1 Functions II -The Sequel Exercises

1. For ϵ representing a real number and N and n representing integers, our definition says that a sequence a_n converges to a limit L says: if for every $\epsilon > 0$ there is $N > 0$ such that for every n, $n > N$ implies $|a_n - L| < \epsilon$.

   (a) express this definition in the language of logic using quantifier symbols, with p being the proposition $n > N$ and q being the proposition $|a_n - L| < \epsilon$.

   (b) formally negate the result of (a) in such a way that a negation symbol does not come before any quantifier or logical connective such as $\wedge$, $\vee$, or $\Rightarrow$.

   (c) express the result of (b) as an English sentence.

2. The *Rational Zeros Theorem* states that if a polynomial $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ has integer coefficients , then any rational zero is of the form $\frac{p}{q}$ where p is some factor of a_0 and q is some factor of a_n

   This theorem allows us to start factoring the polynomial. For instance with the polynomial $f(x) = x^3 + 4x^2 + 3x - 2$, we then know that the rational zeros are of the form ± 2 or ± 1. We then check to see which give zero when substituted for x. We see that -2 gives zero. Then we divide f by $(x - 2)$ remembering that $x = a$ a zero means $(x - a)$ is a factor. When we do this we find that $f(x) = (x+2)(x^2+2x-1)$. If we want to factor f completely into linear factors we now need to factor $(x^2+2x-1$. For this we use the quadratic formula, solving $x^2 + 2x - 1 = 0$. We find that the solutions are $x = -1 \pm \sqrt{2}$, allowing us to write down all the factors of f, that is:

   $$f(x) = (x+2)\Big(x - (-1 - \sqrt{2})\Big)\Big(x - (-1 + \sqrt{2})\Big)$$

   The exercise is , for the polynomial $f(x) = x^3 + 6x^2 + 5x - 12$,

   (a) write down the *possible* rational zeros of f

   (b) write f as a product of a linear and a quadratic factor

   (c) write f as a product of linear factors

3. As mentioned in the lecture, the Fundamental Theorem of Algebra tells us all polynomials can be factored into a product of linear polynomials, with some factors appearing more than once and with some of the form $(x - z)$, where z is a complex number. Here is an example. For $g(x) = x^2 + 2x + 2$, we apply the quadratic formula solving $x^2 + 2x + 2 = 0$ to find the roots. We get

   $$x = \frac{-2 \pm \sqrt{4-8}}{2} = -1 \pm \frac{\sqrt{-4}}{2} = -1 \pm \frac{2\sqrt{-1}}{2} = -1 \pm i.$$

Thus,

$$x^2 + 2x + 2 = \Big(x - (-1 - i)\Big)\Big(x - (-1 + i)\Big).$$

The exercise is

(a) Express $h(x) = x^3 - 1$ as a product of linear factors.

(b) Express $g(x) = x^3 + 7x^2 + 18x + 18$ as a product of linear factors.

4. Find the minimum value of the function $f(x) = x^2 - 8x + 8$ by first completing the square. Show your work.

5. (a) Let $y = x^2$ and suppose its graph is moved 2 units to the right and 2 units up and then reflected about the y axis. Let $\mathcal{G}$ be the new graph. What is the equation whose graph is $\mathcal{G}$.

   (b) Let $y = x^3$ and suppose the graph is moved 1 unit to the left, one unit down, reflected about the x axis and flattened by a factor of 1/2 . What is the equation that describes the new graph?

6. Using a graphing utility graph the function $f(x) = 2x^4 - 6x^2 + 1$ and approximate accurate to 3 decimals the zeros of the function and the points $(x, f(x))$ where the graph attains maximum or a minimum values. Include with your solutions an image of the graph.

7. The cost of producing x yards of a certain fabric is described by the function

$$C(x) = 1300 + 14x - 0.15x^2 + 0.0006x^3$$

dollars, and the company has discovered that it can charge an amount

$$p(x) = 25 - 0.0002x$$

dollars per yard. Assuming that they sell what they produce, how many yards should the company produce in order to maximize profit, and what would the profit be. Use a graphing utility zoom in on the maximum of the profit function $P(x) = R(x) - C(x)$, where $R(x) = xp(x)$ is the revenue function.