Assignment 12

Exercise 1:

(a)

From the first equation, we can get: y = 8 – x.

Now substitute y with 8 – x into the second equation:

2x –3*(8-x)=-1, that is 5x = 23

so x = $\frac{23}{5}$, y = 8 - $\frac{23}{5}$ = $\frac{17}{5}$ are the solutions

(b)

Divide the first equation by 25, the second equation by 5, we get a new system of equations :

$$\begin{cases} x - 3y = 4 \\ -3x + 8y = 12 \end{cases}$$

From the first equation above, we get x = 4+3y

Now substitute x = 4+3y into –3x +8y =12, we get

-3(4+3y)+ 8y = 12, that is –y =24

So y = -24, x = 4+3(-24) = -68 are the solutions

Exercise 2:

Say Jim invested x dollars in the account paying 8% and y dollars in the account paying 5% simple interest, as the total savings is \$15,000, x + y = 15,000; as the interest after one year is \$1050, 8%x+5%y=1050; thus we have a system of equations:

$$\begin{cases} x + y = 15{,}000 \\ 8\%x + 5\%y = 1{,}050 \end{cases}$$

From the first equation, we get y = 15,000-x,

Now substitute this result into the second equation, we get:

8% x + 5% (15,000-x) =1,050, which can be rewritten as 8x + 5(15,000-x)=105,000, that is 3x=3,0000

So x =10,000, y = 15,000-10,000=5,000

\$10,000 was invested in the account paying 8% and \$5,000 is invested in the account paying 5% simple interest

Exercise 3:

(a)

$$\begin{cases} x+y+z=2 \\ -x+3y+2z=8 \\ 4x+y=4 \end{cases} \text{ can be rewritten as } \begin{cases} x+y+z=2 \\ -x+3y+2z=8 \\ 4x+y+0z=4 \end{cases},$$

then the corresponding matrix for above is

$$\left\{\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ -1 & 3 & 2 & 8 \\ 4 & 1 & 0 & 4 \end{array}\right\} \xrightarrow{R1+R2,\&-4R1+R3} \left\{\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 4 & 3 & 10 \\ 0 & -3 & -4 & -4 \end{array}\right\} \xrightarrow{\frac{3}{4}R2+R3} \left\{\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 4 & 3 & 10 \\ 0 & 0 & -\frac{7}{4} & \frac{14}{4} \end{array}\right\}$$

$$\xrightarrow{-\frac{4}{7}R3} \left\{\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 4 & 3 & 10 \\ 0 & 0 & 1 & -2 \end{array}\right\}$$

Reconvert the above last matrix into the system of equations:

$$\begin{cases} x+y+z=2 \\ 0x+4y+3z=10 \\ 0x+0y+z=-2 \end{cases},$$ substitute z =-2 into the second equation, we get 4y-6=10,

that is y = 4; substitute z = -2 and y = 4 into the first equation, we get x + 4 – 2 = 2, that is x = 0

Therefore, the solutions to this system of equations are: x = 0, y = 4, and z = -2

(b)

$$\begin{cases} x+y-3z=-1 \\ y-z=0 \\ -x+2y=1 \end{cases} \text{ can be rewritten as } \begin{cases} x+y-3z=-1 \\ 0x+y-z=0 \\ -x+2y+0z=1 \end{cases},$$

then the corresponding matrix for above is

$$\left\{\begin{array}{ccc|c} 1 & 1 & -3 & -1 \\ 0 & 1 & -1 & 0 \\ -1 & 2 & 0 & 1 \end{array}\right\} \xrightarrow{R1+R3} \left\{\begin{array}{ccc|c} 1 & 1 & -3 & -1 \\ 0 & 1 & -1 & 0 \\ 0 & 3 & -3 & 0 \end{array}\right\} \xrightarrow{-3R2+R3} \left\{\begin{array}{ccc|c} 1 & 1 & -3 & -1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right\}$$

Reconvert the above last matrix into the system of equations:

$$\begin{cases} x+y-3z=-1 \\ 0x+y-z=0 \\ 0x+0y+0z=0 \end{cases}$$, So y =z, substitute this result into the first equation, we get x+y-3y=-1,

that is x = 2y-1

Therefore, this system of equations has infinitely many solutions which can be expressed in y = z, x = 2y-1

(c)

$$\begin{cases} -x+3y+z=4 \\ 4x-2y-5z=-7 \\ 2x+4y-3z=12 \end{cases},$$

The corresponding matrix is:

$$\left\{\begin{array}{ccc|c} -1 & 3 & 1 & 4 \\ 4 & -2 & -5 & -7 \\ 2 & 4 & -3 & 12 \end{array}\right\} \xrightarrow{4R1+R2,\&2R1+R3} \left\{\begin{array}{ccc|c} -1 & 3 & 1 & 4 \\ 0 & 10 & -1 & 9 \\ 0 & 10 & -1 & 20 \end{array}\right\} \xrightarrow{-R2+R3} \left\{\begin{array}{ccc|c} -1 & 3 & 1 & 4 \\ 0 & 10 & -1 & 9 \\ 0 & 0 & 0 & 11 \end{array}\right\}$$

Reconvert the above last matrix into the system of equations:

$$\begin{cases} -x+3y+z=4 \\ 0x+10y-z=9 \\ 0x+0y+0z=11 \end{cases}$$, the last equation is actually 0=11 which does not make any sense.

So this system of equations has no solutions and is inconsistent.

Exercise 4:

Say Howard should use x milliliters of 10%, y milliliters of 20%, and z milliliters of 40% acid.
As the mixture is 100ml of an 18% acid, we have: x +y +z =100,
10%x+20%y+40%z =100*18%=18
As he uses 4 times as much of the 10% solution as the 40% solution, x = 4z
Now, we can combine the above three individual equations into a system of equations:

$$\begin{cases} x+y+z=100 \\ 10\%x+20\%y+40\%z=18 \\ x=4z \end{cases}$$, which can be rewritten as $$\begin{cases} x+y+z=100 \\ x+2y+4z=180 \\ x+0y-4z=0 \end{cases},$$

then the corresponding matrix for above is

$$\left\{\begin{array}{ccc|c} 1 & 1 & 1 & 100 \\ 1 & 2 & 4 & 180 \\ 1 & 0 & -4 & 0 \end{array}\right\} \xrightarrow{-R1+R2,\&-R1+R3} \left\{\begin{array}{ccc|c} 1 & 1 & 1 & 100 \\ 0 & 1 & 3 & 80 \\ 0 & -1 & -5 & -100 \end{array}\right\} \xrightarrow{R2+R3}$$

$$\left\{\begin{array}{ccc|c} 1 & 1 & 1 & 100 \\ 0 & 1 & 3 & 80 \\ 0 & 0 & -2 & -20 \end{array}\right\} \xrightarrow{-\frac{1}{2}R3} \left\{\begin{array}{ccc|c} 1 & 1 & 1 & 100 \\ 0 & 1 & 3 & 80 \\ 0 & 0 & 1 & 10 \end{array}\right\}$$

Reconvert the above last matrix into the system of equations:

$$\begin{cases} x+y+z=100 \\ y+3z=80 \\ z=10 \end{cases}$$,substitute z=10 into the second equation, we get y+30=80,

that is y = 50; substitute z = 10 and y = 50 into the first equation, we get x + 50 +10 = 100, that is x = 40

Therefore, the solutions to this system of equations are: x = 40, y = 50, and z = 10

He should use 40 milliliters of 10%, 50 milliliters of 20% and 10 milliliters of 40% solution.

Exercise 5:

If there are 60 choices for the position of a chair-person, then there should be 99 left for a vice chair, and 98 for a secretary,

so the total ways equal: C(60,1)*C(99,1)*C(98,1)= $60\times 99\times 98=582{,}120$

Suppose we had argued that for the two positions of vice chair and secretary that the possible choices were C(99,2) - namely choose two out of 99. Then the possibility of Sally being chosen for vice chair and Richard for secretary would not have been distinguishable from the possibility of Richard being chosen for vice chair and Sally for secretary, but with the choice of say Selma for chair we would have had two distinct committees -

Committee A: Selma = chair, Sally = vice chair, Richard = secretary

Committee B: Selma = chair, Richard = vice chair , Sally = Secretary

So if we had stated the answer as 60x C(99,2) we would not have distinguished between these two possible committees.

Exercise 6:

If there are 50 choices for a gold, then there should be 49 choices left for a silver, and 48 for a bronze, so this problem is one of permutation,

and the total ways equal: $P(50,3)=\dfrac{50!}{(50-3)!}=117{,}600$

Exercise 7:

Let x denote the number of distinguishable permutations of the 11 letters.

For each such arrangement there are a further 3! arrangements obtained by rearranging the As, 2! by rearranging the Bs, 3! by rearranging the Cs, and 2! by rearranging the Es.

Thus $x\times(3!2!3!2!)=11!$, so $x=\dfrac{11!}{3!\times 2!\times 3!\times 2!}=\dfrac{39916800}{6\times 2\times 6\times 2}=277{,}200$

The number of distinguishable arrangements is 277,200

Exercise 8:

If there are 2 toppings for each pizza, one must be mushrooms, there will be C(14,1) different toppings for each pizza,
If there are 3 toppings for each pizza, one must be mushrooms, there will be C(14,2) different toppings for each pizza… so the table goes like below:

No. of Toppings on each pizza	15	14	13	12	11	10	9	8
Different toppings per pizza	C(14,14)	C(14, 13),	C(14,12)	C(14,11)	C(14,10)	C(14,9)	C(14,8)	C(14,7)

No. of Toppings on each pizza	7	6	5	4	3	2	1
Different toppings per pizza	C(14,6)	C(14, 5),	C(14,4)	C(14,3)	C(14,2)	C(14,1)	C(14,0) (must be mushroom)

There are 3 different style crust, so the number of different pizzas is:

$$C(3,1)\times(C(14,14)+C(14,13)+C(14,12)+C(14,11)+C(14,10)+C(14,9)+C(14,8)+C(14,7)+$$
$$C(14,6)+C(14,5)+C(14,4)+C(14,3)+C(14,2)+C(14,1)+C(14,0))$$
$$=\frac{3!}{2!1!}\times(1+\frac{14!}{13!1!}+\frac{14!}{12!2!}+\frac{14!}{11!3!}+\frac{14!}{10!4!}+\frac{14!}{9!5!}+\frac{14!}{8!6!}+\frac{14!}{7!7!}+\frac{14!}{6!8!}+\frac{14!}{5!9!}+\frac{14!}{4!10!}+\frac{14!}{3!11!}+\frac{14!}{2!12!}+\frac{14!}{1!13!}+1)$$
$$=\frac{3!}{2!}\times(2\times(1+\frac{14!}{1!13!}+\frac{14!}{12!2!}+\frac{14!}{11!3!}+\frac{14!}{10!4!}+\frac{14!}{9!5!}+\frac{14!}{8!6!})+\frac{14!}{7!7!})$$
$$=\frac{3!}{2!}\times(2\times(1+14+91+364+1001+2002+3003)+3432)$$
$$=3\times16384$$
$$=49152$$

Alternatively,

$$C(3,1)\times(C(14,14)+C(14,13)+C(14,12)+C(14,11)+C(14,10)+C(14,9)+C(14,8)+C(14,7)+$$
$$C(14,6)+C(14,5)+C(14,4)+C(14,3)+C(14,2)+C(14,1)+C(14,0))$$
$$=\frac{3!}{2!}\times2^{14}\,(for.C_n^0+C_n^1+C_n^2+\ldots+C_n^{n-2}+C_n^{n-1}+C_n^n=2^n)$$
$$=3\times16384$$
$$=49152$$

So 49,152 different pizzas are possible.

Exercise 9:

There are 4 cards for each card value

The number of ways to get 3 of the same card value: C(4,3) , and there are 13 choices to choose from

The number of ways to get 2 of the same card value: C(4,2) , and there are 12 choices left

So the ways to get a full house equal:

$$C(4,3)\times 13\times (C(4,2)\times 12) = C(4,3)\times C(4,2)\times P(13,2) = \frac{4!}{3!\times 1!}\times\frac{4!}{2!\times 2!}\times\frac{13!}{11!} = 3,744$$