract the focal length.

Exercise 3b: Based on figure 2, construct a ray diagram for a convex lens when the object is placed a distance 2f from the lens.

Exercise 3c: Now plot $(u + v)$ vs. u. Interpret your graph and find the focal length. Consult a TA or course instructor to understand the suggested plot.

Exercise 3d: Compare your results for the focal length from parts 3a and 3c.

Exercise 4: Plot a graph of the transverse magnification $\frac{h'}{h}$ as a function of u. Explain the graph.

7 Experiment III: Finding the focal length of a concave lens

A collimated beam of light appears to diverge from the focus of a concave lens. As a result, we will use a convex lens in combination with the concave lens to find its focal length. Setup the apparatus as shown in figure 8. Place the '+' shaped mask in front of the flashlight as in experiment 2.

Place a convex lens (lens B) in front of the light source. Measure the object distance and the image distance.

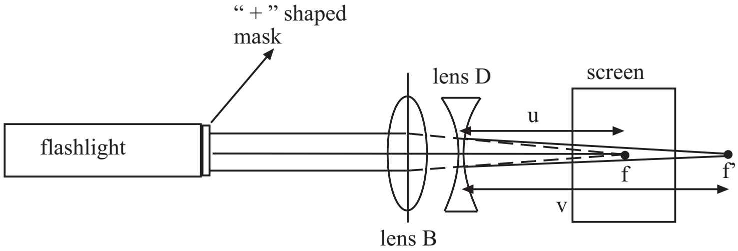

Figure 8: Finding the focal length of a concave lens.

Now introduce the concave lens (lens D) between lens B and the screen. The image formed by lens B acts as a virtual object for the concave lens (lens D). Measure the distance between the lenses B and D and calculate the virtual object distance for lens D. Now, move the screen to the new focus created by the lens system. Measure the image distance for lens D (distance between the concave lens D and the screen).

Exercise 5: Calculate the focal length of the concave lens using equation 3. How does your result compare with the given focal length?

8 Experiment IV: Beam expanding telescopes

WARNING!!: KEEP TRACK OF YOUR LASER BEAM AT ALL TIMES. NEVER POINT THE BEAM AT PEOPLE, OR LOOK IN THE APERTURE OF THE LASER OR BE AT EYELEVEL WITH THE BEAM.

KEEP EYES AWAY FROM DIRECT OR REFLECTED LASER BEAMS. OTHERWISE SERIOUS EYE DAMAGE WILL OCCUR.

YOU SHOULD BE AWARE OF WHERE THE LASER BEAMS STRIKE OPTICAL COMPONENTS. REFLECTIONS FROM OPTICAL COMPONENTS SHOULD BE BLOCKED BY USING PIECES OF CARDBOARD THAT ARE PROVIDED. BE PARTICULARLY CAREFUL WHEN YOU INSERT OR REMOVE LENSES INTO A LASER BEAM.

DO NOT TOUCH THE OPTICAL SURFACES OF LENSES AND MIRRORS. IF THE SURFACES ARE UNCLEAN, PLEASE BRING IT TO THE ATTENTION OF THE

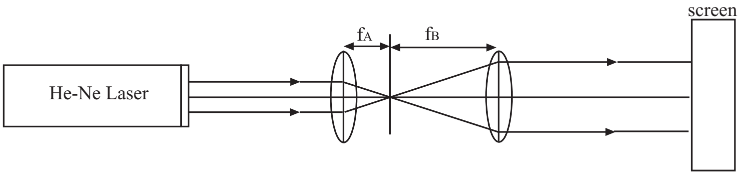

Figure 9: Setup of a refracting telescope.

TA IMMEDIATELY.

USE THE TRANSPARENT LENS TISSUES TO DETECT THE BEAMS.

MAKE SURE ALL MOUNTS ARE SECURELY FASTENED ON THE OPTICAL TABLE.

You will now use a combination of lenses and the **He-Ne laser** to demonstrate two types of beam-expanding telescopes.

Replace the illuminator with the He-Ne laser. Place a piece of graph paper on the screen and measure the diameter of the laser beam on the screen. Next, place lens A in front of the light source at a distance greater than its focal length and note the location of the focused beam. Now place lens B at a distance of approximately f_B from the focussed beam produced by lens A (figure 9). Adjust lens B until the outgoing laser beam is collimated. Measure the diameter of the collimated beam and the distance between the lenses. Switch the order of the lenses and repeat your measurements.

Exercise 6a: For each lens configuration, calculate the ratio of beam sizes and compare it to the value of the magnification using equation 4. Compare the distance between the lenses to the sum of their focal lengths.

Exercise 6b: What happens to the outgoing beam size when the order of the lenses is switched? Illustrate your answer using a ray diagram.

9 Experiment V: Beam expanding telescope, using a concave lens.

Demonstrate a beam-expanding telescope using a concave lens, a convex lens and the He-Ne laser. Measure the incident and outgoing beam sizes and the distance of separation between the lenses.

Exercise 7a: Compare the input and output beam sizes to the theoretical value of the magnification.

Exercise 7b: Compare the distance between the lenses to the sum of their focal lengths.

Exercise 7c: Draw a ray diagram to illustrate how this telescope works.

Note: Once you have completed the experiment, please remove all optical elements from their mounts and place them on the table.

Your lab report should include:

Answers to exercises 1-7 with relevant data tables, graphs, figures and qualitative comments.

Refer to Appendix C for Maple worksheets.