Mathematics 1710 Exercise Set Number 9

1. If an amount of \$1500 is invested at an annual interest rate of $6\frac{1}{3}\%$ what is the value of the investment compounded after 5 years if the investment is compounded
   (a) twice a year - that is semi-annually
   (b) monthly
   (c) daily
   (d) continuously
2. Suppose that \$1000is invested at 6% compounded monthly. How long must you wait until your investment equals 3000?
3. A certain population of salmon is limited by the habitat that allows them to spawn. It has been determined that the population behaves according to the so called *logistic growth model* so that after t years the population has size
$$A(t) = \frac{4300}{0.5 + 25e^{-0.05t}}.$$
   (a) What is the initial population?
   (b) Draw the graph of the function A.
   (c) What is the limiting population as t gets very large?
4. Use the laws of logarithms to rewrite the following expressions so that no log of a power, product, or quotient appears.
   (a) $\log_4\left(\sqrt{\dfrac{x^2}{(x-1)(x+1)}}\right)$
   (b) $\ln\left(\sqrt[3]{\dfrac{x^4y^5}{z^2}}\right)$
5. An ancient humanoid fossil is estimated to contain 5% of the carbon-14 it originally contained. What is the age of the fossil? Carbon 14 has a half-life of 5730 years?
6. The population of wolves in an a region of northern Ontario is said to be growing exponentially at the rate of 12% per year. The current population is thought to be 124. Assuming all factors remain the same, what will the population be in 10 years?
7. Differentiate the following functions
   (a) $f(x) = e^{x^3-2x}$
   (b) $g(x) = \sqrt{1 + xe^{-x^2}}$

(c) $h(x) = \sqrt{x}\ln x$

(d) $f(x) = \log_{10}\left(\frac{x}{x-1}\right)$

8. Functions of the form

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

where σ and μ are special constants are called *normal density functions* and they are used extensively in probability and statistics. The constant σ is called the*standard deviation* and the constant μ is called the *mean*. The factor $\frac{1}{\sigma\sqrt{2\pi}}$ is only a vertical scaling factor so to make things simple lets remove it for this problem. Also set $\mu = 0$. We end up with the function

$$f(x) = e^{\frac{-x^2}{2\sigma^2}}$$

(a) Graph the function for $f(x) = \sigma = 1$ and $\sigma = \frac{1}{2}$.

(b) For $\sigma = 1$ find the maximum and the points of inflection of the curve.

(c) What effect does various values for σ have on the shape of the curve?