Assignment 8 Solutions

Exercise 1:

(a), $f'(x)=10x+4=0$, **so** $x=-\frac{2}{5}$ **is the critical number**

(b), $g'(t)=6t^2+6t-6=6(t^2+t-1)=0$, **so** $t=\frac{-1+\sqrt{5}}{2}, t=\frac{-1-\sqrt{5}}{2}$ **are the critical numbers**

(c), $s'(t)=\frac{10}{3}t^{-\frac{1}{3}}+\frac{5}{3}t^{\frac{2}{3}}=\frac{5}{3}t^{-\frac{1}{3}}(2+t)=\frac{5}{3\sqrt[3]{t}}(2+t)=0$, **so t =-2 and t = 0**(if t =0, $s'(t)$ does not exist) **are the critical numbers.**

Exercise 2:

Set $f'(x)=1.5x^4-6x=1.5x(x^3-4)=0$, so x =0 or $x=\sqrt[3]{4}$

then we can construct a sign analysis table:

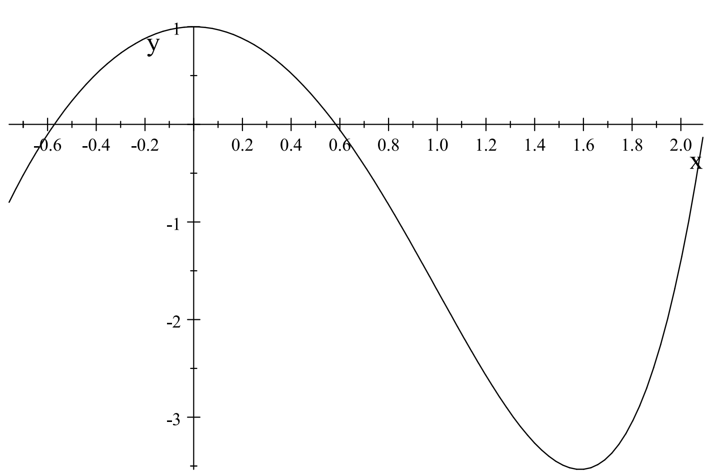
$-\infty$		0		$\sqrt[3]{4}$		∞
1.5x	-		+		+	
x^3-4	-		-		+	
$f'(x)$	+		-		+	

So,

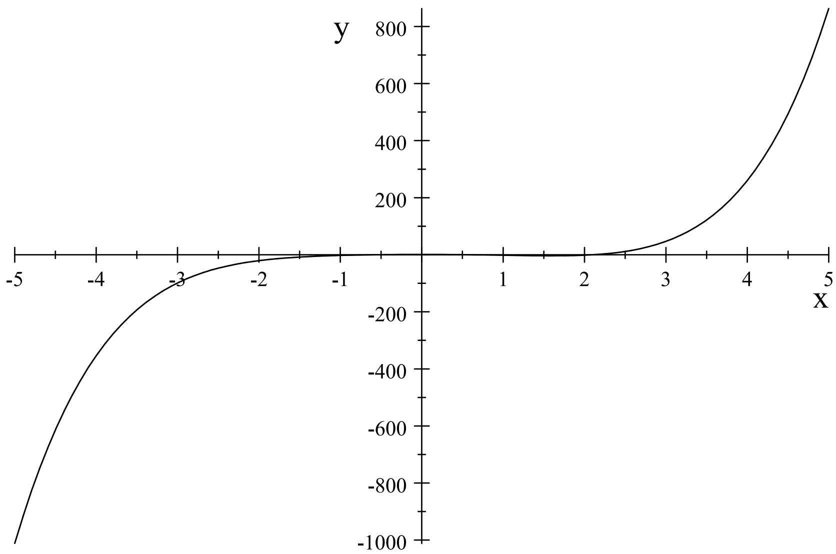
$f'(x)$ **is positive on** $(-\infty,0)$**, while negative on (0,** $\sqrt[3]{4}$**), so this function has a local maximum at x = 0, and the local maximum value is 1**

$f'(x)$ **is negative on (0,** $\sqrt[3]{4}$**), while positive on** $(\sqrt[3]{4},\infty)$**, so this function has a local minimum at x =**$\sqrt[3]{4}$ **, and the local minimum value is**

$0.3\times(\sqrt[3]{4})^5-3\times(\sqrt[3]{4})^2+1=-3.54$



The domain of this function is the entire real line, and the graph of this function is shown below. From the graph, we can see that there is no absolute maximum or absolute minimum



for this function.

Exercise 3:

(a) $f'(x) = 3x^2 - 27 = 3(x^2 - 9) = 3(x+3)(x-3)$

set $f'(x) = 0$, then x =-3 or x = 3, then we can construct a sign analysis table:

	$-\infty$	-3	3	∞
x+ 3	-	+	+	
x-3	-	-	+	
$f'(x)$	+	-	+	

So, this function is increasing on $(-\infty,-3), and (3,\infty)$, **decreasing on (-3,3)**

(b)From the sign analysis table, we can see that:

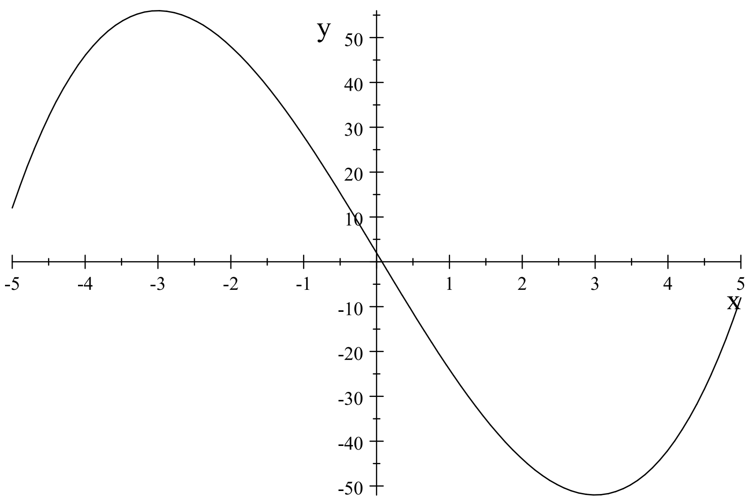
$f'(x)$ is positive on $(-\infty,-3)$, while negative on (-3,3), **so this function has local maximum at x = -3, and the local maximum value of this function is -27+81+2=56**

$f'(x)$ is negative on (-3,3), while positive on $(3,\infty)$, **so this function has local minimum at x =3, and the local minimum value of this function is 27-81+2=-52**

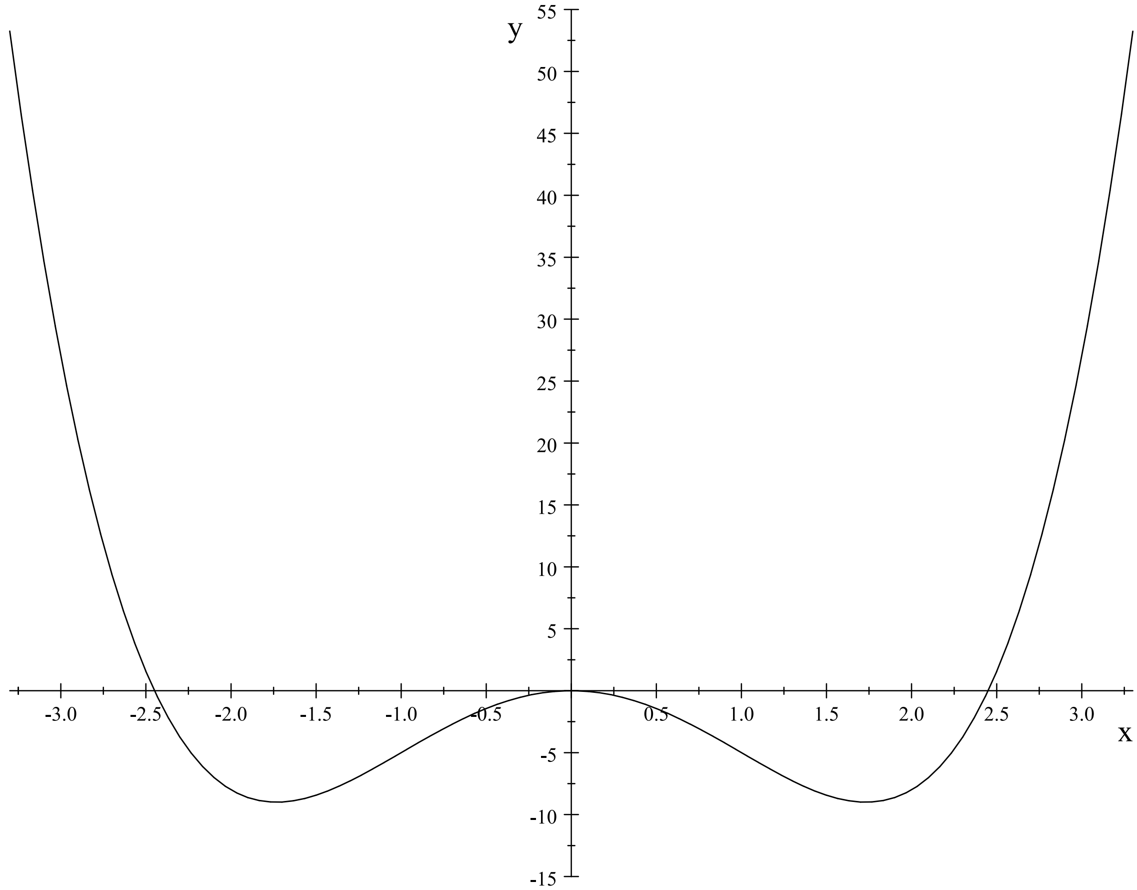
(c) $f''(x) = 6x$ >0 when x>0, $f''(x) = 6x$ <0 when x<0, so **the function is concave down over** $(-\infty,0)$**, concave up over** $(0,\infty)$

(d) as the function concave down over $(-\infty,0)$ and concave up over $(0,\infty)$ so **x = 0 is the inflection point.**

(e)



Exercise 4:



$f'(x) = 4x^3 - 12x = 4x(x^2 - 3) = 4x(x + \sqrt{3})(x - \sqrt{3})$

set $f'(x) = 0$, then x = -$\sqrt{3}$, x = 0, or x = $\sqrt{3}$, then we can construct a sign analysis table:

$-\infty$	-$\sqrt{3}$	0	$\sqrt{3}$	∞
$4x$	-	-	+	+
$(x+\sqrt{3})$	-	+	+	+
$(x-\sqrt{3})$	-	-	-	+
$f'(x)$	-	+	-	+

So, this function is increasing on (-$\sqrt{3}$,0) and ($\sqrt{3}$, ∞), decreasing on ($-\infty$, -$\sqrt{3}$) and (0, $\sqrt{3}$)

From the sign analysis table, we can see that:

$f'(x)$ is negative on **($-\infty$, -$\sqrt{3}$)**, while positive on **(-$\sqrt{3}$,0) ,so this function has local minimum at x =-$\sqrt{3}$, and this local minimum value is 9-18=-9**

$f'(x)$ is negative on **(0, $\sqrt{3}$),** while positive on **($\sqrt{3}$, ∞), so this function has another local minimum at x = $\sqrt{3}$, and this local minimum value is also -9**

$f'(x)$ is positive on **(-$\sqrt{3}$,0)**, while negative on **(0, $\sqrt{3}$), so this function has local maximum at x = 0, and the local maximum is 0**

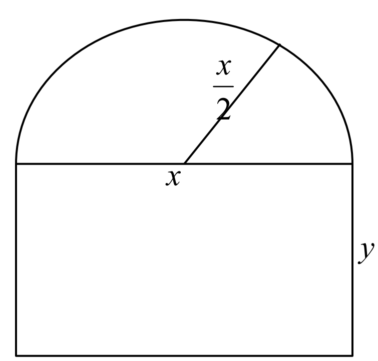
$f''(x) = 12x^2 - 12 = 12(x+1)(x-1)$

set $f''(x) = 12x^2 - 12 = 12(x+1)(x-1) > 0$, then x>1 or x<-1, **so the function is concave up over** $(-\infty, -1)$ **and** $(1, \infty)$

$f''(x) = 12x^2 - 12 = 12(x+1)(x-1) < 0$, then $-1<x<1$, so **the function is concave down over (-1,1)**

As the function concave up over $(-\infty, -1)$, down over(-1,1) and up again over $(1, \infty)$, so **x = -1 and x =1 are the inflection points.**

Exercise 5:



The radius of the semicircle is $\frac{x}{2}$,(0<x<30)

So the circumference of the window is:

$\frac{1}{2} \times 2\pi \times \frac{x}{2} + (2y + x) = 30,$ (0<y<30)

so $y = 15 - \frac{x}{2} - \frac{\pi}{4} x$

The area(S) of this window is:

$$S = xy + \frac{1}{2}\pi(\frac{1}{2}x)^2 = x(15 - \frac{x}{2} - \frac{\pi}{4}x) + \frac{1}{2}\pi \times \frac{1}{4}x^2 = 15x - \frac{x^2}{2} - \frac{\pi}{4}x^2 + \frac{\pi}{8}x^2 = 15x - \frac{x^2}{2} - \frac{\pi}{8}x^2$$

Set $S' = 15 - x - \frac{\pi}{4}x = 0$, then $x = \frac{15}{1 + \frac{\pi}{4}} = \frac{60}{4 + \pi}$

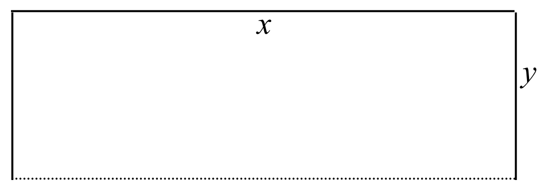
As S' is positive on (0, $\frac{60}{4+\pi}$) and negative on ($\frac{60}{4+\pi}$,30), S has a local maximum at

$x=\frac{60}{4+\pi}\approx 8.4$, thus y = 4.206

So $S=15x-\frac{x^2}{2}-\frac{\pi}{8}x^2=15\times 8.4-8.4^2\div 2-8.4^2\times\frac{\pi}{8}=63.03$ ft^2

So, to admit the greatest amount of light, the window should have an area of 63.03 square feet.

Exercise 6:



The length of the chain is 400 meters

x +2y = 400, so y = 200-0.5x , $(0<x<400, 0<y<400)$

The area (S) is:

$S=xy=x(200-0.5x)=200x-0.5x^2$

Set $S'=200-x=0$, then x = 200

As S' is positive on (0, 200) and negative on (200,400), S has a local maximum at x =200, thus y = 100

So, the dimension of the largest such area is:

$S=200x-0.5x^2=200\times 200-0.5\times 200^2=20{,}000$ $\mathbf{m^2}$