Mathematics 1710 Exercise Set Number 10

1. For each of the following determine the quadrant that the angle lies in. Convert the angle to radians if it is expressed as degrees and convert it to degrees if it is expressed as radians
   (a) 150^0
   (b) $-336^0 30'$
   (c) $\frac{\pi}{7}$ radians
   (d) -4.2π radians
   (e) 780^0
   (f) 2 radians

2. Recall that for a unit circle with center O if given two points P and Q then the arc length from P to Q equals the measurement of the angel $\angle POQ$ in radians. Suppose now that there is a larger circle with radius r and center 0. Let Q' be the intersection of the ray $\overrightarrow{OQ}$ with the larger circle and let P' be the intersection of the ray $\overrightarrow{OP}$ with the larger circle. Then we can say comparing arc lengths to the total circumferences that the fractions one gets must be the same. That is:

$$\frac{\text{arclength}(PQ)}{2\pi} = \frac{\text{arclength}(P'Q')}{2\pi r}$$

   This means that $\text{arclength}(P'Q') = r \times \text{arclength}(PQ)$. With this in mind, here is the problem.

   Given that the earth is roughly a sphere and that the radius is roughly 6378 kilometers, find the distance between Johannesburg, South Africa and Jerusalem, Israel, assuming that the two cities are on the same longitude (Johannesburg due south of Jerusalem) and that their latitudes are respectively $26^o 11' S$ and $31^o 47' N$.

3. Evaluate, when possible and without the use of a calculator the six trigonometric functions for the angular values
   (a) $\theta = \dfrac{5\pi}{4}$
   (b) $\theta = -\dfrac{\pi}{2}$
   (c) $\theta = \dfrac{2\pi}{3}$

4. Without the use of a calculator evaluate

(a) $\sin 7\pi$

(b) $\cos \frac{9\pi}{4}$

(c) $\sin \frac{-4\pi}{3}$

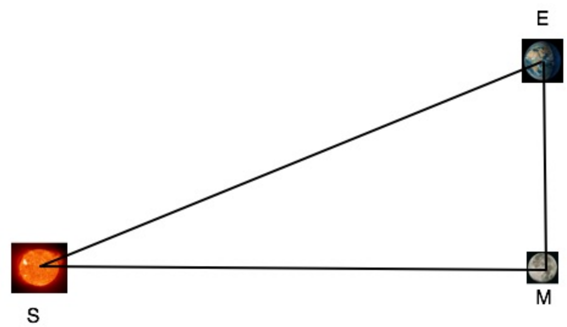

Figure 1: The sun , the moon, the earth

5. When the moon is half full, the angle earth moon-sun $\angle EMS$ is a right angle. At the same time the angle sun-earth-moon $\angle SEM$ is 89.85^o. If the distance from the earth to the moon is approximately 384,403 km. What is the approximate distance from the earth to the sun.

6. Recall that multiplying a function definition by a positive constant causes the graph to be vertically stretched if the constant is greater than 1 and to be vertically shrunk if the constant is less than one but greater than zero. Also recall that to horizontally stretch one needs to multiply the variable by a number between zero and one and to horizontally shrink one needs to multiply the variable by a number greater than one. Suppose that $f(x) = a \sin bx$ and $g(x) = a \cos bx$ where a and b are positive real numbers. The number a is called the *amplitude* and the number $\dfrac{2\pi}{b}$ is called the *period.* In the case that x represents time, the period represents the amount of time for a complete cycle to pass. The *frequency* on the other hand is the reciprocal of the period and represents the number of cycles in a unit of time - say, cycles per second. Any sound causes vibrations of the air which like ripples in a pond form a sine curve. In the case of concert pitch A , this is a sound that represents 440 cycles per second.

   (a) Using a graphing utility, graph $f(x) = a \sin bx$ in the cases

      i. $a = 1$ and $b = 2$

      ii. $a = 1$ and $b = \frac{1}{2}$

      iii. $a = 2$ and $b = 2$

(b) Why is the number $\dfrac{2\pi}{b}$ called the period ?

7. The important points for a cosine or sine graph are those at which the function has value zero, where it has a minimum value and where it has a maximum value. We know that the sine function on the interval $[0, 2\pi]$ has a minimum at $x = 0$, a max at $x = \frac{\pi}{2}$, a zero at $x = \pi$, a min at $x = \frac{3\pi}{2}$ and a zero at $x = 2\pi$. The graph of the sine goes through a single *basic cycle* over the interval $[0, 2\pi]$ and this behavior repeats over all subsequent intervals of length 2π. The cosine graph has similar but different behavior. On the interval $[0, 2\pi]$, it starts with a maximum value at $x = 0$. If you are given for instance $y = a\cos(bx - c)$ or $y = a\sin(bx - c)$, then as before the period is $\frac{2\pi}{b}$, for $b > 0$, and the amplitude is $|a|$. The problem is to find where the basic cycle starts and finishes. We know that the number c causes a horizontal translation. Here is what you do. Look at the equations $bx - c = 0$ and $bx - c = 2\pi$ and solve for x. The solutions give the beginning and ending points of the basic cycle.

   Here are the problems.

   For each of $y = 2\sin(x - \frac{\pi}{3}$, and $y = 3\cos\left(\frac{\pi x}{2} + \frac{\pi}{2}\right)$,

   (a) find the starting point and ending point of the basic cycle

   (b) identify the amplitude and the period

   (c) sketch a graph of two periods of the function showing zeros and maximum and minimum points

   (d) verify (c) with a graph using a graphing utility.