Lab 7: Fabry-Perot Interferometer

1 Introduction

Refer to Appendix D for photos of the apparatus

A Fabry-Perot interferometer is a device that uses multiple beam interference of light for high resolution spectroscopy. The Fabry-Perot interferometer is useful because the conditions for which the device produces constructive interference are very strict so that its resolution can be very high. In fact, this instrument can have a much higher resolution than a Michelson interferometer. It is typically used to resolve sources of light that have narrowly separated wavelengths.

In this lab we will characterize the properties of the interferometer. We will determine the free spectral range and the finesse of the device. The free spectral range tells us the range of observable wavelengths. The finesse is a measure of the resolving power of the instrument. To determine these properties, you will need to record the interference pattern.

EXERCISES 1, 2 & 9 PERTAIN TO THE BACKGROUND CONCEPTS AND EXERCISES 3-8 AND 10-13 PERTAIN TO THE EXPERIMENTAL SECTIONS.

2 Background

The Fabry-Perot interferometer consists of two parallel (highly) reflective surfaces separated by a small gap. This is in fact the basic geometry of a resonant laser cavity in which the active lasing medium fills the space between reflectors.

The Fabry-Perot interferometer is able to produce sharp interference peaks because it uses multiple beam interference. This accounts for the higher resolution of the interferometer. In comparison, a Michelson interferometer combines just two beams and observes the interference pattern, hence is produces a lower resolution. Consider a ray of light incident at an angle
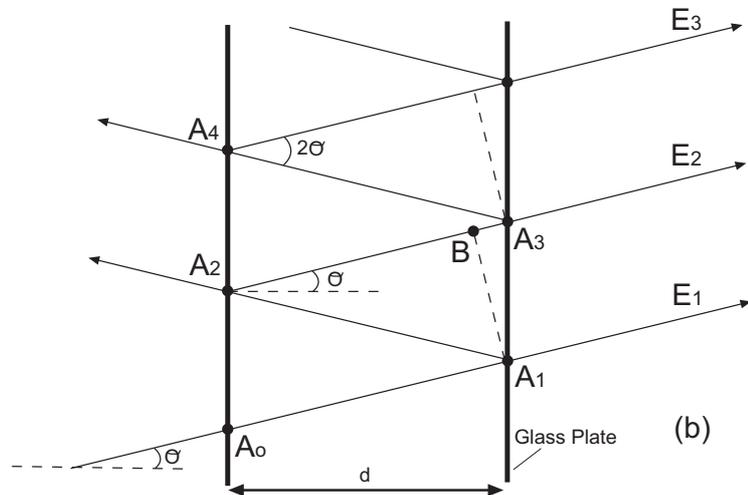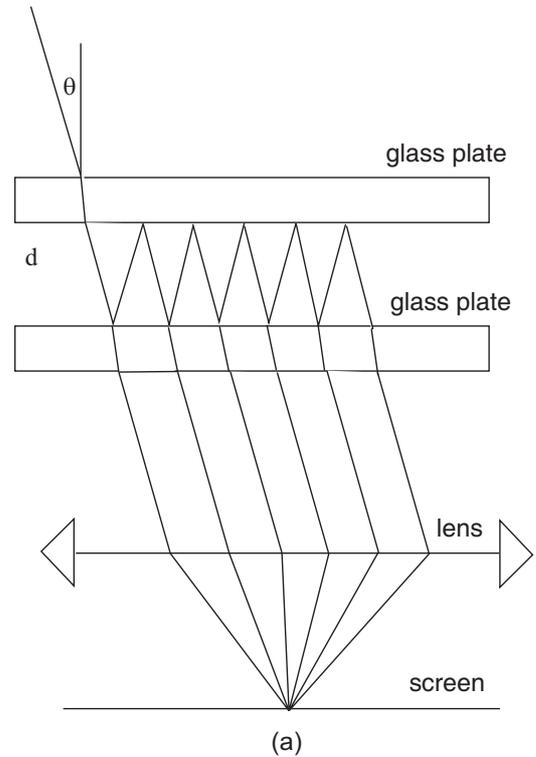

Figure 1: (a)Fabry-Perot setup (b) Fabry-Perot cavity.

θ, as illustrated in figure 1(b). The path difference p, between laser beams I and II can be obtained from the figure,

$$p = A_1 A_2 + A_2 B \tag{1}$$

From the figure we find that,

$$A_2 B = A_1 A_2 \cos 2\theta \tag{2}$$

Plugging equation 2 into 1 we get,

$$p = A_1 A_2 (1 + \cos 2\theta)$$
$$p = A_1 A_2 (2 \cos^2 \theta)$$

$$p = 2d \cos \theta \tag{3}$$

where $d = A_1 A_2 \cos \theta$. If the cavity has a medium with index n, the path difference between consecutively reflected beams is,

$$p = 2nd \cos \theta \tag{4}$$

where d is the plate separation, n is the index of refraction between the plates (1 for air) and θ is the angle of incidence of the beam. The factor of 2 is due to the two reflections between plates. Since all the multiple reflections come out parallel to each other, a lens can be used to combine the beams and observe the interference pattern.

In order to understand the characteristics of the cavity we will derive equations using the transmission and reflection coefficients of the incident light beam. Consider the following variables,

- t, t' are the amplitude transmission coefficients at each of the glass plates.

- r is the amplitude reflection coefficient of the glass plates.

The transmission coefficient is the fraction of the incident electric field that is transmitted and the reflection coefficient is the fraction that is reflected.

Now we can write the transmitted intensity of the emerging light beams, as shown in figure 1(b). Notice that the equations below correspond to the amplitudes of the emerging beams. For example, N = 1 defines a beam that travels right through the two glass plates and N = 2 defines a beam that is transmitted after being reflected once.

$N = 1 \qquad E_1 = E_0 tt' e^{i\omega t}$
$N = 2 \qquad E_2 = E_0 trr t' e^{i(\omega t - \delta)}$
$N = 3 \qquad E_3 = E_0 tr^4 t' e^{i(\omega t - 2\delta)}$

Therefore,

$N \qquad E_N = E_0 tt' r^{2(N-1)} e^{i(\omega t - (N-1)\delta)}$

Here, E_0 is the amplitude of the incident electric field. The exponential shows that the electric field is time dependant and δ is the phase difference between adjacently transmitted beams. The total electric field is,

$$E_{total} = a + aR' + aR'^2 + ... = \frac{a}{1 - R'}$$

where $a = E_0 tt' e^{i\omega t}$ and $R' = r^2 e^{-i\delta}$.

Therefore the total transmitted electric field is,

$$E_{total}{}^{transmitted} = \frac{E_0 tt' e^{i\omega t}}{1 - r^2 e^{-i\delta}}$$

The intensity of the transmitted electric field can be obtained by multiplying the transmitted electric field by its complex conjugate $I_{transmitted} = E_{total} E^*{}_{total}$. The transmitted intensity becomes,

$$I_{transmitted} = \frac{I_0 (tt')^2}{(1 + r^4) - 2r^2 \cos \delta}$$

Here, I_0 is the intensity of the incident electric field. Using the identity $\cos \delta = 1 - \sin^2(\frac{\delta}{2}$ and $tt' + r^2 = 1$ the transmitted intensity becomes,

$$I_{transmitted} = \frac{I_0}{1 + (\frac{2r}{1 - r^2})^2 \sin^2 \frac{\delta}{2}}$$

The equation $tt' + r^2 = 1$ is a statement of energy conservation and it implies that there are no losses in the interferometer. To simplify the above equation we will define a new variable F, called the coefficient of finesse. F depends on the coefficient of reflection of the glass plates, r.

$$I_{transmitted} = \frac{I_0}{1 + F \sin^2 \frac{\delta}{2}} \tag{5}$$

where $F = \frac{4R}{(1-R)^2}$ and $R = r^2$.

As you can see from figure 1 the last beam travels much further than the first. If the separation of the

plates is changed by x the path of the last beam is changed by $(Nx)2n\cos\theta$, where N is the number of reflections between the plates. This means that the interference pattern is very sensitive to small changes in the plate separation. The sensitivity of the interferometer to changes in d means that the peaks in the resulting interference pattern are very sharp. The transmittance of the interferometer is defined as the ratio of the transmitted light intensity to the incident light intensity. It can be derived from equation 5. For normal incidence ($\theta = 0$), $p = 2nd$ and the transmittance is given by the equation,

$$T = \frac{1}{1 + F\sin^2\frac{\delta}{2}} \qquad (6)$$

where,

$$\delta = 2\pi\frac{p}{\lambda} \qquad (7)$$

Constructive interference will occur when the phase difference between adjacent beams is an integer multiplied by 2π. From equation 7 we see that this means the path difference p must be an integral number of wavelengths. This situation is similar to a cavity in which standing waves occur when the length of the cavity is an integral number of half wavelengths (see figure 4). In this case, the round trip is an integral number of wavelengths so that the wave will reinforce itself. The transmitted waves will be in phase and result in constructive interference.

A derivation of these equations will be covered in PHYS2060 and can be found in "Introduction to Optics" by Pedrotti and Pedrotti, Prentice Hall 1993 (York Library: QC 355.2 P43 1993) chapter 11.

The transmittance of the interferometer is plotted as a function of the phase difference δ in figure 2. Note that δ is proportional to the path difference p. p is related to the mirror spacing d. Therefore, the spectrum shown in figure 2 can be regarded as the intensity of transmitted light as a function of plate spacing. Note that the transmission spectrum consists of a series of discrete peaks. The peaks correspond to the plate spacing for which an integral number of half wavelengths fit within the Fabry-Perot cavity. Looking at equation 6, it seems that the easiest way to measure the finesse of the interferometer is to find the minimum of the signal, half way between the peaks. This can be achieved by adjusting the plate separation. At this separation the sine term in equation 6 goes to 1 and the transmittance is just 1/(1+F). If you could measure the minimum transmittance you could solve T=1/(1+F) and determine the coefficient of finesse.
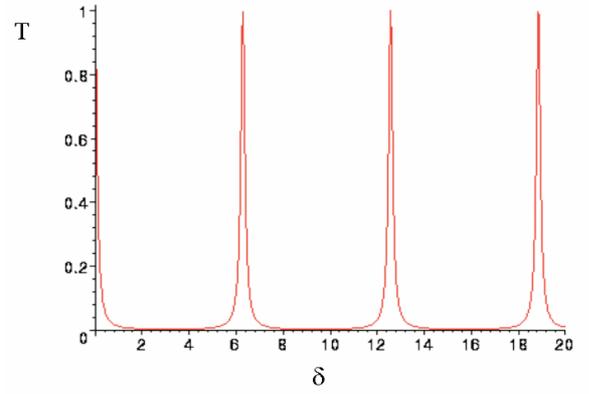

Figure 2: Transmittance as a function of phase, r = 0.95.

The problem with this method is that when the reflectance of the Fabry-Perot mirror is very high (for example ($\sim$ 99%), it can be seen from equations 5 and 6 that the coefficient of finesse is also very high ($\sim$ 10000). As a result, the transmittance goes to a minimum ($\sim$ 1/10000). The transmittance is therefore very small and difficult to measure.

Since the width of the transmitted peak is related to the coefficient of finesse, a better method of finding the coefficient of finesse involves measuring the width of this peak. In order to make this measurement the transmitted intensity is recorded as a function of the plate separation.

Resolving Power: Many light sources produce a discrete rather than a continuous spectrum of wavelengths. These discrete wavelengths will appear as successive sets of peaks in the transmitted spectrum. The resolving power of the device tells us the minimum difference between two wavelengths that can be distinguished. We will use the Rayleigh criterion to define this property. The criterion states that the separation of the peaks corresponding to two closely separated wavelengths λ_1 and λ_2 must be at least the full width half maximum for the two peaks to be resolved (see figure 3). From figure 3 we can see that this condition is satisfied when the (diffraction) minimum associated with one spectral line corresponds to the maximum of the second spectral line. If the coefficient of finesse is high (as in figure 2), the transmitted intensity between peaks is nearly zero. It is therefore a good approximation to assume that the half maximum occurs when T=0.5. If we let T=0.5 equation 6 can be rearranged so that,

$$\sin\frac{\delta}{2} = \frac{1}{\sqrt{F}} \qquad (8)$$

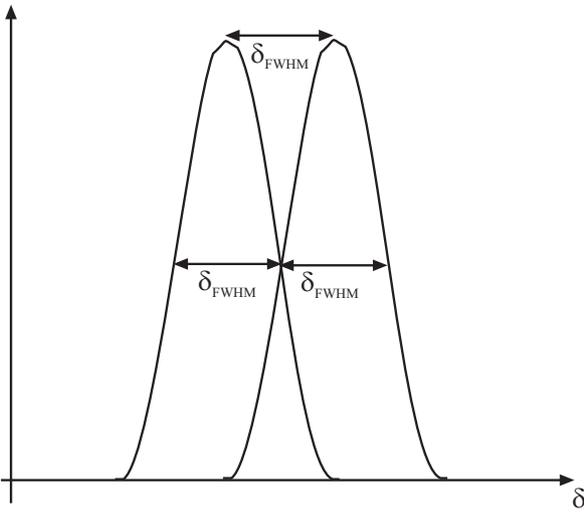Since a transmission peak corresponds to $\delta = 0$, the

Figure 3: Rayleigh Criterion.

solution for δ from the above equation gives you the phase change when going from the peak to the half maximum. For high coefficient of finesse, the transmission peaks are sharp. It is therefore easy to see that the phase change from peak to half maximum will be small. For small δ, we can use the approximation that $\delta = \sin(\delta)$ so that equation 8 simplifies to $\delta = \frac{2}{\sqrt{F}}$. This gives us the phase change from the peak to the point of half maximum. Therefore the phase change between two peaks is,

$$\delta_{min} = \frac{4}{\sqrt{F}} \qquad (9)$$

which is also called the FWHM (Full Width Half Maximum). This is a measure of the resolving power of the instrument, which is clearly related to the coefficient of finesse. We can relate this phase difference to a change in wavelength. To do this, we can combine equations 4 and 7 (for $n = 1$ and $\theta = 0$) to get $\delta = 4\pi d/\lambda$. For small changes in this wavelength we can differentiate the above expression to get: $\Delta\delta = 4\pi d(\Delta\lambda)/\lambda^2$. If we solve this expression for $\Delta\lambda$ we get $\Delta\lambda_{min} = \delta_{min}\lambda^2/4\pi d$. If we combine this expression with equation 9 we can get an expression relating minimum resolvable wavelength and the coefficient of finesse,

$$\Delta\lambda_{min} = \frac{\lambda^2}{\pi\sqrt{F}d} \qquad (10)$$

Exercise 1a: From the equation $\lambda = 4\pi d/\delta$ and equation 10 derive an expression for $\frac{\lambda}{\Delta\lambda_{min}}$.
Exercise 1b: Similarly, from the equation $\delta = 4\pi d/\lambda$ and equation 9 derive an expression for $\frac{\delta}{\delta_{min}}$.
Exercise 1c: These expressions define a quantity called the resolving power. Show that they are both equal to $\frac{\pi\sqrt{F}d}{\lambda}$.

Finesse: Next, we will derive a quantity that is related to the resolving power called the Finesse. It is defined as the ratio of the separation of adjacent maxima over the FWHM. Therefore using equation 9 and considering 2π as the separation of adjacent peaks (which can vary by changing d) we get,

$$f = \frac{2\pi}{4/\sqrt{F}} = \frac{\pi\sqrt{F}}{2} \qquad (11)$$

Exercise 2: Using equation 11 show that your answer to exercise 1 can be written as,

$$\frac{\lambda}{\Delta\lambda_{min}} = \frac{\delta}{\delta_{min}} = fm$$

Free spectral range: Another useful property of a Fabry-Perot interferometer is the free spectral range. The free spectral range tells us the range of wavelengths that can be observed. From figure 2, you can see that the interference pattern of the interferometer is repeated as the path difference changes. The transmission of the interferometer has a maximum when the path difference between consecutive reflections is an integer number of wavelengths

We can understand the properties of the interferometer in terms of the standing waves in the cavity. For these standing waves, a node must occur at the ends of the cavity (see figure 4). There are two ways the number of nodes in the standing waves in the cavity can change. If the wavelength of light changes, a different number of nodes will be accommodated within the cavity. If, however, the wavelength remains constant the cavity length itself must change until one more (or less) node is accommodated.

The first way to define free spectral range is to call it the change in wavelength between maxima in the transmittance. If the cavity length remains constant, a free spectral range is the difference in wavelength between adjacent modes that give you constructive interference. In figure 4 these correspond to λ_2 and λ_3. Since the cavity length is the same, it is true that,

$$m\lambda_2 = (m+1)\lambda_3 \qquad (12)$$

The difference between these two wavelengths is the free spectral range,

$$\lambda_{fsr} = \lambda_2 - \lambda_3 = \frac{\lambda}{m} \qquad (13)$$

Here we assume that λ_2 and λ_3 are nearly the same and m is the index of a cavity mode or the number of
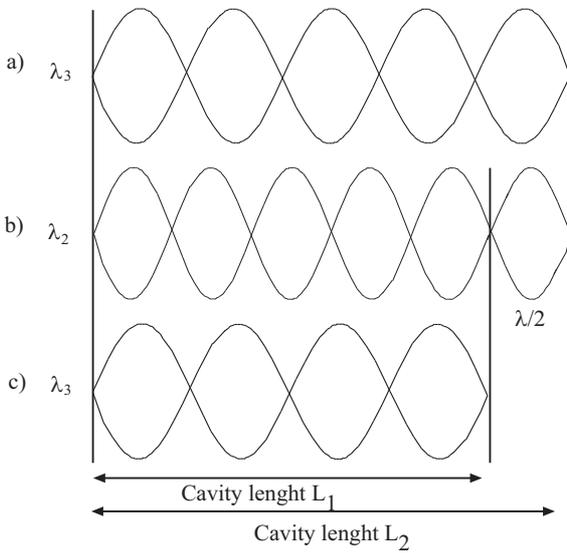
Figure 4: Change of one free spectral range.

half-wavelengths that fit in the cavity. In other words,

$$m = \frac{2d}{\lambda} \qquad (14)$$

Using equations 13 and 14, we can find the free spectral range in terms of measurable quantities to be,

$$\lambda_{fsr} = \frac{\lambda^2}{2d} \qquad (15)$$

The free spectral range can also be defined in terms of the change in frequency between maxima in the transmittance,

$$\nu_{fsr} = \frac{c}{\lambda_3} - \frac{c}{\lambda_2} = \frac{c(m+1)}{2d} - \frac{cm}{2d} = \frac{c}{2d} \qquad (16)$$

Here, c is the speed of light. The second way to define the free spectral range is to call it the change in the cavity length between transmission maxima, keeping the wavelength constant (making the cavity resonant with the next higher or lower mode). This is shown in figure 4 where the cavity changes and λ_2 ranges from mode m to $(m+1)$. The cavity length changes by $\lambda/2$. Figure 4c corresponds to the resonant conditions (maximum transmission) for wavelength λ_3. A different wavelength λ_2 also satisfies the resonant condition at the same cavity length as shown in figure 4b (cavity length L_1). When the cavity length increases to L_2, one more node of each wavelength can fit inside the cavity (figures 4a and 4b).

3 Suggested Reading

Refer to the relevant chapters,

A.E. Siegman, **Lasers** *Chapters 11* (University Science Books, 1986). Pedrotti and Pedrotti, **Introduction to Optics** (Prentice Hall 1993).

4 Apparatus

- Photodiode
- Voltmeter
- BNC cable with adapter to banana
- Fabry-Perot interferometer
- He-Ne laser
- 2 mirrors
- Lens
- 3 stands for mirrors and lens
- Optical breadboard
- 3/16" Allen key

WARNING!!: KEEP TRACK OF YOUR LASER BEAM AT ALL TIMES. NEVER POINT THE BEAM AT PEOPLE, OR LOOK IN THE APERTURE OF THE LASER OR BE AT EYELEVEL WITH THE BEAM.

KEEP EYES AWAY FROM DIRECT OR REFLECTED LASER BEAMS. OTHERWISE SERIOUS EYE DAMAGE WILL OCCUR.

YOU SHOULD BE AWARE OF WHERE THE LASER BEAMS STRIKE OPTICAL COMPONENTS. REFLECTIONS FROM OPTICAL COMPONENTS SHOULD BE BLOCKED BY USING PIECES OF CARDBOARD THAT ARE PROVIDED. BE PARTICULARLY CAREFUL WHEN YOU INSERT OR REMOVE LENSES INTO A LASER BEAM.

DO NOT TOUCH THE OPTICAL SURFACES OF LENSES AND MIRRORS. IF THE SURFACES ARE UNCLEAN, PLEASE BRING IT TO THE ATTENTION OF THE TA IMMEDIATELY.

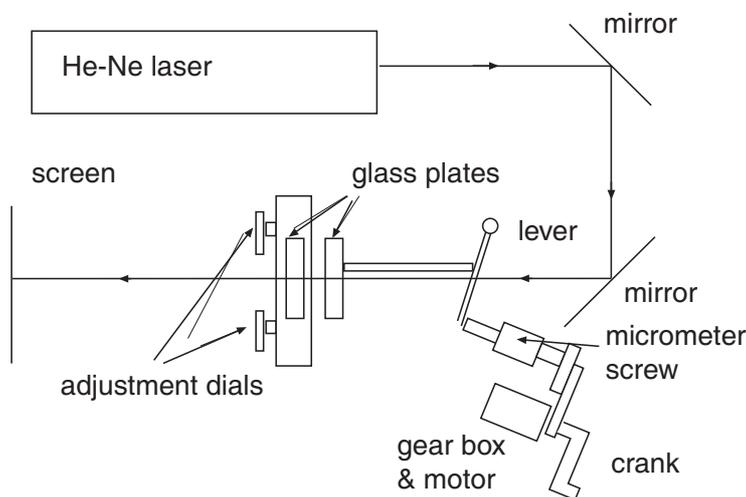USE THE TRANSPARENT LENS TISSUES TO DETECT THE BEAMS.

Figure 5: Fabry-Perot setup.

Figure 6: Aligning the interferometer.

MAKE SURE ALL MOUNTS ARE SE-CURELY FASTENED ON THE OPTICAL TABLE.

5 Experiment I: Aligning the Fabry-Perot Interferometer

The first step in the alignment is to have the laser incident exactly perpendicular to the first glass plate of the Fabry-Perot cavity. To ensure this is the case, adjust the mirrors so the reflected laser beam goes back on itself into the laser. You should see the slightly expanded beam reflected back on to the laser aperture. Now that the beam is incident perpendicular to the first glass plate, we must adjust the second glass plate so that it is exactly parallel to the first. To accomplish this adjust the gray dials on the second glass plate. If the plates are not parallel you will see a row of dots from each reflection between the plates. Move the dials and get a feel for how each dial adjusts the plate. Once you are comfortable with the dials try getting all the dots to overlap on a point. When you are close you will start to see an interference pattern. If the plates are not parallel, you will see lines that indicate that the path difference changes as you go across the beam. Adjust one of the dials until the direction of the lines changes. This indicates that the plates are parallel along the axis controlled by the dial. Now adjust the other dial until the pattern of lines changes direction again. After several iterations, you will see a uniform spot without the line pattern. Take your time to adjust the plates. If they are not aligned properly, yo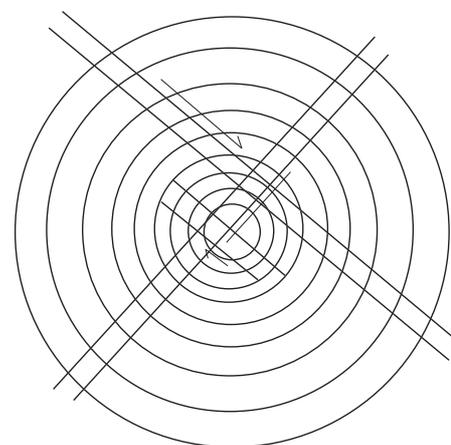ur results will suffer. Perform the alignment as described above. Rotate the crank that controls the length of the cavity and observe bright flashes in the transmission on a screen.

6 Experiment II: Calibrating the interferometer

Once you have aligned your interferometer you must calibrate the crank. To do this start by turning it and counting the number of flashes per full rotation of the crank. Call this number "f1" (the units are flashes/turn). When the crank is turned, it rotates a micrometer screw. The screw then adjusts the end of a lever that moves one of the glass plates. To calibrate the device, turn the crank 10 full rotations and observe how much this moves the screw. Call the movement of the screw per turn of the crank "s", (the units are m/turn). The distance the glass plate moves is a fraction of the distance that the screw moves because the screw is attached to a lever (see figure 7).

To find this fraction measure the distance from the fulcrum of the lever to the tip of the screw (call this "b"). Now measure the distance from the fulcrum to the rod attached to the glass plate (call this "a") (see figure 7). Clearly, the lever system will translate the mirror by a fraction of the distance moved when the screw is turned. This is due to the difference in distances from the pivot or fulcrum. Since $b >> a$, the lever moves the adjustable mirror by the fraction $(a/b)x$, where x is the displacement of the screw.

Exercise 3a: Make the calibration described above and use it to find how far the glass plate moves per degree of rotation of the crank as measured by the
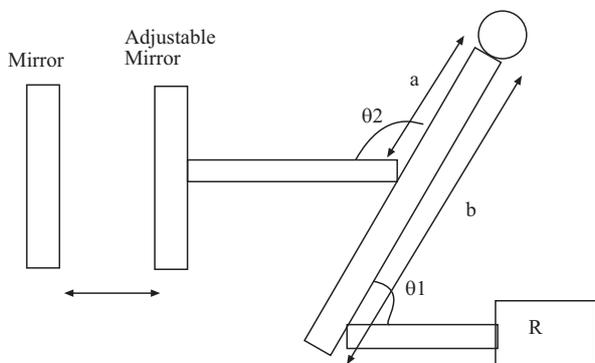
Figure 7: Lever system.

protractor (call this conversion "c", the units of c are m/degree).

Exercise 3b: In this experiment, the angles θ_1 and θ_2 are $\sim 90^o$ (i.e. $\sin 90^o = 1$) which simplifies the calibration. How would the calibration be dependent on θ if the angle was less then 90°?

Exercise 4: From this conversion, and from your value of f_1, find the distance W that the glass plate moves between each peak in the transmittance. To determine when the flash occurs monitor the transmitted light intensity on a photo-diode. The output of the photo-diode can be recorded on a digital multimeter. You don't need the actual value of intensity just the intensity relative to the peak intensity. For better accuracy, use a black sheet to block as much of the room light as possible.

Recall that peaks occur when the phase difference, δ, is an integer number of 2π. From equations 4 and 7 we see that if $n = 1$ and $\theta = 0$ the peaks will occur when the distance between the mirrors is an integral number of half-wavelengths. This means the measurement in exercise 4 gives you a value for half the wavelength.

Exercise 5: Given that the wavelength of the light is 6328 $\mathring{A}$ how does your measurement compare with the excepted value?

7 Experiment III: Finding the finesse

The finesse of the device tells us its resolution (see equation 11). To find this we need to find not just the spacing of the peaks, as we did in part 2, but also the shape of the peak. To find this shape we will need to measure the intensity of the light transmitted as a function of the plate spacing using the photo-diode

and voltmeter. Position the photo diode so the transmitted laser beam is incident on the photosensitive region. Now rotate the crank so that you can see the voltage change periodically as the crank is rotated. Position the crank at the 0° position and record the voltage on the voltmeter. Now rotate the crank by 5° (you can rotate by less if you feel that you can do this accurately) and record the voltage at this position. Continue this at 5° intervals until you have passed 2 or 3 peaks. **Now block the laser and determine the background signal.**

Exercise 6: Make the above measurements and plot the voltmeter reading as a function of plate spacing. Before you plot your results, subtract the background signal from all your data points.

Exercise 7: From your plot, find the full width half maximum, w, of the transmitted peak.

Exercise 8: Measure the distance between the plates and using the accepted wavelength of the laser, calculate the free spectral range using equation 15.

You can understand the behavior of the Fabry-Perot cavity from different points of view. When the cavity length is changed, a peak occurs every time the glass plate is moved by half the wavelength, $\lambda/2$. You measured this to be W. This motion of the cavity changes the phase between consecutive reflections by 2π, or one free spectral range. This phase change could also occur if the wavelength were changed by λ_{fsr}.

In exercise 7, we determined the distance that the mirror moves to resolve one transmission peak (this is the full width half maximum). Let us call this w. w also corresponds to the phase change δ_{min} given in equation 9. This phase change could also occur if the wavelength were changed by $\Delta\lambda_{min}$. These quantities are related by the equation,

$$\frac{\delta_{min}}{2\pi} = \frac{\Delta\lambda_{min}}{\lambda_{fsr}} = \frac{w}{W} \qquad (17)$$

Exercise 9: Show that by combining equations 10, 15 and 17 it is true that,

$$\frac{w}{W} = \frac{2}{\pi\sqrt{F}} \qquad (18)$$

Exercise 10: Use equation 17 and the values for w and W from exercise 6 to find δ_{min} and $\Delta\lambda_{min}$.

Exercise 11: Use equation 18 to calculate the coefficient of finesse for your values of w and W.

Exercise 12: What is the approximate coefficient of reflection corresponding to this coefficient of finesse?

Exercise 13a: Using your experimental results for d, λ and F and your answers for exercises 1 and 2 calculate the resolving power fm. Refer to exercises 1 and 2.

Exercise 13b: Using $\Delta\lambda_{min}$ calculated in exercise 10, find $\Delta\nu_{min}$.

Note: Once you have completed the experiment, please remove all optical elements and the detector from their mounts and place them on the optical table.

Your lab report should include:

Answers to exercises 1-13 with relevant data tables, graphs, figures and qualitative comments.

Refer to Appendix C for Maple worksheets.