1 Functions I - Exercises

1. It is a fact that two lines are perpendicular to one another if and only if the slope of one is the negative reciprocal of the other. That is: given any non-zero real number m and any pair of real numbers b and c, the two lines with equations $y = mx + b$ and $y = -\frac{1}{m}x + c$, are perpendicular .

Knowing this, show that if a and b are any two real numbers and if $a \neq b$, then the line connecting the points (a, b) and (b, a) is perpendicular to the line with equation $y = x$

2. It is also a fact that two lines are parallel to one another if and only if they have the same slope.

Knowing this, and given a line whose equation is $4x - 2y = 3$, write down the slope-intercept equation of the line that passes through the point $(2, 1)$ and is parallel to the given line.

3. A manufacturer has bought some equipment for making his widgets. The equipment costs $\$36,500$. The maintenance of the equipment and the raw materials for the widgets requires roughly $\$5.25$ for each hour of operation and the operator of the equipment is paid $\$11.50$ an hour.

(a) Write a linear equation expressing the total cost $C(t)$ for operating the equipment for a total of t hours. Do not forget the initial investment.

(b) Assuming that the machinery produces 10 widgets an hour and they sell for $\$2.70$ a piece, write down an equation showing total revenue $R(t)$ after t hours.

(c) Find the break-even point - namely the value of t when revenue equals cost - that is $C(t) = R(t)$

This is a case in which we wish to find the "zero" of the profit function $P(t) = R(t) - C(t)$ - namely the spot where the line represented by $P(t)$ crosses the x axis

4.
(a) Let $f(x) = x^3 + 4$ and let $g(x) = \sqrt[3]{x+1}$. Calculate $f \circ g,\ g \circ f,\ and\ \ f \circ f$

(b) Let $f(x) = \frac{3-x}{4}\ and\ g(x) = \frac{x+5}{2}$. Calculate $f \circ g,\ g \circ f$

(c) Find the inverse of the function $f(x) = \frac{x-2}{x+2}$.

5. After much experimentation it has been discovered that the frequency (*vibrations per second* of a vibrating string which is under a constant tension satisfies the equation $f(x) = \frac{k}{x}$ where x is the length of the string and k is some constant dependent on the tension and

the thickness of the string. The violin A string which is 12 inches long vibrates 440 times per second. How much shorter must this string be if it were to vibrate 660 times per second?

6. The Italian astronomer/mathematician Galileo Galilei who lived from 1564-1642 discovered that the speed of a falling object is independent of its weight. In particular after a lengthy process which entailed dropping canon balls off the leaning tower of Pisa, he discovered that the distance $d(t)$ in feet the cannon ball traveled t seconds after it had been let fall was $d(t) = 16t^2$.

Knowing this, if a ball has been let fall from a height of 400 feet, how many seconds does it take for it to hit the ground?