Journal of International Money and Finance
20 (2001) 191–218

Journal of International Money and Finance

www.elsevier.nl/locate/econbase

Systematic jump risks in a small open economy: simultaneous equilibrium valuation of options on the market portfolio and the exchange rate

Melanie Cao [a, b,*]

[a] *Department of Economics, Queen's University, Kingston, Ontario, Canada K7L 3N6*
[b] *Department of Finance, Schulich School of Business, York University, Toronto, Ontario, Canada M3J 1P3*

Abstract

The valuation of stock options and currency options has witnessed an explosion of new development in the past 20 years. These models, set up either in a partial equilibrium or a general equilibrium framework, have certainly enriched our understanding of option valuation in one way or the other. However, the main drawback of these models is that stock options and currency options are analyzed in separate contexts. The co-movement of the stock market and the currency market is absent from the option valuation analysis. Such co-movement is extremely important and is best illustrated by the Southeast Asian financial crisis.

To overcome this drawback, this paper uses an equilibrium model to investigate the joint dynamics of the exchange rate and the market portfolio in a small open monetary economy with jump-diffusion money supplies and aggregate dividends. It is shown that the exchange rate and the market portfolio are strongly correlated since both are driven by the same economic fundamentals. Furthermore, options on the exchange rate and the market portfolio are evaluated in the same equilibrium context. The analysis shows that parameters describing the same economic fundamentals have very different effects on currency and stock options © 2001 Elsevier Science Ltd. All rights reserved.

JEL classification: F31

Keywords: Open economy; General equilibrium; Market portfolio; Exchange rate; Options; Jumps; Siegel's paradox

* Correspondence address: Department of Finance, Schulich School of Business, York University, Toronto, Ontario, Canada M3J 1P3. Tel.: +1-416-736-2100 ext 33801; fax: +1-416-736-5687.

E-mail address: mcao@ssb.yorku.ca (M. Cao).

PII: S0261-5606(00)00053-X

1. Introduction

Derivatives valuation has witnessed an explosion of new development in the past 20 years. Examples for stock option valuations include Black and Scholes (1973), Merton (1976), Cox and Ross (1976), Hull and White (1987), Bailey and Stulz (1989) and Naik and Lee (1990). Examples for currency option models include Biger and Hull (1983), Garman and Kohlhagen (1983), Grabbe (1983), Chesney and Scott (1989), Amin and Jarrow (1991), Heston (1993), Bates (1996) and Bakshi and Chen (1997). The references listed here are by no means exhaustive. These models, set up either in a partial equilibrium or a general equilibrium framework, have certainly enriched our understanding of option valuation in one way or the other.

However, the main drawback of these models is that stock options and currency options are analyzed in separate contexts. The co-movement of the stock market and the currency market is absent from the option valuation analysis. Such co-movement is extremely important and is best illustrated by the recent Southeast Asian financial crisis, which has swamped small economies like Thailand, Indonesia, Malaysia and Korea.

During the crisis, the dramatic currency devaluations were always accompanied by sharp decreases in their corresponding stock markets. As shown in Table 1, the 1997 average return on Southeast Asia's currency and the stock market is about -45%. The 1998 drastic devaluation of the Russia ruble and Russia's stock market only adds more evidence to the co-movement. Such evidence suggests that the stock market and the currency market are affected by the same fundamental economic factors. Failure to incorporate such simultaneous reactions to changes in the same fundamental economic factors would misguide the derivative valuations.

The second drawback of the existing models is best summarized by Jorion (1988, pp. 427-428):

> Many financial models rely heavily on the assumption of a particular stochastic process, while relatively little attention is paid to the empirical fit of the postulated distribution. As a result, models like option pricing models are applied indiscriminately to various markets such as the stock market and the foreign exchange market when the underlying processes may be fundamentally different.

Table 1
Summary of currency and stock index performance

Country	1997 returns on currency (%)	1997 returns on the stock index (%)
Thailand	−45	−54
Indonesia	−56	−37
Malaysia	−35	−52
Korea	−47	−38
Average	−46	−45

Obviously, the information arrival process in the foreign exchange market differs from that in the stock market, since exchange rates are directly influenced by monetary polices that do not have apparent counterparts in the stock market. It is important to directly investigate the effect of monetary policy changes on exchange rates and hence on currency options. Such analysis can only be carried out in a general equilibrium framework where the relation between exchange rates and monetary policies can be endogenized. In fact, indiscriminately applying the Black and Scholes (1973) formula to both stock options and currency options yields the opposite pricing bias pattern. The Black-Scholes formula generally overprices out-of-the-money stock call options and underprices in-the-money stock call options (MacBeth and Merville, 1979), but it usually underprices out-of-the-money currency calls (Bodurtha and Courtadon, 1987).

Another problem of applying stock option models to currency options is that the assumptions for stock option models may not be valid for currency options. For example, a number of scholars, such as Bodurtha and Courtadon (1987), Jorion (1988) and Dumas et al. (1995), suggest that currency options should be priced with Merton's (1976) mixed jump–diffusion stock option model since jumps have been found in exchange rates.[1] The key problem with this application is that the jump risk in Merton's model is assumed to be uncorrelated with the market. Such an assumption of uncorrelated jump risk may be reasonable if the concern were firm specific stocks, but is problematic for currency markets. Since the exchange rate reflects one nation's purchasing power relative to another nation, the exchange rate is inherently correlated with aggregate fundamental forces that affect the market.

The main objective of this paper is to overcome these drawbacks, by simultaneously analyzing option valuations for the exchange rate and the market portfolio in a small open economy with *systematic and non-systematic jump risks*. I employ a continuous-time extension of the Lucas (1978) asset pricing model to a small open monetary economy, where money has a non-trivial role in the agents' utility function. Based on utility maximization, the equilibrium analysis endogenizes the precise relationship between the exchange rate and the market portfolio which are functions of the same fundamental forces. The explicit modelling of the relationship between the exchange rate and monetary policies also helps to uncover the distinct nature of the exchange rate process that differs from the stock price process. Under the logarithmic utility function, the equilibrium exchange rate, expressed as the relative price of foreign currency in terms of home currency, is affected by the domestic money supply, aggregate dividends and the level of investments in foreign assets. In contrast to the exchange rate, the real price of the domestic stock is affected by aggregate dividends and the level of investments in foreign assets. This equilibrium formulation also enables me to price options on the exchange rate and stock accordingly. Comparative analysis shows that currency options and stock options are affected differently by the parameters underlying economic fundamentals. In addition, this paper

[1] See Akgiray and Booth (1988), Jorion (1988), Tucker (1991) and Ball and Roma (1993) for empirical evidence on the jumps in exchange rates.

also addresses the analog of the so-called Siegel's paradox in currency option valuation with systematic jump risks, which is illustrated by Dumas et al. (1995).[2]

The current model is obviously different from the existing partial equilibrium option models in which the exchange rate or the stock price is exogenous. As pointed out by Bailey and Stulz (1989), the arbitrary choice of the exogenous process for any security price in the partial equilibrium models is unlikely to be consistent with the equilibrium conditions or to provide important insights into how derivative prices may respond to changes in any fundamental economic variables.

Though the current model shares the equilibrium approach with the existing equilibrium option models, the key difference is that the current model simultaneously analyzes currency option and stock option valuation in a consistent manner. Moreover, the focus here is on a small open economy, which is different from a closed pure-exchange economy as in Naik and Lee (1990) for stock option valuation, or a two-country setting as in Bakshi and Chen (1997) for currency option valuation.

The remainder of this paper is organized as follows. Section 2 describes the economy and presents the equilibrium results. Section 3 examines the endogenized exchange rate and the price of the market portfolio, and derives equilibrium prices for European currency and stock options from the view of the domestic risk-averse agent. Section 4 identifies the adjustments on the risk-neutral process of the exchange rate that help to solve the analog of Siegel's paradox in currency options. Section 5 extends the model to allow for a correlation between the money supply and aggregate dividends. Section 6 concludes the paper and the appendices provide the necessary proofs.

2. A small open monetary economy

Consider a small open economy with perfect capital mobility between itself (termed the domestic country) and the rest of the world (termed the foreign country). This economy consists of a single risk-averse representative agent whose lifetime horizon is infinite. I adopt the standard formulation of a small open economy used in the existing literature with the following characteristics.[3] First, the agent in the small economy has perfect access to the international goods and assets markets. Since the small economy has little influence on the foreign country, it takes the price of any foreign asset as given. Second, the domestic currency and domestic assets held by the foreign country are assumed to be negligible, implying that the supplies of these assets are cleared by domestic demands. Third, domestic aggregate consumption is financed through both domestic aggregate output (dividends) and the

[2] The paradox (Siegel, 1972) originally illustrates the discrepancy between the forward exchange rate and the expected future exchange rate. That is, when the exchange rate is expressed as the price of the domestic currency in terms of the foreign currency, the forward exchange rate is always less than the expected future rate.

[3] For a reference to a deterministic model of a small open economy, see Obstfeld (1981). An example in the stochastic environment is Grinols and Turnovsky (1994).

return to holding foreign assets (which is paid in consumption goods). When the sum of aggregate dividend and the return to foreign assets exceeds aggregate consumption, the goods market is cleared by an increased holding of foreign assets (i.e., a current account surplus); when the sum of aggregate dividends and the return to foreign assets falls short of aggregate consumption, the residual is financed by a reduction in the holding of foreign assets (i.e., a current account deficit). This feature distinguishes a small open economy from a closed economy.

I will first describe the primitives of the economy and then solve the agent's maximization problem. Equilibrium asset prices, including the domestic nominal interest rate and the exchange rate, are determined by requiring goods, money and financial markets to clear, as in Lucas (1982).

2.1. Structure of the economy

There is a single good traded worldwide with no barriers, which can be used for consumption and investment. The nominal price of the good at home at time t is denoted p_t. Let P^* be the foreign price level measured in the foreign currency. According to the law of one price in the good market, p equals the spot exchange rate times P^*. Since the home country is small, it takes P^* as given and so we can simplify the discussions by normalizing $P^*=1$.[4] Then, p_t equals the spot exchange rate expressed as the relative price of the foreign currency in terms of the home currency.

The home government controls the domestic money supply, which is taken as given by each domestic agent. The real money balances held by the domestic agent at time t are defined as $m_t=M_t/p_t$, where M_t is the nominal quantity of domestic money demanded by home agents. To assign a non-trivial role to money, I follow Sidrauski (1967) to assume that real money balances yield utility to agents in addition to their purchasing power. In particular, the agent's period utility function, $U(c_t, m_t, t)$, depends positively on the agent's real money balances, m_t, as well as on consumption, c_t. The rationale is that larger real money balances reduce the transaction time in the goods market and hence allow the agent to enjoy more leisure. As long as leisure yields positive marginal utility to the agents, real money balances yield utility.[5]

The government's purchase of goods and services is assumed to be constant and so the change in the money supply is injected into the economy as lump-sum monet-

[4] Allowing P^* to follow a stochastic process complicates the analysis without changing the qualitative results, provided that the process for P^* is independent of the processes for domestic dividends and domestic money supply.

[5] The money-in-the-utility approach is also technically convenient in a continuous-time setting. On the other hand, the cash-in-advance approach depends crucially on the timing of events and hence on the discrete-time structure, as stated in Sargent (1987, p. 157). For the cash-in-advance constraint to bind, all financial markets must be temporarily shut down when consumption goods are purchased with money. In a continuous-time setting where agents can instantaneously sell goods and assets for money, the cash-in-advance constraint imposed by Bakshi and Chen (1997) is technically difficult to implement.

ary transfers. As in Lucas (1982), I assume that the agent is endowed with one unit of a claim on these monetary transfers. Denote the real price of this equity claim at time t as L_t. The money transfer measured in real terms, l, can be understood as the "dividend" for this claim. Therefore, L is the present value of future real monetary transfers. Note that monetary transfers are lump-sum and hence are taken as given by individual agents. The dynamics of the domestic money supply are described in the following assumption.

Assumption 1 The domestic money supply, M^s, is assumed to evolve according to the following mixed diffusion-jump process:

$$\frac{\mathrm{d}M^s}{M^s}=(\mu_m-\lambda_m k_m)\,\mathrm{d}t+\sigma_m\,\mathrm{d}z_1+(Y_m-1)\,\mathrm{d}Q_m,\ \forall\ t\in(0,\infty). \tag{2.1}$$

Here, μ_m is the instantaneous expected growth rate of the money supply; σ_m^2 is the instantaneous variance of the growth rate, conditional on no arrivals of new important shock and $\mathrm{d}z_1$ is a one-dimensional Gauss-Wiener process. The element $\mathrm{d}Q_m$ is a jump process with a jump intensity parameter λ_m and Y_m-1 is the random variable percentage change in the money supply if the Poisson event occurs. The logarithm of Y_m is normally distributed with mean θ_m and variance ϕ_m^2. The expected jump amplitude, $k_m=E(Y_m-1)$, is equal to $\exp(\theta_m+\phi_m^2/2)-1$. Also, $\bar{k}_m=E((1/(Y_m))-1)$ is equal to $\exp(-\theta_m+\phi_m^2/2)-1$. The random variables $\{z_{1t},\ t\geq 0\}$, $\{Q_{mt},\ t\geq 0\}$ and $\{Y_{mj},\ j\geq 1\}$ are assumed to be mutually independent. Also, Y_{mj} is independent of $Y_{mj'}$ for $j\neq j'$. The parameters $(\mu_m,\ \sigma_m,\ \lambda_m,\ \theta_m,\ \phi_m)$ are constant.

The above money supply process incorporates both frequent fluctuations in the money supply, which correspond to the diffusion part $\mathrm{d}z_1$, and infrequent large shocks to the money supply, which correspond to the jump part $\mathrm{d}Q_m$. Both capture changes in government monetary policies.

There is only one domestic risky stock, which represents the ownership of the home productive technology for the single good. The total supply of this risky stock is normalized to one. Denote its real price at time t as S_t and the dividend as δ_t. The dividend stream $\{\delta_t\}$ can be understood as aggregate dividends in this small economy, which are exogenously given as:[6]

$$\frac{\mathrm{d}\delta}{\delta}=\mu(\delta)\,\mathrm{d}t+\sigma(\delta)\,\mathrm{d}z_2+(Y_\delta-1)\,\mathrm{d}Q_\delta, \tag{2.2}$$

where $\mathrm{d}z_2$ is a one-dimensional Gauss-Wiener process and $\mathrm{d}Q_\delta$ is an independent jump process, described more precisely later.

The specification of the aggregate dividend process corresponds to an economy which is infrequently subject to real shocks of unpredictable magnitude. The shocks

[6] Although dividends are not continuously distributed in reality, one may be able to find reasonable proxies for aggregate dividends used here. Aggregate output and dividends on stock indices are the examples.

on dividends could result from output shocks or shocks due to technological innovations. For most of the discussion, the dividend process and the money supply process are assumed to be independent, measured with respect to a given probability space $(\Omega, \mathscr{F}, \mathscr{P})$. Section 5 will extend the discussion to allow for a correlation between the two processes.

There are foreign pure discount bonds available for trading to the home agent at any time. A foreign pure discount bond pays 1 unit of consumption goods at maturity and 0 at all other times. The agent can internationally diversify his portfolio by holding the foreign bonds and the domestic financial assets. That is, the net trading in assets between this small economy and the foreign country is positive and time-varying. Since the country is small, the real price of the foreign bond at time t, F_t, is taken as exogenous by the home agent. The dynamics of F_t are assumed below:

Assumption 2 F_t evolves as $\mathrm{d}F=rF\,\mathrm{d}t$, where r is a positive constant.

The processes for the money supply, the foreign bond price and the aggregate dividend are the primitives of the economy. Together with the specification of the utility function described below, they induce equilibrium prices for other assets. Among these other assets, there are domestic nominal pure discount bonds in zero net supply, with nominal rate of return i. A domestic nominal discount bond pays 1 unit of domestic currency at maturity and 0 at all other times. Denote B_t as the nominal price of the discount bond at time t. Then, $\mathrm{d}B=iB\,\mathrm{d}t$, where i is endogenously determined in equilibrium. The real price of the domestic bond at time t, b_t, is given as $b_t=B_t/p_t$. In addition, there are many other contingent claims on the risky domestic stock and the spot exchange rate available for trading at any time in the economy. These contingent claims are all in zero net supply. Denote the real prices of the contingent claims at time t by a vector x_t and the corresponding vector of real dividends by δ_t^x.

2.2. The agent's optimization problem

The representative agent's information structure is given by the filtration $\mathscr{F}_t\equiv\sigma(M_\tau^s, \delta_\tau;\ 0\leq\tau\leq t)$. As described earlier, the period utility at time t is $U(c_t, m_t, t)$, where $U(\cdot, \cdot, t)$: $\mathscr{R}_+^2\rightarrow\mathscr{R}$ is increasing and strictly concave and satisfies the following properties:

$$\lim_{x_j\rightarrow\infty} U_j(x_1, x_2)=0 \text{ and } \lim_{x_j\rightarrow 0} U_j(x_1, x_2)=\infty,\ j=1, 2.$$

The agent's intertemporal utility is described by

$$V(c, m)=E_0\int_0^\infty U(c_t, m_t, t)\,\mathrm{d}t.$$

Initially, the agent is endowed with N_0^F units of the foreign bond, one share of the domestic risky stock, money holdings M_0 and one share of the equity claim for domestic monetary transfer. His consumption over time is financed by a continuous trading strategy $M_t, N_t, \forall\, t \geq 0$, where M_t is the money holding at time t and $N_t = (N_t^L, N_t^F, N_t^S, N_t^b, N_t^{x'})'$ is a vector which represents the portfolio holdings consisting of all the financial assets traded in financial markets at time t. For example, N_t^F is the quantity of foreign bonds held by the domestic agent at time t. Denote the real prices of all financial assets at time t by a vector $X_t = (L_t, F_t, S_t, b_t, x'_t)'$ and the corresponding vector of real dividends by q_t. The cumulative dividends up to t are defined as $D_t = \int_0^t q_\tau \, d\tau$. At any point $\tau \geq 0$, the agent's wealth is $W_\tau = N_\tau X_\tau + M_\tau / p_\tau$ and the flow budget constraint is

$$c_\tau \, d\tau = M_\tau \, d\left(\frac{1}{p_\tau}\right) + N_\tau^X (dD_\tau + dX_\tau) - dW_\tau. \tag{2.3}$$

This constraint intuitively states that the sum of the wealth increase (dW_τ) and consumption flow ($c_\tau \, d\tau$) is bounded by the dividend and capital gain from the portfolio $\{M_\tau, N_\tau\}$.

With this flow budget constraint, one can use the technique of optimal control to derive the partial differential equations that are satisfied by the assets prices. In the presence of the jump components in the money supply process and the dividend process, these partial differential equations turn out to be very complicated. In contrast, the Euler equation approach appears much simpler and is adopted here.[7] To do so, transform the flow budget constraint into an integrated one (see Duffie, 1992, p. 110, for a similar formulation):

$$\int_0^t c_\tau \, d\tau = \frac{M_0}{p_0} + \int_0^t M_\tau \, d\left(\frac{1}{p_\tau}\right) - \frac{M_t}{p_t} + N_0^X X_0 + \int_0^t N_\tau^X (dD_\tau + dX_\tau) - N_t^X X_t. \tag{2.4}$$

The agent chooses an optimal portfolio trading strategy $\{M_t, N_t, \forall\, t \geq 0\}$ so as to maximize his expected lifetime utility. Precisely, he solves:

$$\max_{\{c_t, M_t, N_t\}} E \int_0^\infty U(c_t, m_t, t) \, dt \text{ s.t. (2.4) holds.}$$

The expectation is taken with respect to the filtration specified earlier. The Euler equations are:

[7] The Euler equation approach has been used in Naik and Lee (1990) and the two approaches are equivalent in the sense that they lead to the same asset prices.

$$\frac{1}{p_t}=\frac{1}{U_c(c_t,\,m_t,\,t)}E_t\left(\int_t^\infty U_m(c_\tau,\,m_\tau,\,\tau)\frac{1}{p_\tau}\,d\tau\right), \tag{2.5}$$

$$X_t=\frac{1}{U_c(c_t,\,m_t,\,t)}E_t\left(\int_t^\infty U_c(c_\tau,\,m_\tau,\,\tau)\,dD\tau\right). \tag{2.6}$$

That is, the reciprocal of the exchange rate equals the expected discounted sum of the future real wealth of one dollar, with the state price deflator being the marginal rate of substitution between consumption and the real money balance. The price of any other asset equals the expected discounted sum of dividends, with the stochastic state price deflator being the marginal rate of substitution between consumption at different dates.

As is typical for a small open economy, the exogenous foreign interest rate, the rate of time preference and the parameters describing consumption must satisfy certain restrictions in order to ensure the existence of an equilibrium. Such a restriction can be obtained by examining an agent's trade-off between consuming at time t and purchasing the foreign bond. The net utility gain from purchasing bond is

$$\frac{dF/F}{dt}+E_t\left(\frac{dU_c/U_c}{dt}\right),$$

where $[(dF/F)/dt]=r$ is the rate of return to holding the bond and $E_t((dU_c/U_c)/dt)$ is the utility loss due to the delay in consumption. Since optimality requires the net utility gain to be zero, the equilibrium restriction is $r=-E_t((dU_c/U_c)/dt)$.[8]

2.3. *Equilibrium exchange rate and asset prices under logarithmic utility*

Market clearing conditions are described as follows. The domestic currency held by the foreign country is assumed to be negligible, implying that the money market is cleared by domestic money demand,[9] i.e., $M_s=M$. Similarly, the demand for the risky stock equals the supply of shares, which is one share, and the demand for the claim on monetary transfers equals the supply, which is also one. Equilibrium prices are such that the representative agent holds neither the domestic nominal bonds nor any other contingent claims, because the net supply of each such asset is zero. Note that the supply of a domestic asset (or money) equals the domestic demand for the asset (or money). This equality holds here not because the economy is closed but rather because the economy is small relative to the outside world and so the foreign

[8] This restriction can be formally derived from the Euler equation (2.6). When there is no uncertainty, this restriction becomes the well-known equality between the real interest rate (r) and the rate of time preference (ρ).

[9] One can easily introduce a stochastic percentage of the domestic currency held by the foreign country, however including an additional stochastic percentage will not change the main results in the current paper, but only complicates the analysis.

demand for its asset (its money) is considered to be negligible, as discussed at the beginning of Section 2.

On the other hand, the goods market clearing condition is quite different here from that in a closed economy. Since the country can have current account surplus or deficit, as discussed in the introduction, aggregate consumption does not necessarily equal the aggregate dividend generated from the domestic stock. Since the country can export the goods to the foreign country to increase its holdings on foreign bonds, the total expenditure on goods is $c\,\mathrm{d}t+\mathrm{d}f$, where $f_t=N_t^{\mathrm{F}}F_t$ is the value of foreign bonds. The total supply of goods is the sum of domestic dividends, $\delta\,\mathrm{d}t$, and return to holding foreign bonds, $rf\,\mathrm{d}t$. Thus, the goods market clearing condition is

$$\mathrm{d}f=(\delta+rf-c)\,\mathrm{d}t.$$

This goods market clearing condition also differs from that in Lucas' (1982) two-country assets pricing model and its application in currency options by Bakshi and Chen (1997). In these models, the equilibrium portfolio of each country is identical to its initial endowment, so that the net trading in assets between the two countries is zero in equilibrium. In contrast, here the net trading in foreign bonds must be non-zero in equilibrium as δ and c vary over time. This difference not only makes it more challenging to solve for the equilibrium portfolio but also leads to important differences in the behavior of the exchange rate: since the exchange rate clears the goods market, the net trading volume affects the exchange rate.

For analytical tractability, I assume that preferences are given by:

Assumption 3 The risk-averse agent's period utility is described by

$$U(c_t, m_t, t)=\mathrm{e}^{-\rho t}[\alpha \ln c_t+(1-\alpha) \ln m_t],\ \alpha\in(0, 1). \tag{2.7}$$

The goods market clearing condition implies that the real wealth, f_t+S_t, is equal to the expected present value of future consumption stream, c_t/ρ. This condition, together with (2.6), helps to determine the equilibrium price of the domestic risky stock, S_t, and the equilibrium quantity of the foreign bonds held by the domestic agent, f_t (see Appendix A for a proof).

Proposition 2.1 *Under Assumptions 1–3, the equilibrium real price of the domestic risky stock at time t,* S_t, *is* $S_t=S(\delta_t)=\delta_t/\rho,\ \forall\, t\in(0,\infty)$ *and the equilibrium value of foreign bonds held by the domestic agent is* $f_t=N_t^{\mathrm{F}}F_t=f_0\mathrm{e}^{(r-\rho)t}$.

Given the logarithmic utility function in Assumption 3, the real price of the risky stock is only affected by aggregate dividend. Precisely, the stock price equals the present value of future dividends discounted at the rate of time preference. The quantity of foreign bonds held by the domestic agent in equilibrium evolves at a constant rate of $r-\rho$. Equivalently, the level of investment in foreign bonds at time t in equilibrium is determined as $N_t^{\mathrm{F}}=N_0^{\mathrm{F}}\mathrm{e}^{-\rho t}$. Therefore, the market portfolio in this small open economy is internationally diversified and consists of the domestic risky stock and f_t amount of foreign bonds.

Using (2.5), (2.6) and the money market clearing condition $M_s=M$, we can derive the equilibrium exchange rate, the nominal interest rate and the real price of the claim on monetary transfers, L_t (see Appendix B for a proof).

Proposition 2.2 *Under Assumptions 1-3, the equilibrium exchange rate is*

$$p_t=\frac{\alpha}{1-\alpha}\cdot\frac{iM_t}{\delta_t+\rho f_t}=\frac{\alpha}{1-\alpha}\cdot\frac{iM_t}{c_t}.$$

The nominal interest rate is $i=(\rho+\beta_m)$ where $\beta_m\equiv\mu_m-\sigma_m^2-\lambda_m k_m-\lambda_m\bar{k}_m$ *is defined through* $E_t(M_t/M_T)=e^{-\beta_m(T-t)}$. *The equilibrium real price at any time t of the claim for monetary transfers is* $L_t=(i-\rho)m_t/\rho$.

In contrast to the real price of the risky stock, the exchange rate is determined by the money supply, aggregate dividends and the level of investment in the foreign bonds, as in a typical small open economy model. This is a consequence of the representative agent's optimal condition, $U_m/U_c=i$, which states intuitively that the marginal rate of substitution between the real money balance and consumption must equal the opportunity cost of holding money (the foregone nominal interest income). Under the logarithmic utility function form, this general relation implies that the flow of services derived from holding money is proportional to the level of consumption. That is, $i(M_t/p_t)=((1-\alpha)/\alpha)c_t$, which leads to the expression for equilibrium exchange rate in Proposition 2.2.

The nominal interest rate is constant and equal to the sum of the rate of time preference and the expected growth rates of money supply and money holding after adjusting the uncertainties in Proposition 2.2. This relation arises from the agent's optimal trade-off between consuming today and purchasing a nominal bond today. Holding a nominal bond for an arbitrarily short period of time and then spending the return on consumption goods has a net gain

$$i+E_t\left(\frac{dp^{-1}/p^{-1}}{dt}\right)+E_t\left(\frac{dU_c/U_c}{dt}\right),$$

where $E_t((dp^{-1}/p^{-1})/dt)$ is the capital loss resulted from inflation and $E_t((dU_c/U_c)/dt)$ is the utility loss from the delay in consumption. Since optimality requires the agent to be indifferent between consuming now and holding a nominal bond at the margin,

$$i=-E_t\left(\frac{dp^{-1}/p^{-1}}{dt}\right)-E_t\left(\frac{dU_c/U_c}{dt}\right).$$

Under the logarithmic utility function and the exchange rate p in Proposition 2.2, this implies $i=(\rho+\beta_m)$.

Also, the real price of the claim on monetary transfers is proportional to real money balances, i.e., the present value of future real monetary transfers is proportional to current real money balances in equilibrium.

Since $c_t = \delta_t + \rho f_t$ and since real prices of the stock and foreign bonds are independent of the money supply process, equilibrium consumption is independent of the money supply process. The domestic agent consumes the dividends generated from the domestic risky stock and the foreign bond. Since the foreign bond price evolves exogenously in equilibrium, equilibrium consumption is determined by the stock dividend process. Under the general process for dividends (2.2), consumption follows a complicated stochastic process. This makes it difficult to compare the results of the current model with those in previous models such as Garman and Kohlhagen (1983) and Merton (1976), who assume that the exchange rate follows a diffusion or jump-diffusion process. To facilitate comparison, let us restrict the dividend process by the following assumption, which allows me to derive currency option pricing formulas that encompass Garman and Kohlhagen (1983) and Merton (1976) as special cases.

Assumption 4 The dividend process (2.2) evolves as:

$$d\delta = (\mu_\delta - \lambda_\delta k_\delta)(\delta + \rho f)\, dt - \rho(r-\rho) f\, dt + \sigma_\delta(\delta + \rho f)\, dz_2 + (Y_\delta - 1)(\delta + \rho f)\, dQ_\delta.$$

Assumption 4 implies the following mixed jump-diffusion process for consumption:

$$\frac{dc}{c} = (\mu_\delta - \lambda_\delta k_\delta)\, dt + \sigma_\delta\, dz_3 + (Y_\delta - 1)\, dQ_\delta. \tag{2.8}$$

Here, μ_δ is the instantaneous expected growth rate; σ_δ^2 is the instantaneous variance of the growth rate, conditional on no arrivals of new important shock. The element dQ_δ is a jump process with a jump intensity parameter λ_δ and $Y_\delta - 1$ is the random variable percentage change in aggregate consumption if the Poisson event occurs. The logarithm of Y_δ is normally distributed with mean θ_δ and variance ϕ_δ^2. The expected jump amplitude, $k_\delta = E(Y_\delta - 1)$, is equal to $\exp(\theta_\delta + \phi_\delta^2/2) - 1$. Also $\bar{k}_\delta = E((1/Y_\delta) - 1)$, is equal to $\exp(-\theta_\delta + \phi_\delta^2/2) - 1$. The random variables $\{z_{2t},\ t \geq 0\}$, $\{Q_{\delta t},\ t \geq 0\}$ and $\{Y_{\delta j},\ j \geq 1\}$ are assumed to be mutually independent. Also, $Y_{\delta j}$ is independent of $Y_{\delta j'}$ for $j \neq j'$. The parameters $(\mu_\delta, \sigma_\delta, \lambda_\delta, \theta_\delta, \phi_\delta)$ are constant.

Under the logarithmic utility function and the above assumption, the restriction on the foreign interest rate, discussed at the end of Section 2.2, becomes $r = \rho + \beta_\delta$, where $\beta_\delta \equiv \mu_\delta - \sigma_\delta^2 - \lambda_\delta k_\delta - \lambda_\delta \bar{k}_\delta$ is defined through $E_t(c_t/c_T) = e^{-\beta_\delta (T-t)}$.

3. Pricing currency and stock options

3.1. Dynamics of the exchange rate

Let us examine the dynamics followed by the exchange rate from the domestic agent's perspective. Since p_t is a function of M^s and c, applying Ito's lemma yields

$$\frac{dp}{p} = (\mu_p - \lambda_m k_m - \lambda_\delta \bar{k}_\delta)\, dt + \sigma_m\, dz_1 - \sigma_\delta\, dz_2 + (Y_m - 1)\, dQ_m + (Y_\delta^{-1} - 1)\, dQ_\delta, \tag{3.1}$$

where $\mu_{\mathrm{p}}=\mu_{\mathrm{m}}-\beta_{\delta}$. Under the equilibrium conditions for the nominal interest rate and the rate of time preference, the exchange rate dynamics can be rewritten as:

$$\frac{\mathrm{d}p}{p}=(i-r+\sigma_{\mathrm{m}}^{2}+\lambda_{\mathrm{m}}\bar{k}_{\mathrm{m}}-\lambda_{\delta}\bar{k}_{\delta})\,\mathrm{d}t+\sigma_{\mathrm{m}}\,\mathrm{d}z_{1}-\sigma_{\delta}\,\mathrm{d}z_{2}+(Y_{\mathrm{m}}-1)\,\mathrm{d}Q_{\mathrm{m}}+\left(\frac{1}{Y_{\delta}}-1\right)\mathrm{d}Q_{\delta}.$$

The key feature of the above exchange rate is that it is derived endogenously from the underlying processes for the money supply, aggregate dividend and the foreign bond price. This endogeneity is in stark contrast with the arbitrariness in the existing partial equilibrium currency option models mentioned in the introduction. Clearly, the exchange rate is affected by the domestic government monetary policy, aggregate dividend and the level of foreign investment.

The domestic government monetary policy and aggregate dividends affect the real price of the domestic risky stock and the exchange rate differently. The difference is crystal clear under the logarithmic utility. The real price of the domestic risky stock is solely determined by aggregate dividends and the level of investment in foreign assets, where monetary policies play no role. The exchange rate incorporates jump components from both aggregate consumption and the money supply, while the stock price is only affected by the jump risk from the aggregate dividend. Thus, the current model is able to explain why discontinuities in exchange rate movements are more prevalent than in stock prices, a feature empirically documented by Jorion (1988). Examining the sample paths of exchange rates and the NYSE stock market index, Jorion finds that exchange rates display significant jump components, while discontinuities are harder to detect in the stock market.

Specifically, the expected growth rate of the exchange rate, μ_{p}, is associated with the drifts of the money supply and aggregate consumption. It is also affected by the instantaneous variance of the growth rate of consumption and the jump component in consumption. The exchange rate dynamics incorporate the two independent jump components from the money supply and aggregate dividend. Obviously, the jump in the exchange rate generated by aggregate dividends must be priced. The instantaneous variance of the growth rate of the exchange rate is the sum of the variances in the money supply and consumption, $\sigma_{\mathrm{m}}^{2}+\sigma_{\delta}^{2}$. On the other hand, the stock price is completely described by the parameters underlying aggregate consumption. These requirements suggest that cross-equation restrictions must be imposed on the coefficients when the processes for the exchange rate and the stock price are to be estimated.[10]

[10] In a sequel work studying the empirical implication of the systematic jump risks in currency option prices, I have used these cross-equation restrictions for estimating parameters of the underlying processes for exchange rates and stock indices (see Cao, 1997).

3.2. Domestic risk-averse agent's valuation of currency and stock options

Now consider the valuation of European style currency options. According to the agent's maximization condition (2.6), for any contingent claim with maturity T and dividend q_T, its real price at time $t \leq T$, $x_t(T)$, is

$$x_t(T) = \frac{1}{U_c(c_t, m_t, t)} E_t(q_T U_c(c_T, m_T, T)).$$

For a European call written on the spot exchange rate with a striking price K that matures at time T, its nominal price at time $t \leq T$, $CC_t^D(p_t, T)$, is

$$CC_t^D(p_t, T) = p_t e^{-\rho(T-t)} c_t E_t\left(\frac{1}{c_T} \frac{\max(p_T - K, 0)}{p_T}\right).$$

Similarly, for a European put written on the spot exchange rate with a striking price K that matures at time T, its nominal price at time $t \leq T$, $CP_t^D(p_t, T)$, is

$$CP_t^D(p_t, T) = p_t e^{-\rho(T-t)} c_t E_t\left(\frac{1}{c_T} \frac{\max(K - p_T, 0)}{p_T}\right).$$

The joint density function for (c_T, M_T^s) conditional on (c_t, M_t^s), $f(c_T, M_T^s, T | c_t, M_t^s, t)$, is known. We can explicitly compute the prices of the European call and put, since the exchange rate is a function of c and M^s. To facilitate the presentation of equilibrium prices of call and put options, let C_{GK} and P_{GK} be, respectively, the currency call and put prices derived by Garman and Kohlhagen (1983) with the following expressions

$$C_{GK}(p_t, \tau; K, r_F, r_D, \sigma_E) = p_t e^{-r_F \tau} N(d_1) - K e^{-r_D \tau} N(d_2),$$

$$P_{GK}(p_t, \tau; K, r_F, r_D, \sigma_E) = K e^{-r_D \tau} N(-d_2) - p_t e^{-r_F \tau} N(-d_1),$$

where

$$\tau = T - t,\ d_1 = \frac{\ln p_t/K + (r_D - r_F + \frac{1}{2}\sigma_E^2)\tau}{\sigma_E \sqrt{\tau}},\ d_2 = d_1 - \sigma_E \sqrt{\tau}.$$

Then, the option prices in the current model are described as follows (see Appendix C for a proof):

Proposition 3.1 *Under Assumptions 1-4, equilibrium nominal prices of currency call and put are:*

$$CC_t^D(p_t, T) = \sum_{n_\delta=0}^{\infty} \sum_{n_m=0}^{\infty} P(\lambda_\delta, \lambda_m) C_{GK}(p_t, \tau; K, r_\delta, i_m, \sigma_{\delta,m}), \tag{3.2}$$

and

$$CP_t^{\mathrm{D}}(p_t, T)=\sum_{n_\delta=0}^{\infty}\sum_{n_{\mathrm{m}}=0}^{\infty}P(\lambda_\delta, \lambda_{\mathrm{m}})P_{\mathrm{GK}}(p_t, \tau; K, r_\delta, i_{\mathrm{m}}, \sigma_{\delta,\mathrm{m}}), \tag{3.3}$$

where $P(\cdot, \cdot)$ is defined as

$$P(a, b)=\frac{\mathrm{e}^{-a\tau}(a\tau)^{n_\delta}}{n_\delta!}\cdot\frac{\mathrm{e}^{-b\tau}(b\tau)^{n_{\mathrm{m}}}}{n_{\mathrm{m}}!},$$

and where:

$$r_\delta=r+\lambda_\delta\bar{k}_\delta+\frac{n_\delta\left(\theta_\delta-\frac{1}{2}\phi_\delta^2\right)}{\tau}=\rho+\mu_\delta-\lambda_\delta k_\delta-\sigma_\delta^2+\frac{n_\delta\left(\theta_\delta-\frac{1}{2}\phi_\delta^2\right)}{\tau},$$

$$i_{\mathrm{m}}=i+\lambda_{\mathrm{m}}\bar{k}_{\mathrm{m}}+\frac{n_m\left(\theta_{\mathrm{m}}-\frac{1}{2}\phi_{\mathrm{m}}^2\right)}{\tau}=\rho+\mu_{\mathrm{m}}-\lambda_m k_{\mathrm{m}}-\sigma_{\mathrm{m}}^2+\frac{n_{\mathrm{m}}\left(\theta_{\mathrm{m}}-\frac{1}{2}\phi_{\mathrm{m}}^2\right)}{\tau},$$

$$\sigma_{\delta,\mathrm{m}}=\sqrt{\sigma_\delta^2+\frac{n_\delta\phi_\delta^2}{\tau}+\sigma_{\mathrm{m}}^2+\frac{n_{\mathrm{m}}\phi_{\mathrm{m}}^2}{\tau}}.$$

Consider the call price for example. C_{GK} is an increasing function of the conditional domestic interest rate, i_{m}, and the conditional exchange rate volatility, $\sigma_{\delta,\mathrm{m}}$, but a decreasing function of the conditional foreign interest rate, r_δ. The currency option prices depend intuitively on the fundamental parameters. First, an increase in the conditional consumption volatility, σ_δ, or the volatility of jump size, ϕ_δ, induces a lower r_δ and a higher $\sigma_{\delta,\mathrm{m}}$: the joint consequence is a higher currency call price. Second, a higher conditional volatility of money supply, σ_{m}, or higher volatility of the corresponding jump, ϕ_{m}, does not necessarily imply a higher call price. This is because an increase in σ_{m} or ϕ_{m} reduces i_{m} and increases $\sigma_{\delta,\mathrm{m}}$ simultaneously, while the increase in $\sigma_{\delta,\mathrm{m}}$ tends to increase the call price and the reduction in i_{m} tends to reduce the call price. Further, the call value is positively related to the instantaneous expected growth rate of the money supply, μ_{m}, and negatively related to the instantaneous expected growth rate of aggregate consumption, μ_δ. The effects of parameters $(\lambda_\delta, \lambda_{\mathrm{m}}, \theta_\delta, \theta_{\mathrm{m}})$ on currency call prices are ambiguous.

Note that if there were no jump component in aggregate dividends, the currency call and put prices in Proposition 3.1 would reduce to Merton's (1976) price equations. In this case, the only jump uncertainty underlying the exchange rate would be from the money supply and this jump uncertainty is not priced.

The Euler equation (2.6) can also be used to price European style options on the domestic risky stock. Denote the real price of a call (put) on the risky stock at time t with a striking price k and an expiration date T by $C_t(k, S_t, T)$ $(P_t(k, S_t, T))$. As shown in Appendix D, the stock option prices are completely described by the parameters underlying aggregate dividend. The explicit valuations are stated in the following proposition.

Proposition 3.2 *Under Assumptions 1-4, $C_t(k, S_t, T)$ and $P_t(k, S_t, T)$ are:*

$$C_t(k, S_t, T)=\sum_{n_\delta=0}^{\infty}\frac{e^{-\lambda_\delta\tau}(\lambda_\delta\tau)^{n_\delta}}{n_\delta!}C_{GK}\left(S_t+f_t,\tau;k+f_t e^{(r-\rho)\tau},\rho,r_\delta,\sigma_\delta^2+\frac{n_\delta\phi_\delta^2}{\tau}\right)$$

and

$$P_t(k, S_t, T)=\sum_{n_\delta=0}^{\infty}\frac{e^{-\lambda_\delta\tau}(\lambda_\delta\tau)^{n_\delta}}{n_\delta!}P_{GK}\left(S_t+f_t,\tau;k+f_t e^{(r-\rho)\tau},\rho,r_\delta,\sigma_\delta^2+\frac{n_\delta\phi_\delta^2}{\tau}\right),$$

where r_δ is defined in Proposition 3.1.

In contrast to currency options, real prices of stock options are independent of the uncertainty underlying the domestic money supply. Although aggregate consumption affects both the stock price and the exchange rate, the parameters describing the dynamics of consumption affect stock options and currency options differently. For example, the instantaneous growth rate of consumption, μ_δ, positively affects the price of a call on the stock but negatively affects the price of a call on the exchange rate. An increase in σ_δ or ϕ_δ increases the currency call prices as discussed earlier, but does not necessarily increase the stock call price. For the call price on the stock, increasing σ_δ or ϕ_δ implies a higher instantaneous stock volatility $\sigma_\delta^2+(n_\delta\phi_\delta^2/\tau)$, which in turn induces a higher call price. However, an increase in σ_δ or ϕ_δ also reduces r_δ at the same time. Since r_δ is positively related to the call price, the joint effect of a lower r_δ and a higher $\sigma_\delta^2+(n_\delta\phi_\delta^2/\tau)$ on the call price, is ambiguous. This further illustrates the difference between currency options and stock options.[11]

Note that the market portfolio in this small open economy consists of the domestic stock and the foreign bond. If the domestic agent did not hold the foreign bond in equilibrium, this small open economy would be similar to a closed economy in which the market portfolio is the domestic stock. In this case, the stock option formulas in Proposition 3.2 would reduce to those on the market portfolio in Naik and Lee (1990) with jump risks and logarithmic utility.

4. Foreign agent's risk-neutral valuation

I now use the above framework to examine the analog of Siegel's paradox in currency option valuation. The purpose is to identify the necessary restrictions that

[11] The common belief is that an increase in stock volatility will be accomplished by an increase in call price according to the risk-neutral based Black–Scholes model (1973). Bailey and Stulz (1989) show that this common belief is not necessarily supported in an equilibrium context, which is confirmed by our results.

must be imposed on the risk-neutral process of the exchange rate if foreign agents use the risk-neutral approach.

The analog of Siegel's paradox in currency option valuation refers to the violation of the parity conditions between domestic and foreign investors' valuations. A call option from the domestic agent's point of view is a put option from the foreign investor's perspective. A call gives the domestic agent the right to buy the foreign currency from the foreign agent. On the other hand, a put option from the point of view of the foreign agent is an option to sell the domestic currency to obtain the foreign currency. In fact, the expression of "the call option value from the domestic agent's view" is the same as the expression of "the put option value from the foreign agent's view". The foreign agent's risk-neutral valuation of the put option is

$$CP_t^{\mathrm{F}}(1/p_t, T)=\mathrm{e}^{-r(T-t)}E_t^{\mathrm{F}}\left(\max\left(1-\frac{K}{p_T}, 0\right)\right),$$

where $E_t^{\mathrm{F}}(\cdot)$ is the risk-neutral expectation operator conditional on the information at time t available to the foreign investor. According to the law of one price, $CP_t^{\mathrm{F}}(1/p_t, T)$ converted into the domestic currency at the spot exchange rate should be the same as $CC_t^{\mathrm{D}}(p_t, T)$. That is

$$p_tCP_t^{\mathrm{F}}(1/p_t, T)=CC_t^{\mathrm{D}}(p_t, T). \tag{4.1}$$

Similarly, the put value from the domestic agent's point should equal the call value from the foreign agent's point, once the price is converted into the domestic currency at the spot exchange rate. That is

$$p_tCC_t^{\mathrm{F}}(1/p_t, T)=CP_t^{\mathrm{D}}(p_t, T), \tag{4.2}$$

where

$$CC_t^{\mathrm{F}}(1/p_t, T)=\mathrm{e}^{-r(T-t)}E_t^{F}\left(\max\left(\frac{K}{p_T}-1, 0\right)\right).$$

The relations in (4.1) and (4.2) are unique to currency options. As pointed out by Dumas et al. (1995), if both the domestic and foreign investors assume their own risk neutral processes, even in the case where the jump component in the exchange rate is uncorrelated with the consumption, applying Merton's formula generates an analog to Siegel's paradox that either (4.1) or (4.2) is violated. The reason is that both investors use different probability measures for the exchange rate. To see this, let x be the risk-neutral exchange rate expressed as the relative price of the foreign currency in terms of the home currency. The risk-neutral process is usually assumed to be

$$\frac{\mathrm{d}x}{x}=(i-r-\lambda_xE(Y_x-1))\,\mathrm{d}t+\sigma\,\mathrm{d}w_x+(Y_x-1)\,\mathrm{d}Q_x,$$

where the difference between the domestic and the foreign interest rate, $i-r$, is the risk-neutral drift rate. The foreign agent observes the same exchange rate dynamics but instead expresses the spot rate as $y=1/x$, the relative price of the home currency expressed in terms of the foreign currency. The risk-neutral process for y is usually assumed by the foreign investor to be

$$\frac{dy}{y}=(r-i-\lambda_y E(Y_y-1))\,dt+\sigma\,dw_y+(Y_y-1)\,dQ_y,$$

where the difference between the foreign and the domestic interest rate, $r-i$, is the risk-neutral drift rate. Obviously,

$$\frac{dx^{-1}}{x^{-1}}=\left(r-i+\sigma_x^2+\lambda_x E\left(\frac{(Y_x-1)^2}{Y_x}\right)-\lambda_x E\left(\frac{1}{Y_x}-1\right)\right)dt-\sigma_x\,dw_x+\left(\frac{1}{Y_x}-1\right)dQ_y\neq\frac{dy}{y},$$

with $\sigma_x=\sigma_y$ and $Y_y=1/Y_x$. The extra term, $\sigma_x^2+\lambda_x E[(Y_x-1)^2/Y_x]$, appears in the drift for dx^{-1}/x^{-1}. Bardhan (1995) calls this extra term the "directional adjustments" and suggests that the foreign investor use dx^{-1}/x^{-1} as his risk-neutral process for y, or vice verse.[12] Strictly speaking, dx^{-1}/x^{-1} is not the risk-neutral process for y since the drift for y is no longer the risk-neutral drift $r-i$. Instead, the drift is $r-i+\sigma_x^2+\lambda_x E[(Y_x-1)^2/Y_x]$. One may interpret dx^{-1}/x^{-1} as the domestic risk-neutral process for y. Bardhan's directional adjustments would eliminate the paradox if the jump risk in the exchange rate were uncorrelated with consumption. However, they are insufficient to eliminate the paradox when the exchange rate is correlated with consumption, as in our case.

To examine the necessary restrictions on the risk-neutral process of the exchange rate when the jump component in the exchange rate is correlated with aggregate consumption, denote $\omega_t=1/p_t=(1-\alpha/\alpha i)(c_t/M_t)$. The actual process for ω viewed by both domestic and foreign investors is

$$\frac{d\omega}{\omega}=(r-i+\sigma_\delta^2+\lambda_\delta\bar{k}_\delta-\lambda_m\bar{k}_m)\,dt+\sigma_\delta\,dz_2-\sigma_m\,dz_1+(Y_\delta-1)\,dQ_\delta \tag{4.3}$$

$$+\left(\frac{1}{Y_m}-1\right)dQ_m.$$

If the foreign agent uses the risk-neutral valuation to price the currency options, we can identify the restrictions on the risk-neutral process for ω by comparing the risk-neutral valuation of the options with (3.2) and (3.3). Denote the risk-neutral process for ω as follows:

$$\frac{d\omega^*}{\omega^*}=(r-i-\lambda_\delta^* k_\delta^*-\lambda_m^*\bar{k}_m^*)\,dt+\sigma_\delta\,dz_2^*-\sigma_m\,dz_1^*+(Y_\delta^*-1)\,dQ_\delta^* \tag{4.4}$$

$$+\left(\frac{1}{Y_m^*}-1\right)dQ_m^*.$$

[12] The "directional adjustments" are sometimes referred to as the quanto adjustments or the convexity effects.

The following proposition details the foreign agents' risk-neutral valuations of the corresponding currency options (see Appendix E for a proof):

Proposition 4.1 *Under the risk-neutral process of the exchange rate (4.4), the foreign agents' valuations of* $CP_t^F(1/p_t, T)$ *and* $CC_t^F(1/p_t, T)$ *are:*

$$CP_t^F(1/p_t,T)=\sum_{n_\delta=0}^{\infty}\sum_{n_m=0}^{\infty}P(\lambda_\delta^*(k_\delta^*+1),\lambda_m^*)C_{GK}\left(1,\tau;\frac{K}{p_t},r_\delta^*,i_m^*,\sigma_{\delta,m}^*\right) \tag{4.5}$$

and

$$CC_t^F(1/p_t,T)=\sum_{n_\delta=0}^{\infty}\sum_{n_m=0}^{\infty}(\lambda_\delta^*(k_\delta^*+1),\lambda_m^*)P_{GK}\left(1,\tau;\frac{K}{p_t},r_\delta^*,i_m^*,\sigma_{\delta,m}^*\right), \tag{4.6}$$

where $C_{GK}(\cdot)$, $P_{GK}(\cdot)$ *and* $P(\cdot,\cdot)$ *are defined in the previous section and where:*

$$r_\delta^*=r+\lambda_\delta^*k_\delta^*+\frac{n_\delta\left(\theta_\delta^*+\frac{1}{2}\phi_\delta^{*2}\right)}{\tau},$$

$$i_m^*=i+\lambda_m^*k_m^*+\frac{n_m\left(\theta_m^*-\frac{1}{2}\phi_m^{*2}\right)}{\tau},$$

$$\sigma_{\delta,m}^*=\sqrt{\sigma_\delta^2+\sigma_m^2+\frac{n_\delta\phi_\delta^{*2}}{\tau}+\frac{n_m\phi_m^{*2}}{\tau}}.$$

In order to ensure the parity conditions (4.1) and (4.2), the following restrictions on the risk-neutral process (4.4) must be satisfied:

$$\begin{aligned}
&\lambda_m^*=\lambda_m, && \lambda_\delta^*=\lambda_\delta(1+\bar{k}_\delta),\\
&\bar{k}_m^*=\bar{k}_m, && k_\delta^*=E(Y_\delta^*-1)=-\frac{\bar{k}_\delta}{k_\delta+1},\\
&\theta_m^*=\theta_m, && \theta_\delta^*=\theta_\delta-\phi_\delta^2,\\
&\phi_m^*=\phi_m, && \phi_\delta^*=\phi_\delta.
\end{aligned} \tag{4.7}$$

Under these restrictions, the actual probability is transformed into the risk-neutral or the equivalent martingale measure. In this case, the risk-neutral process can be expressed as:

$$\frac{d\omega^*}{\omega^*}=\frac{d\omega}{\omega}-\sigma_\delta^2\,dt-Y_\delta(1-e^{-\phi_\delta^2})\,dQ_\delta.$$

In light of (4.3) and (4.4), this implies $dz_2^*=dz_2-\sigma_\delta\,dt$, $dz_1^*=dz_1$, $(Y_\delta^*-1)\,dQ_\delta=(Y_\delta\,e^{-\phi_\delta^2}-1)\,dQ_\delta$. In fact, no adjustment is needed for the money supply process since it is assumed to be independent of the consumption process. For the consumption process, one needs to adjust not only the risk from the diffusion (dz_2) and jump intensity parameters (λ_δ, k_δ), but also that from the jump size (θ_δ). The adjustments on (dz_2, λ_δ, k_δ) are the directional adjustments suggested by Bardhan (1995) for the case where the jump in the exchange rate is not correlated with aggregate consumption. The additional adjustment on θ_δ reflects the fact that the jump risk in exchange rate is related to aggregate consumption. Note that in the special case where the jump size in consumption is certain, i.e., $\phi_\delta=0$, no adjustment is needed for the jump size and so the jump component in consumption can be hedged away (see Bardhan, 1995).

The above adjustments are specific to the utility function (2.7), but the general message of the exercise should be valid for a wider class of utility functions. That is, if the jump components in the exchange rate are related to those in consumption, the appropriate risk-neutral or the equivalent martingale process for the exchange rate should be based on an equilibrium model in an international context in order to ensure the parity conditions (4.1) and (4.2). Adjustments for the risk-neutral process must be made on all uncertainties, including the Brownian motion, the jump intensity and the jump size. Making only the directional adjustments is not enough.

5. An extension of the model

The above discussions have employed the assumption that the government monetary policy is independent of aggregate dividend. In this section, I extend the previous framework to incorporate a correlation between the money supply and aggregate dividend. This correlation arises when the government uses the monetary policy to react to shocks in aggregate output. I capture this possible active monetary policy by allowing for a correlation between the shock dz_1 in the money supply and the shock dz_2 in aggregate dividends to be correlated, with a correlation coefficient ρ_{12}.[13]

With this correlation structure, the jump component in the money supply is still independent of aggregate dividends. Because of the separability between consumption and real money balances in the utility function, the exchange rate, the nominal interest rate, the restriction on the rate of time preference, the risky stock price and the equilibrium quantity of foreign bonds held by the domestic agent are the same as in previous sections. More importantly, the stock option valuation in Proposition 3.2 is unchanged and so is still independent of the money supply. In contrast, the correlation between dz_1 and dz_2 affects currency option valuations from the domestic agent's view. To see this, one can verify that Proposition 3.1 still holds with the following modification:

[13] I thank John Hull for suggesting this extension. Although in principle one can also allow the money supply and aggregate dividends to be correlated through the jumps, analyzing this type of correlation is not tractable.

$$\sigma^2_{\delta,\mathrm{m}}=\sigma^2_{\delta}+\frac{n_\delta\phi^2_\delta}{\tau}-2\rho_{12}\sqrt{\left(\sigma^2_\delta+\frac{n_\delta\phi^2_\delta}{\tau}\right)\left(\sigma^2_\mathrm{m}+\frac{n_\mathrm{m}\phi^2_\mathrm{m}}{\tau}\right)}+\sigma^2_\mathrm{m}+\frac{n_\mathrm{m}\phi^2_\mathrm{m}}{\tau}.$$

Since the parameter ρ_{12} influences the currency option price only through $\sigma_{\delta,\mathrm{m}}$, a call on the exchange rate with $\rho_{12}<0$ will have a higher value than when $\rho_{12}=0$, because the call price is an increasing function of $\sigma_{\delta,\mathrm{m}}$.

One can also examine the analog of Siegel's paradox through the hypothetical exercise in Section 4. The risk neutral valuations in Proposition 4.1 are modified through the conditional instantaneous variance below:

$$\sigma^{*2}_{\delta,\mathrm{m}}=\sigma^2_{\delta}+\frac{n_\delta\phi^{*2}_\delta}{\tau}-2\rho_{12}\sqrt{\left(\sigma^2_\delta+\frac{n_\delta\phi^{*2}_\delta}{\tau}\right)\left(\sigma^2_\mathrm{m}+\frac{n_\mathrm{m}\phi^{*2}_\mathrm{m}}{\tau}\right)}+\sigma^2_\mathrm{m}+\frac{n_\mathrm{m}\phi^{*2}_\mathrm{m}}{\tau}.$$

The restrictions imposed on the risk-neutral process of the exchange rate are the same as in (4.7). The risk-neutral process now is expressed as:

$$\frac{\mathrm{d}\omega^*}{\omega^*}=\frac{\mathrm{d}\omega}{\omega}-(\sigma^2_\delta-\rho_{12}\sigma_\delta\sigma_\mathrm{m})\,\mathrm{d}t-Y_\delta(1-\mathrm{e}^{-\phi^2_\delta})\,\mathrm{d}Q_\delta.$$

In light of (4.3), this implies $\mathrm{d}z^*_2=\mathrm{d}z_2-\sigma_\delta\,\mathrm{d}t$, $\mathrm{d}z^*_1=\mathrm{d}z_1-\rho_{12}\sigma_\delta\,\mathrm{d}t$, $(Y^*_\delta-1)\,\mathrm{d}Q_\delta=(Y_\delta\mathrm{e}^{-\phi^2_\delta}-1)\,\mathrm{d}Q_\delta$. Compared with the adjustments made for the risk-neutral process (4.4) where the correlation is zero, an additional adjustment on $\mathrm{d}z_1$ in the magnitude of $-\rho_{12}\sigma_\delta\,\mathrm{d}t$ is needed to reflect the fact that the money supply is correlated with aggregate consumption. In this case, the exchange rate is correlated with aggregate consumption, not only directly, but also indirectly through the correlation between the money supply and aggregate consumption. Both correlations must be priced for currency options.

6. Conclusion

This paper uses an equilibrium model to investigate the joint dynamics of the exchange rate and the market portfolio in a small open monetary economy with jump-diffusion money supplies and jump-diffusion aggregate dividends. It is shown that the exchange rate and the market portfolio are strongly correlated since both are driven by the same economic fundamentals. In particular, the exchange rate is affected by government monetary policies, aggregate dividends and the level of foreign investments, while the real price of the domestic market portfolio is determined only by the aggregate consumption. Furthermore, options on the exchange rate and the market portfolio are evaluated in the equilibrium context. The analysis shows that parameters describing the same economic fundamentals have very different effects on currency and stock options. The equilibrium conditions imply that cross-equation restricts should be imposed when the distributions of the exchange rate and the market portfolio are estimated. Such cross-equation restrictions have important implications for currency and stock option markets.

An empirical investigation of the model's predictions is a natural step to take and has been completed in Cao (1997). The detailed empirical procedure is not presented here because of lack of space. Some of the findings can be summarized here. Using the equilibrium conditions imposed on the joint distribution of the exchange rate and the price of the domestic market portfolio, I empirically estimate the parameters underlying the joint distribution through the maximum likelihood method. The likelihood ratio tests strongly reject the hypothesis that there is no systematic jumps in the exchange rate. Further, I applied the estimated parameters to currency option pricing. With parameters estimated from the joint movements of the exchange rate and the stock market, it is shown that the current model can perform better than both the GK pure diffusion model and Merton's non-systematic jumps model. For example, for short-maturity call options written on the three exchange rates (C$/US$, US$/DM and C$/DM), the current model provides a 28% upward correction on the price generated by the GK model, a magnitude close to eliminating the price bias (29%) suggested by previous evidence (Bodurtha and Courtadon, 1987).

Acknowledgements

This research is supported by the Social Sciences and Humanities Research Council of Canada. I am grateful to Phelim Boyle, Benoit Carmichael, Dan Bernhardt, John Hull, Robert Jarrow, Shouyong Shi and Alan White for discussions and comments on an earlier version of this paper. I have also benefited from comments and suggestions of seminar participants at Concordia, Houston, Iowa, McGill, Queen's, Toronto, Vanderbilt, Virginia, Waterloo and York, the 1996 NFA, 1997 SED and 1997 FEMES meetings. All remaining errors are mine alone.

Appendix A. Proof of Proposition 2.1

The risky stock price and the foreign bond price must satisfy the first order condition (2.6). Thus

$$S_t=\frac{1}{U_{c_t}}E_t\left(\int_t^{\infty}U_{c_T}\delta_T\,\mathrm{d}T\right)=c_tE_t\left(\int_t^{\infty}\mathrm{e}^{-\rho(T-t)}\frac{\delta_T}{c_T}\,\mathrm{dT}\right),\ F_t=\mathrm{e}^{-r(T-t)}=\frac{1}{U_{c_t}}E_t(U_{C_T}). \tag{A1}$$

Since the real wealth in equilibrium, $S+f$, equals the expected present value of the future consumption stream, c/ρ, thus $c=\rho S+\rho f$. From the flow budget constraint (2.3), we have

$$\mathrm{d}f=(\delta+rf-c)\,\mathrm{d}t=(\delta+rf-\rho S-\rho f)\,\mathrm{d}t. \tag{A2}$$

The stock price S_t and the quantity of foreign bonds held by the domestic agent f are solved from Eqs. (A1) and (A2). The solutions are $S=\delta/\rho$ and $f_t=f_0\mathrm{e}^{(r-\rho)t}$.■

Appendix B. Proof of Proposition 2.2

Since the money supply process (2.1) is independent of the consumption process (2.8), the joint distribution of (M_T, c_T) conditional on (M_t, c_t) is:

$$f(M_T, c_T, |M_t, c_t, t)=g(M_T, T|M_t, t)h(c_T, T|c_t, t),$$

where

$$g(M_T, T|M_t, t)=\sum_{n_m=0}^{\infty}\frac{e^{-\lambda_m\tau}(\lambda^m\tau)^{n_m}}{n^m!}\frac{1}{\sqrt{2\pi\Sigma_m}}e^{-\frac{(\ln M_T-\psi_m)^2}{2\Sigma_m}},$$

$$h(c_T, T|c_t, t)=\sum_{n_\delta=0}^{\infty}\frac{e^{-\lambda_\delta\tau}(\lambda_\delta\tau)^{n_\delta}}{n_\delta!}\frac{1}{\sqrt{2\pi\Sigma_\delta}}e^{-\frac{(\ln c_T-\psi_\delta)^2}{2\Sigma_\delta}},$$

with

$$\psi_m=\ln M_t+(\mu_m-\lambda_m k_m-\frac{1}{2}\sigma_m^2)(T-t)+n_m\theta_m,\ \Sigma_m=\sigma_m^2(T-t)+n_m\phi_m^2,$$

$$\psi_\delta=\ln c_t+(\mu_\delta-\lambda_\delta k_\delta-\frac{1}{2}\sigma_\delta^2)(T-t)+n_\delta\theta_\delta,\ \Sigma_\delta=\sigma_\delta^2(T-t)+n_\delta\phi_\delta^2.$$

According to the first order condition (2.5) and utility function (2.7),

$$\frac{1}{p_t}=\frac{1}{U_{c_t}}E_t\left(\int_t^{\infty}U_{m_T}\frac{1}{p_T}\mathrm{d}T\right)=\frac{1-\alpha}{\alpha}c_t e^{\rho t}\int_t^{\infty}e^{-\rho T}E_t\left(\frac{1}{M_T}\right)\mathrm{d}T.$$

Since $E_t(\frac{1}{M_T})=\frac{1}{M_t}e^{-(\mu_m-\sigma_m^2-\lambda_m k_m-\lambda_m\bar{k}_m)(T-t)}$, then we have

$$p_t=\frac{\alpha}{1-\alpha}\frac{M_t}{c_t}(\rho+\mu_m-\lambda_m k_m-\sigma_m^2-\lambda_m\bar{k}_m),\ \forall\ t\in(0,\infty).$$

The first order conditions (2.5) and (2.6) imply $i=U_m/U_c$. Under the logarithmic utility function (2.7), $i=(\alpha/(1-\alpha))(m_t/c_t)$. Therefore, $i=\rho+\mu_m-\lambda_m k_m-\sigma_m^2-\lambda_m\bar{k}_m$.

Also the expected present value of services (im) generated by money equals m_t+L_t. That is,

$$m_t+L_t=\frac{1}{U_{c_t}}E_t\left(\int_t^{\infty}U_{c_T}\frac{iM_T}{p_T}\mathrm{d}T\right)=\frac{im_t}{\rho}.$$

Therefore, $L_t=\frac{i-\rho}{\rho}m_t$. ■

Appendix C. Proof of Proposition 3.1

For a European call written on the spot exchange rate with a striking price K that matures at time T, its nominal price at time $t \leq T$, $CC_t^{\mathrm{D}}(p_t, T)$, is

$$CC_t^{\mathrm{D}}(p_t, T) = p_t \mathrm{e}^{-\rho(T-t)} c_t E_t\left(\frac{1}{c_T}\frac{1}{p_T}\max(p_T - K, 0)\right).$$

Since $p = (\alpha i/(1-\alpha))(M/c) = A(M/c)$, then

$$CC_t^{\mathrm{D}}(p_t, T) = p_t \mathrm{e}^{-\rho\tau} c_t E_t\left(\max\left(\frac{1}{c_T} - \frac{K}{A}\frac{1}{M_T}, 0\right)\right) = p_t \mathrm{e}^{-\rho\tau} c_t \int_{-\infty}^{\infty}\left(\int_{\frac{K}{A}c_T}^{\infty}\left(\frac{1}{c_T}\right.\right.$$

$$\left.\left. - \frac{K}{A}\frac{1}{M_T}\right) g(M_T|M_t)\,\mathrm{d}M_T\right) h(c_T|c_t)\,\mathrm{d}c_T.$$

Tedious exercises show that

$$\int_{-\infty}^{\infty}\left(\int_{\frac{K}{A}c_T}^{\infty}\frac{1}{c_T} g(M_T|M_t)\,\mathrm{d}M_T\right) h(c_T|c_t)\,\mathrm{d}c_T$$

$$= \sum_{n_\delta=0}^{\infty}\sum_{n_m=0}^{\infty} P(\lambda_\delta, \lambda_{\mathrm{m}}) \mathrm{e}^{-\psi_\delta + \frac{1}{2}\Sigma_\delta} \int_{-\infty}^{\infty} z(v)\,\mathrm{d}v \int_{\underline{w1}}^{\infty} z(w)\,\mathrm{d}w,$$

where $P(\cdot,\cdot)$ is defined in Proposition 3.1, $z(\cdot)$ is the standard normal density and $\underline{w1} = (-d_1^{\delta,\mathrm{m}} - \varphi v)/(\sqrt{1-\varphi^2})$ with

$$d_1^{\delta,\mathrm{m}} = \frac{\ln p_t/K + (r_\delta - i_{\mathrm{m}} + \frac{1}{2}\sigma_{\delta,\mathrm{m}}^2)\tau}{\sigma_{\delta,\mathrm{m}}\sqrt{\tau}}, \quad \varphi = -\sqrt{\frac{\Sigma_\delta}{\Sigma_\delta + \Sigma_{\mathrm{m}}}} = -\frac{\sqrt{\Sigma_\delta}}{\sigma_{\delta,\mathrm{m}}\sqrt{T-t}}.$$

$\sigma_{\delta,\mathrm{m}}$ is defined in Proposition 3.1. According to Abramowitz (1965), the probability function for a bivariate normal with correlation φ is defined as

$$\int_a^{\infty} z(v)\,\mathrm{d}v \int_{\frac{b-\varphi v}{\sqrt{1-\varphi^2}}}^{\infty} z(w)\,\mathrm{d}w = L(a, b, \varphi).$$

Thus

$$\int_{-\infty}^{\infty} z(v)\,\mathrm{d}v \int_{\underline{w1}}^{\infty} z(w)\,\mathrm{d}w = L(-\infty, -d_1^{\mathrm{c,m}}, \varphi) = N(d_1^{\mathrm{c,m}}),$$

where $N(a)=\int_{-\infty}^{a} z(v)\,\mathrm{d}v$. Therefore,

$$\int_{-\infty}^{\infty}\left(\int_{\frac{K}{A}c_T}^{\infty}\frac{1}{c_T}g(M_T|M_t)\,\mathrm{d}M_T\right)h(c_T|c_t)\,\mathrm{d}c_T=\sum_{n_\delta=0}^{\infty}\sum_{n_m=0}^{\infty}P(\lambda_\delta,\lambda_{\mathrm{m}})\mathrm{e}^{-\psi_\delta+\frac{1}{2}\Sigma_\delta}N(d_1^{\delta,\mathrm{m}}).$$

Similarly,

$$\int_{-\infty}^{\infty}\left(\int_{\frac{K}{A}c_T}^{\infty}\frac{K}{A}\frac{1}{M_T}g(M_T|M_t)\,\mathrm{d}M_T\right)h(c_T|c_t)\,\mathrm{d}c_T=\frac{K}{A}\sum_{n_\delta=0}^{\infty}\sum_{n_m=0}^{\infty}P(\lambda_\delta,\lambda_{\mathrm{m}})\mathrm{e}^{-\psi_{\mathrm{m}}+\frac{1}{2}\Sigma_{\mathrm{m}}}N(d_2^{\delta,\mathrm{m}}),$$

where $d_2^{\delta,\mathrm{m}}=d_1^{\delta,\mathrm{m}}-\sigma_{\delta,\mathrm{m}}\sqrt{\tau}$. Rearranging terms, we have

$$CC_t^{\mathrm{D}}(p_t,T)=\sum_{n_\delta=0}^{\infty}\sum_{n_m=0}^{\infty}P(\lambda_\delta,\lambda_{\mathrm{m}})C_{\mathrm{GK}}(p_t,\tau;K,r_\delta,i_{\mathrm{m}},\sigma_{\delta,\mathrm{m}}).$$

For a European currency put, we have

$$CP_t^{\mathrm{D}}(p_t,T)=p_t\mathrm{e}^{-\rho(T-t)}c_tE_t\left(\max\left(\frac{K}{A}\frac{1}{M_T}-\frac{1}{c_T},0\right)\right)=p_t\mathrm{e}^{-\rho(T-t)}c_t\int_{-\infty}^{\infty}\left(\int_{-\infty}^{\frac{K}{A}c_T}\left(\frac{K}{A}\frac{1}{M_T}-\frac{1}{c_T}\right)g(M_T|M_t)\,\mathrm{d}M_T\right)h(c_T|c_t)\,\mathrm{d}c_T.$$

The same tedious exercises will give us

$$CP_t^{\mathrm{D}}(p_t,T)=\sum_{n_\delta=0}^{\infty}\sum_{n_m=0}^{\infty}P(\lambda_\delta,\lambda_{\mathrm{m}})P_{\mathrm{GK}}(p_t,\tau;K,r_\delta,i_{\mathrm{m}},\sigma_{\delta,\mathrm{m}}).\blacksquare$$

Appendix D. Proof of Proposition 3.2

For a European call written on the stock with a striking price k that matures at time T, its real price at time $t\leq T$, $C_t(k,S_t,T)$, is

$$C_t(k,S_t,T)=\mathrm{e}^{-\rho\tau}c_tE_t\left(\frac{1}{c_T}\max(S_T-k,0)\right).$$

Since $S_t=\frac{\delta_t}{\rho}=\frac{c_t-\rho f_t}{\rho}$ and $f_t=f_0\mathrm{e}^{(r-\rho)t}$, then

$$C_t(k, S_t, T) = e^{-\rho\tau} c_{tE_t}\left(\frac{1}{c_T}\max\left(\frac{c_T}{\rho} - f_T - k, 0\right)\right)$$

$$= e^{-\rho\tau} c_t \int_{-\infty}^{\infty} \max\left(\frac{1}{\rho} - \frac{f_t e^{(r-\rho)\tau} + k}{c_T}, 0\right) h(c_T|c_t)\, dc_T.$$

Tedious exercises show that

$$C_t(k, S_t, T) = \sum_{n_\delta=0}^{\infty} \frac{e^{-\lambda_\delta\tau}(\lambda_\delta\tau)^{n_\delta}}{n_\delta!} C_{GK}\left(S_t + f_t, \tau; k + f_t e^{(r-\rho)\tau}, \rho, r_\delta, \sigma_\delta^2 + \frac{n_\delta\phi_\delta^2}{\tau}\right).$$

Similarly, we have

$$P_t(k, S_t, T) = \sum_{n_\delta=0}^{\infty} \frac{e^{-\lambda_\delta\tau}(\lambda_\delta\tau)^{n_\delta}}{n_\delta!} P_{GK}\left(S_t + f_t, \tau; k + f_t e^{(r-\rho)\tau}, \rho, r_\delta, \sigma_\delta^2 + \frac{n_\delta\phi_\delta^2}{\tau}\right). \blacksquare$$

Appendix E. Proof of Proposition 4.1

Based on the risk-neutral process (4.4), the distribution of ω_T conditional on ω_t is:

$$G^*(\omega_T, T|\omega_t, t) = \sum_{n_\delta=0}^{\infty} \sum_{n_m=0}^{\infty} P(\lambda_\delta^*, \lambda_m^*) \frac{1}{\sqrt{2\pi\Sigma^*}} e^{-\frac{(\ln\omega_T - \psi^*)^2}{2\Sigma^*}},$$

where

$$\psi^* = \ln\omega_t + \left(r - i - \frac{1}{2}\sigma_m^2 - \frac{1}{2}\sigma_\delta^2 - \lambda_m^* k_m^* - \lambda_\delta^* k_\delta^*\right)\tau + n_\delta\theta_\delta^* - n_m\theta_m^*,$$

$$\Sigma^* = (\sigma_m^2 + \sigma_\delta^2)\tau + n_m\phi_m^{*2} + n_\delta\phi_\delta^{*2}.$$

For the European put and call currency option from the perspective of the foreign agent, $CP_t^F(\omega_t, T)$ and $CC_t^F(\omega_t, T)$, we can compute according to the risk-neutral probability density. That is

$$CP_t^F(\omega_t, T) = e^{-r(T-t)} E_t^F(\max(1 - \omega_T K, 0)),$$

$$CC_t^F(\omega_t, T) = e^{-r(T-t)} E_t^F(\max(\omega_T K - 1, 0)).$$

Then it is straightforward to prove proposition (4.1).■

References

Abramowitz, M., 1965. Handbook of Mathematical Functions. Dover Publications, New York.
Akgiray, V., Booth, G., 1988. Mixed diffusion-jump process modelling of exchange rate movement. The Review of Economics and Statistics 70, 631–637.

Amin, K., Jarrow, R., 1991. Pricing foreign currency options under stochastic interest rates. Journal of International Money and Finance 10, 310–329.

Bailey, W., Stulz, R., 1989. The pricing of stock index options in a general equilibrium model. Journal of Financial and Quantitative Analysis 24, 1–12.

Bakshi, G.S., Chen, Z., 1997. Equilibrium valuation of foreign exchange claims. Journal of Finance 52, 799–826.

Ball, C., Roma, A., 1993. A jump diffusion model for the European monetary system. Journal of International Money and Finance 12, 475–492.

Bardhan, I., 1995. Exchange rate shocks, currency options and the Siegel paradox. Journal of International Money and Finance 14, 441–458.

Bates, D., 1996. Dollar jump fears, 1984-1992: distributional abnormalities implicit in currency futures options. Journal of International Money and Finance 15, 65–93.

Biger, N., Hull, J., 1983. The valuation of currency options. Financial Management 12, 24–28.

Black, F., Scholes, M., 1973. The pricing of options and corporate liabilities. Journal of Political Economy 81, 637–655.

Bodurtha, J.N. Jr., Courtadon, G.R., 1987. Tests of an American option pricing model on the foreign currency option market. Journal of Financial and Quantitative Analysis 22, 153–167.

Cao, M., 1997. Empirical implication of systematic jump risks in currency option pricing. Manuscript, Queen's University, Canada.

Chesney, M., Scott, L., 1989. Pricing European currency options: a comparison of the modified Black-Scholes model and a random variance model. Journal of Financial and Quantitative Analysis 24, 267–284.

Cox, C.J., Ross, S.A., 1976. The valuation of options for alternative stochastic processes. Journal of Financial Economics 3, 145–166.

Duffie, D., 1992. Dynamic Asset Pricing Theory. Princeton University Press, Princeton, NJ.

Dumas, B., Jennergren, L.P., Näslund, B., 1995. Siegel's paradox and the pricing of currency options. Journal of International Money and Finance 14, 213–223.

Garman, M.B., Kohlhagen, S.W., 1983. Foreign currency option values. Journal of International Money and Finance 2, 231–237.

Grabbe, O., 1983. The pricing of call and put options on foreign exchange. Journal of International Money and Finance 2, 239–253.

Grinols, E.L., Turnovsky, S.J., 1994. Exchange rate determination and asset prices in a stochastic small open economy. Journal of International Economics 36, 75–97.

Heston, S., 1993. A closed-form solution for options with stochastic volatility with applications to bond and currency options. Review of Financial Studies 6, 327–343.

Hull, J., White, A., 1987. The pricing of options on assets with stochastic volatilities. Journal of Finance 42, 281–300.

Jorion, P., 1988. On jump processes in the foreign exchange and stock markets. Review of Financial Studies 1 (4), 427–445.

Lucas, R., 1978. Asset prices in an exchange economy. Econometrica 46, 1429–1445.

Lucas, R., 1982. Interest rates and currency prices in a two-country world. Journal Monetary Economics 10, 335–360.

MacBeth, J.D., Merville, L.J., 1979. An empirical examination of the Black-Scholes call option pricing model. Journal of Finance 34, 1173–1186.

Merton, R., 1976. Option pricing when underlying stock returns are discontinuous. Journal of Financial Economics 3, 125–144.

Naik, V., Lee, M., 1990. General equilibrium pricing of options on the market portfolio with discontinuous returns. Review of Financial Studies 3 (4), 493–521.

Obstfeld, M., 1981. Macroeconomic policy, exchange rate dynamics, and optimal accumulation. Journal of Political Economy 89, 1142–1161.

Sargent, T.J., 1987. Dynamic Macroeconomic Theory. Harvard University Press, Cambridge.

Sidrauski, M., 1967. Rational choice and patterns of growth in a monetary economy. America Economic Review 57 (2), 534–544.

Siegel, J., 1972. Risk, interest rates and the forward exchange. Quarterly Journal of Economics 86, 303–309.

Tucker, A.L., 1991. Exchange rate jumps and currency options pricing. In:. Sarkis, J.K. (Ed.), Recent Developments in International Banking and Finance, vols. IV and V. North Holland, pp. 441–459.