Mathematics 1710 Exercise Set Number 8

1. A critical number of a function f is a number x at which the derivative $f'(x) = 0$ or $f'(x)$ does not exist. Find the critical numbers of the following functions.

   (a) $f(x) = 5x^2 + 4x$

   (b) $g(t) = 2t^3 + 3t^2 - 6t + 4$

   (c) $s(t) = 5t^{\frac{2}{3}} + t^{\frac{5}{3}}$

2. Find the precise absolute maximum, the absolute minimum and all local maximum or minimum values of the function below using techniques of calculus. Plot a graph of the function and use the graph as a means of distinguishing absolute maximums and minimums and local maximums and minimums.

$$f(x) = 0.3x^5 - 3x^2 + 1$$

3. For the function $f(x) = x^3 - 27x + 2$:

   (a) find the intervals where the function is increasing and the intervals where the function is decreasing

   (b) find the local maximums and minimums of the function

   (c) find the interval over which the function is concave down and the interval over which the function is concave up

   (d) find the inflection point

   (e) graph the function

4. For the function $f(x) = x^4 - 6x^2$, as in the previous problem, graph the function and find the: intervals where the function is increasing and where it is decreasing, local maximums and minimums, intervals over which it is concave down and over which it is concave up, inflection points

5. A Norman window has the shape of a rectangle surmounted by a semicircle so that the diameter of the circle is the width of the window. If the perimeter of the window is 30 feet, what are the dimensions of the window so that the greatest amount of light can be admitted.

6. A construction crew repairing a section of the 401 has 400 meters of chain linked fence for creating a fenced in area adjacent to the highway for storing heavy equipment. There is already a fence parallel with the highway whose function is to keep stray cows from crossing, so that the area to be fenced needs fencing only on 3 sides. What are the dimensions of the largest such area that uses all 400 meters of fencing?

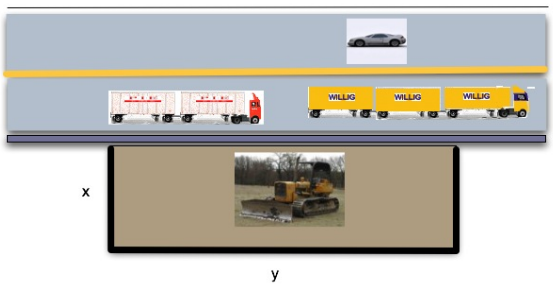

Figure 1: fenced area adjacent to highway