Why Don't Firms Reward their CEOs with Relative Compensation?*

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This version: October, 2014

Abstract

The existing economic theories on managerial compensation renders the relative performance evaluation (RPE) as the desirable mechanism to reward CEOs. However, in reality, very few firms use the RPE scheme, which constitutes a puzzle as noted by Murphy (1999) and others. This paper sheds some light on this puzzle. I show that the RPE scheme is not always the desirable mechanism in a competitive market with multiple firms and multiple agents. In other words, the apparent desirability of the RPE mechanism is a result of simplistic model setup. Once realistic features are built into the model, the equilibrium could be compatible with either an absolute performance evaluation scheme (APE) or an RPE scheme. Therefore, that the preponderance of firms use APE is not necessarily a puzzle at all.

I reach the above conclusion via an equilibrium model within which the optimal incentive contract is derived in a market with many firms and many CEOs. The main innovative features absent in the existing models include the inter-firm interactions through the CEOs' endogenized reservation utilities and the possibility for CEOs to quit. In this rich setting, I show that either form of the compensation scheme can prevail depending on the economic condition. As a by-product of the inquiry, I also show that a mixture of APE and RPE can actually better motivate the CEO and therefore enhance the firm's profit than either of the simple form.

* The research supports from the *Schulich School of Business* at York University are gratefully acknowledged.

1. Introduction

The existing economic theory on managerial compensation suggests that the proper mechanism to compensate CEOs is to reward them based on their performance, especially on their relative performance benchmarked to an aggregate performance measure (see Holmstrom 1982). Given this result, academic researchers (such as Jensen and Murphy 1990b) and public activists (such as Crystal 1991) have advocated relative performance evaluations (hereafter RPE) for CEOs. However, very few firms use such a relative scheme in reality (Meulbroek 2001b) and most compensation schemes are based on firms' absolute performances. For example, stocks and options granted to executives depend on companies' absolute stock performances. The lack of using RPE has been identified as a key unsolved puzzle by Abowd and Kaplan, (1999) Murphy (1999) and Prendergast (1999). This puzzle arises because the observed industry practice is not consistent with the prediction of a framework with a single principal and a single agent. This framework takes the CEO's reservation value as exogenous and assumes that a firm's action is independent of other firms' actions. However, firms do interact with each other in reality. Also, the exogenously specified reservation utility doesn't reflect the economic state, because the CEO's reservation utility should depend on his outside opportunities which are affected by the aggregate states. These two reasons call for an equilibrium analysis that allows for interdependence among firms' decisions and endogenizes the reservation utility.

In an equilibrium framework with many firms and CEOs, it is not clear how each firm should compensate the CEO. Would the optimal incentive mechanism necessarily favor the RPE compensation scheme? If not, what should the optimal reward scheme be? To address these questions, I construct a market equilibrium model to analyze the optimal incentive contract with many firms and many CEOs. Firms use either the absolute performance evaluation (hereafter APE) or the RPE to reward their CEOs. If CEOs are not satisfied with their compensations, they can quit and look for outside opportunities. The value of a job that a CEO can find in the market is determined endogenously by other firms' compensation schemes. Thus, a CEO's reservation utility (or his outside option) is endogenous in this paper. Moving from one job to another is costly and inconvenient, which is modelled as a disutility to the CEO. A CEO quits only if his outside option exceeds the utility he can derive from the current compensation scheme, after considering the moving cost. Also, quitting is costly for the firm, because the firm must use resources to find a replacement. So a firm would offer a contract that retains the CEO under "normal" circumstances, and depends on other firms' contracts through the CEO's outside options. This inter-firm linkage allows all firms' contracts to be determined in equilibrium. I focus on symmetric equilibria where all firms offer the same type of contracts, i.e., either the APE or the RPE contract.

There are two types of equilibria considered here: one features CEOs' quitting decisions and the other does not have quitting. The no-quitting equilibrium exists when firms' hiring costs are high. The quitting equilibrium exists when firms' hiring costs are low in which case firms are better off in some states by hiring a new CEO with a lower compensation payout. In both equilibria, APE or RPE can become the equilibrium incentive scheme, depending on economic conditions. In particular, the APE scheme prevails when the aggregate economy is facing a possible downturn. However, when the economy is more likely to boom, the RPE scheme prevails. Relative to that with quitting, the equilibrium contract with no-quitting offers a high fixed cash wage and a low equity reward. Furthermore, the reservation utility depends on aggregate shocks in both equilibria. In particular, a high expected aggregate state or a high aggregate volatility can result in a high reservation utility, which induces a high compensation. Similarly, if a firm's expected state and volatility are high, then the equilibrium reservation utility is high. Also, the reservation utility increases with the disutility of moving and decreases with the effort aversion attitude.

The features of the equilibrium reservation utility support the conjecture of Himmelberg and Hubbard (2000). They argued that CEOs' reservation utilities should rise if aggregate shocks increase the demand for CEOs and consequently induce high compensations. More importantly, the analysis in this paper shows that the RPE is not always a better scheme in a market equilibrium with multiple firms and agents. In this sense, the lack of using RPE in reality may not constitute a puzzle, since it can be consistent with an equilibrium.

Since the market equilibrium does not favor one type of reward scheme over the other, it is natural to investigate whether there exists a compensation scheme which dominates both the APE and the RPE schemes under all circumstances. I propose a mixture of the APE and RPE reward mechanisms. The equilibrium analysis shows that this mixed scheme can provide a better motivation to CEOs and yield higher profits to firms. Intuitively, the APE component captures the influence of the aggregate economy on CEOs' reservation utilities and ensures the competitiveness of the compensation scheme while the RPE component motivates the CEO to work hard at a lower cost to firm.

This paper is related to Cao and Wang (2013). Similar to their model, the current model recognizes that an agent may choose not to participate in a contract in certain states of the world. Furthermore, the current model determines the agent's outside option and optimal compensation in a market equilibrium. However, Cao and Wang's objective is to study the effects of systematic and idiosyncratic risks on the pay-to-performance sensitivity in a APE context. The current paper examines the APE and RPE simultaneously in a market equilibrium. Such analysis is new to the existing literature. Given the endogenous nature of the reservation utility, the current paper shows that the RPE is not always a better reward mechanism than the APE scheme. In fact, a compensation mixing the APE and RPE mechanisms is a better incentive contract.

The rest of the paper is organized as follows. Section 2 sets up the model. Section 3 analyzes an individual firm's optimal compensation scheme while taking other firms' contracts as given. The compensation scheme takes two possible forms: one is a linear contract based on the absolute performance of the firm's stock price and the other is a linear contract based on the relative performance of the firm's stock. Section 4 characterizes the market equilibrium compensation polices and identifies economic conditions under which either APE or RPE scheme is the equilibrium outcome. Section 5 presents a mixed compensation scheme and shows that it dominates a pure APE or RPE scheme. Section 6 concludes the paper. Proofs are collected in Appendix.

2. The Model Economy

Consider a one-period economy where the aggregate economy has two possible states. A high economic state Y_H occurs with probability P_H^Y and a low economic state Y_L with probability $P_L^Y = 1 - P_H^Y$. The expected aggregate shock is μ_y and the variance σ_y^2. There are many firms in the economy, each having a CEO responsible for carrying out a project on behalf of the principal of the firm. The value of each firm i depends on the aggregate state Y, the firm's own risk X_i

and its CEO's effort e_i, as follows:

$$S_i \equiv S(e_i, X_i, Y) = Y X_i e_i.$$

The linear technology adopted here is similar to that used in Cao and Wang (2013), and Edmans, Gabaix, and Landier (2009).

Notice that different firms' values are correlated with each other through the aggregate shock. To simplify the analysis, I assume that each firm's risk is i.i.d and is independent of the aggregate shock. Furthermore, I assume that all X have the same distribution over two possible states. A good state X_H is realized with probability P_H^X and a bad state X_L with probability $P_L^X = 1 - P_H^X$. The expected value of X is μ_x and the variance is σ_x^2. The shocks are realized before the CEO chooses his effort.

As is standard, I assume that each principal is risk-neutral and maximizes the expected residual value of the firm, and each CEO has the following exponential utility function:

$$U(W, e) = 1 - U_0 \exp[-\gamma \left(W - ce^2\right)],$$

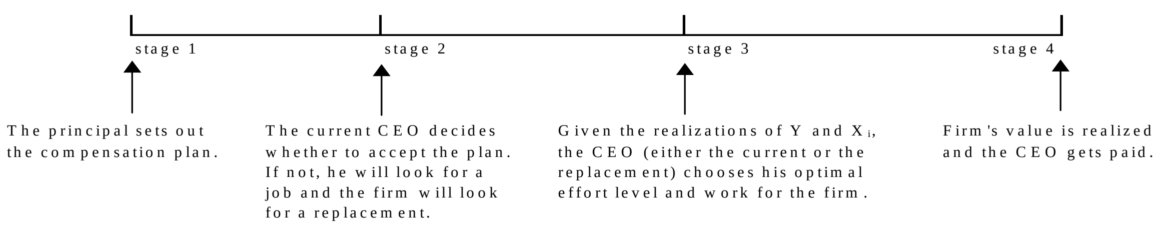
where W is the CEO's total compensation and e is the CEO's effort level. $\gamma > 0$ is the coefficient of risk aversion and $c > 0$ is a constant reflecting the CEO's effort aversion.

The objective of firm i's principal is to design a compensation scheme which maximizes the firm's expected residual value. Each firm takes other firms' compensation schemes as given and optimally chooses its own. The incentive package can be based on either APE or RPE. The APE scheme consists of a fixed cash payment and a percentage of equity sharing. The RPE scheme is based on firm i's relative performance benchmarked against the aggregate performance. Precisely, it consists of a fixed cash payment plus a reward based on firm i's relative performance.

Firm i's compensation contract must satisfy an incentive constraint and a participation constraint. The incentive constraint ensures that the contract induces the CEO to select the optimal effort level. The participation constraint requires that the contract induces the CEO to stay with the firm; that is, the contract provides a reservation utility $\underline{V}$ that is at least as high as the expected utility the CEO could obtain from his outside job opportunity. Let p be the exogenous probability with which a CEO can obtain a job from an outside firm, say firm j $(\neq i)$.[1] The reservation utility of firm i's CEO is $\underline{V} = pE(U(W_j, e_j))$. Although firm i takes firm j's compensation scheme as given, all firms' compensation schemes must be determined together in the equilibrium. Thus, the reservation utility is an equilibrium object, through which different firms' compensation decisions are related.

In this paper, I focus on symmetric equilibria where all firms offer the same type of contracts, i.e., either the APE or the RPE. The first equilibrium is the no-quitting equilibrium where no CEO quits his current job. Every firm's incentive contract ensures that the participation constraint is satisfied in all possible states for its CEO. The second equilibrium has quitting in some states, in which case firms' incentive contracts do not guarantee the participation constraint.

Figure 1: The Sequence of Events for Firm i



There are a few stages to the game in this one-period framework. To summarize the decision process, I outline the detailed stages below:

[1]To simplify analysis, I assume the probability of finding a job by the CEO is exogenous. This probability is less than one since there exists matching friction in the CEO labor market.

At stage one, firm i's principal offers a compensation scheme to its current CEO. The contract can be based on either APE or RPE. At stage two, the current CEO decides whether to accept this compensation scheme. If he does, he will work for the firm. Otherwise, he searches for another job. I assume that the disutility associated with the job change is D_m. If the CEO quits, firm i needs to hire a replacement, which entails an hiring cost H. At stage three, the CEO (either the current or the replacement) chooses his optimal effort level, after observing the realized aggregate and firm-specific shocks. At stage four, firm i's value is realized. The CEO gets paid according to the compensation scheme and the principal claims the residual value of the firm.

3. Firm i's Optimal Incentive Contract

In this paper, I focus on symmetric equilibria where all firms offer the same type of incentive contracts. Denote the compensation offered by firm i as W_i. Then the APE scheme takes the following linear form:

$$W_i = a_A + b_A S_i,$$

where a_A is the fixed salary and b_A is the percentage of equity sharing based on firm i's value, S_i. The RPE scheme is based on firm i's performance relative to the aggregate performance. Specifically, the RPE scheme takes the following form:

$$W_i = a_R + b_R \frac{S_i}{Y},$$

where a_R is the fixed salary and b_R is the dollar amount rewarded to the CEO after filtering the aggregate performance. Equivalently, b_R/Y can be understood as the percentage of equity sharing. The two schemes differ in that the CEO's equity sharing under the RPE scheme (b_R/Y) varies with the realizations of the aggregate shock, but it does not under the APE scheme.

3.1. Firm i's Optimal Contract when All Firms Use the APE scheme

To analyze firm i's incentive contract, I consider the first symmetric scenario where all firms use the APE scheme. Taking other firms' APE schemes as given, the objective of firm i's principal is to determine an optimal APE contract which maximizes his expected profit after paying the CEO. Because the principal offers the contract before the aggregate and firm-specific shocks are realized, the contract is contingent on such shocks.

There is an important distinction between the current equilibrium model with multiple firms and agents and the principal-agent model with a single firm and a single agent. In the latter, the agent's participation constraint is satisfied in all states. In the current model, this is not necessarily the case. It is possible for firm i to have an optimal contract which does not satisfy the participation constraint in some states. For example, if the hiring cost is not high, firm i may be better off with a new CEO if the new CEO accepts a lower contract in some states. In this case, the participation constraint is not satisfied and consequently the current CEO quits. In the current set-up, there are only four possible states facing firm i. There are, $\{(X_L, Y_L)$, (X_L, Y_H), (X_H, Y_L), $(X_H, Y_H)\}$, which yields six possible scenarios regarding a CEO's quitting decisions:

The first scenario is that the participation constraint is satisfied in all possible states $\{(X_L, Y_L)$, (X_L, Y_H), (X_H, Y_L), $(X_H, Y_H)\}$. In this case, the CEO stays with firm i with certainty.

The second scenario is that the anticipation constraint is satisfied in three possible states $\{$ (X_L, Y_H), (X_H, Y_L), $(X_H, Y_H)\}$, and the CEO quits only if the aggregate shock is low and the firm-specific shock is bad (i.e., $Y = Y_L$ and $X = X_L$). The quitting probability is $P_L^X P_L^Y$.

The third scenario is that the participation constraint is satisfied in two possible states $\{(X_H, Y_L)$, $(X_H, Y_H)\}$. That is, the CEO stays with firm i only if firm i is in good state, regardless of the aggregate state. The quitting probability is P_L^X.

The fourth scenario is that the participation constraint is satisfied in two possible states $\{(X_H, Y_H), (X_L, Y_H)\}$. That is, the CEO stays with firm i only if the aggregate state is good. The quitting probability is P_L^Y.

The fifth is that the participation constraint is only satisfied in state (X_H, Y_H). Put it differently, the CEO stays with firm i only if the aggregate shock is good and firm i is in good state (i.e., $Y = Y_H$ and $X = X_H$). The quitting probability is $1 - P_H^X P_H^Y$.

The sixth is that the participation constraint is not satisfied in any state and the CEO quits with certainty.

To make the model realistic, I restrict attention to the first three scenarios. Scenarios 4 and 5 imply a non-zero possibility that no CEO would work in an economy with a low state, which may result in shutting down the economy. Scenario 6 indicates a non-zero possibility of completely shutting down the economy regardless of the states. Such outcomes are not undesirable, nor realistic. In contrast, under Scenario 2, a CEO may quit when both the aggregate and the firm's states are low. Or, under Scenario 3, a CEO may quit when the firm's state is low. Since it is very unlikely for all firms to be in a low state at the same time, CEOs' quitting behavior will not result in shutting down of the economy. Table 1 below summarizes the key features of the three realistic scenarios.

Now I begin with the discussion of firm i's optimal compensation contract. Because a CEO may not stay with the firm in all states, the firm takes the possible quitting event into account when offering the contract. To facilitate the presentation of firm i's problem, I define an indicating function 1_{gh}^k for a no-quitting state (X_g, Y_h) in Scenario k as

Table 1: Possible Quitting Scenarios

Scenario	No-Quitting States	Quitting States	Quitting Probability
$k=1$	$\{(X_L, Y_L),\ (X_L, Y_H),\ (X_H, Y_L),\ (X_H, Y_H)\}$	$\varnothing$	0
$k=2$	$\{(X_L, Y_H),\ (X_H, Y_L),\ (X_H, Y_H)\}$	$\{(X_L, Y_L)\}$	$P_L^X P_L^Y$
$k=3$	$\{(X_H, Y_L),\ (X_H, Y_H)\}$	$\{(X_L, Y_L),\ (X_L, Y_H)\}$	P_L^X

$$1_{gh}^k = \begin{cases} 1 & \text{when the state } (X_g, Y_h) \text{ is a no-quitting state in Scenario } k \\ & \text{where } g = H, L \text{ and } h = H, L; \\ & \\ 0, & \text{otherwise.} \end{cases}$$

Note that when the CEO is induced to stay in a particular state, the principal obtains the residual value of the firm. However, if the CEO quits, then firm i hires another CEO. In this case, the principal receives a profit J_A as an average firm does in the market with the APE scheme. Therefore, principal i's problem in Scenario k ($k = 1, 2, 3$) is formulated as

$$\max_{a_{Ak}, b_{Ak}} \pi_{Ak} = \sum_{g=H,L} \sum_{h=H,L} P_g^X P_h^Y \left\{ \left[S(e_i^*, X_i, Y) - a_{Ak} - b_{Ak} S(e_i^*, X_i, Y) \right] 1_{gh}^k + J_A (1 - 1_{gh}^k) \right\}$$

s.t.

$$e_i^* = \underset{e}{\arg\max} \left\{ 1 - U_0 \exp\left[-\gamma (a_{Ak} + b_{Ak} S(e, X_i, Y) - ce^2) \right] \right\}$$ Incentive constraint is satisfied in all no-quitting states,

$$1 - U_0 \exp\left\{ -\gamma [a_{Ak} + b_{Ak} S(e_i^*, X_i, Y) - ce_i^{*2}] \right\} \geq \underline{V}_A - D_m$$ Participation constraint is satisfied in all no-quitting states.

The level $\underline{V}_A$ is the CEO's reservation utility derived from working for an average firm with the APE scheme. D_m is the disutility resulted from the job change.

The above problem can be solved in two steps. The first step solves for the CEO's best response to the firm's compensation scheme, i.e., the CEO's optimal effort level given firm i's APE compensation scheme. The second step solves for the optimal compensation scheme anticipating the CEO's optimal effort level. Appendix A derives firm i's optimal compensation scheme and its expected profit for the three scenarios outlined in Table 1. Table 2 on the next page summarizes the results.

It is easy to show that $b_{A1} < b_{A2} < b_{A3}$ and $a_{A1} > a_{A2} > a_{A3}$. This indicates that the likelihood of the CEO quitting increases with the percentage of equity sharing and decreases with the fixed cash compensation. When firm i offers the lowest percentage of equity sharing b_{A1} and the highest cash payment a_{A1}, the CEO stays with certainty because this contract satisfies both constraints in all states. The CEO's optimal action (staying with the firm) is induced by the CEO's risk aversion attitude.[2] As the cash payment drops from a_{A1} to a_{A2} and the equity sharing jumps from b_{A1} to b_{A2}, the probability of quitting jumps from 0 to $P_L^Y P_L^X$. In the latter case, the CEO quits his current job when the aggregate state is low and firm i's state is bad. Otherwise, he works for firm i. As the cash rewards reduces further to a_{A3} and the equity sharing increases to b_{A3}, the probability of quitting increases to P_L^X. In this case, the CEO quits when firm i's state is bad, regardless of the aggregate state. He stays when firm i is in the good state.

3.2. Firm i's Optimal RPE Incentive when All Firms Use the RPE scheme

The objective of principal i is to choose a RPE scheme which maximizes his expected profit while taking other firms' RPE schemes as given. When the CEO quits and firm i hires another CEO, the principal receives a profit J_R as an average firm does in the market with the RPE scheme. Since the RPE depends on firm-specific states only, there are only two realistic cases: the first case corresponds to Scenario 1 where the participation constraint is satisfied in all four possible

[2] The best risk-sharing contract is such that the principal takes all risks since he is risk-neutral while the CEO takes no risk and is fully paid with cash since he is risk-averse.

Table 2: Firm i's Optimal APE-Based Incentive scheme and its CEO's Response

Scenario	Optimal Compensation & Principal's Profit	CEO's Response
1	$a_{A1} = [\ln U_0 - \ln(1 + D_m - \underline{V}_A)]/\gamma - (b_{A1} Y_L X_L)^2/4c$ $b_{A1} = \frac{E(X^2Y^2)}{2E(X^2Y^2) - X_L^2 Y_L^2}$ $\pi_{A1} = b_{A1} E(X^2Y^2)/4c - \frac{1}{\gamma}[\ln U_0 - \ln(1 + D_m - \underline{V}_A)]$	working for firm i with prob. 1
2	$a_{A2} = [\ln U_0 - \ln(1 + D_m - \underline{V}_A)]/\gamma - (b_{A2} Y_H X_L)^2/4c$ $b_{A2} = \frac{E(X^2Y^2 - P_L^Y P_L^X Y_L^2 X_L^2)}{2E(X^2Y^2 - P_L^Y P_L^X Y_L^2 X_L^2) - (1 - P_L^Y P_L^X) X_L^2 Y_H^2}$ $\pi_{A2} = P_L^Y P_L^X (J_A - H) + b_{A2} E(X^2Y^2 - P_L^Y P_L^X Y_L^2 X_L^2)/4c$ $-(1 - P_L^Y P_L^X)[\ln U_0 - \ln(1 + D_m - \underline{V}_A)]/\gamma$	quitting firm i with prob. $P_L^Y P_L^X$
3	$a_{A3} = [\ln U_0 - \ln(1 + D_m - \underline{V}_A)]/\gamma - (b_{A3} Y_L X_H)^2/4c$ $b_{A3} = \frac{E(Y^2)}{2E(Y^2) - Y_L^2}$ $\pi_{A3} = P_L^X (J_A - H) + b_{A3} P_H^X X_H^2 E(Y^2)/4c$ $-P_H^X [\ln U_0 - \ln(1 + D_m - \underline{V}_A)]/\gamma$	quitting firm i with prob. P_L^X

states and the CEO stays with firm i with certainty; the second corresponds to Scenario 3 where the participation constraint is satisfied when firm i is in good state. The quitting probability is P_L^X. Therefore, principal i's problem with the RPE scheme in Scenario k ($k = 1, 3$) is formulated as

$$\max_{a_{Rk}, b_{Rk}} \pi_{Rk} = \sum_{g=H,L} \sum_{h=H,L} P_g^X P_h^Y \left\{ \left[S(e_i^*, X_i, Y) - a_{Rk} - b_{Rk} \frac{S(e_i^*, X_i, Y)}{Y} \right] 1_{gh}^k + J_R(1 - 1_{gh}^k) \right\}$$

s.t

$$e_i^* = \underset{e}{\arg\max} \left\{1 - U_0 \exp[-\gamma(a_{Rk} + b_{Rk}\tfrac{S(e_i^*,X_i,Y)}{Y} - ce^2)]\right\}$$ Incentive constraint is satisfied in all no-quitting states,

$$1 - U_0 \exp\left\{-\gamma[a_{Rk} + b_{Rk}\tfrac{S(e_i^*,X_i,Y)}{Y} - ce_i^{*2}]\right\} \geq \underline{V}_R - D_m$$ Participation constraint is satisfied in all no-quitting states.

The level $\underline{V}_R$ is the CEO's reservation utility derived from a RPE scheme offered by an average firm. Appendix B derives firm i's optimal compensation scheme and its expected profit. Table 3 on the next page presents the results.

Obviously, $b_{R1} < b_{R3}$ and $a_{R1} > a_{R3}$. That is, the CEO of firm i stays with his current job when the fixed cash payment is at its highest, a_{R1}, and the equity sharing rule is at its lowest, b_{R1}. This result is similar to the one obtained under the APE scheme and is driven by the CEO's risk aversion. The likelihood of the CEO's quitting increases with the equity sharing and decreases with the cash compensation. As the cash payment deduces to a_{R3} and the equity sharing

Table 3: Firm i's RPE Optimal Incentive scheme and its CEO's Response

Scenario	Optimal Compensation & Principal's Profit	CEO's Response
1	$a_{R1} = [\ln U_0 - \ln(1 + D_m - \underline{V}_A)]/\gamma - (b_{R1}X_L)^2/4c$ $b_{R1} = \mu_y \frac{E(X^2)}{2E(X^2) - X_L^2}$ $\pi_{R1} = b_{R1}\mu_y E(X^2)/4c - [\ln U_0 - \ln(1 + D_m - \underline{V}_A)]/\gamma$	working for firm i with prob. 1
3	$a_{R3} = [\ln U_0 - \ln(1 + D_m - \underline{V}_A)]/\gamma - (\mu_y X_H)^2/4c$ $b_{R3} = \mu_y$ $\pi_{R3} = P_L^X(J - H) + P_H^X(\mu_y X_H)^2/4c$ $-P_H^X[\ln U_0 - \ln(1 + D_m - \underline{V}_A)]/\gamma$	quitting firm i with prob. P_L^X

increases b_{R3}, the probability of quitting increases from 0 to P_L^X. That is, the CEO quits firm i when it is in a bad state, regardless of the aggregate state. He stays when firm i is in a good state.

4. Market Equilibrium with No Quitting

Now I turn to the determination of the equilibrium incentive contract. The first equilibrium is one where no CEO quits his current job,[3] which I call the no-quitting equilibrium. As before, I analyze the symmetric equilibria with APE and RPE separately. To verify that a particular compensation scheme forms a market equilibrium, I investigate the net gain to a single firm from switching to the alternative compensation scheme. The conditions under which such net gain is non-positive support the particular scheme as a market equilibrium.

4.1. Equilibrium Compensation when Only the APE scheme is Used

Suppose that all firms other than firm i offer an APE scheme $W_j = A_A + B_A S_j$, where $j \neq i$. Then a potential CEO working for firm j chooses his optimal effort e_j^{A*} so as to maximize his utility.

$$\max_{e_j} \left\{ 1 - U_0 \exp\left[-\gamma\left(A_A + B_A S(e_j, X_j, Y) - ce_j^2\right)\right]\right\}.$$

The optimal effort level under the APE scheme is $e_j^{A*} = \frac{B_A Y X_j}{2c}$ and the CEO's utility is $U_j^A = 1 - U_0 \exp[-\gamma(A_A + B_A Y^2 X_j^2/4c)]$.

Now consider firm i's CEO. Since firm i's current CEO has a probability p to obtain a job from an average firm j, his expected reservation utility under the APE scheme is

$$\underline{V}_A = pE(U_j^A) = p - pU_0 E\left[\exp\left(-\gamma A_A - \gamma B_A Y^2 X^2/4c\right)\right]. \tag{4.1}$$

Clearly, the CEO's optimal effort does not depend on the fixed amount of cash compensation under the APE scheme. The optimal effort depends positively on the percentage of equity sharing

[3] A market equilibrium with quitting behavior is considered as an extension in Section 4.4. The conclusions derived from the equilibrium with quitting are qualitatively the same as those under the no-quitting equilibrium.

and negatively on his effort-aversion. Moreover, the CEO would devote high effort to the project when the economy is in the high state and firm i is in the good state. On the other hand, his reservation utility increases when the probability of getting a job is high or the fixed cash compensation and the equity sharing offered by the potential job are high. More importantly, his reservation utility is higher when the state of the aggregate economy is high and firm i's state is good. I define the no-quitting market equilibrium as follows:

Definition 4.1. *When every firm only uses the APE scheme, the no-quitting APE equilibrium contract consists of a pair (A_A, B_A) which ensures that no CEO quits his current job and each principal obtains the highest profit.*

This definition suggests that the no-quitting APE equilibrium scheme requires firm i's compensation scheme (a_{A1}, b_{A1}) to be the same as that of an average firm's (A_A, B_A) and principal i's expected profit π_{A1} be the same as an average firm's J_A. In addition, the equilibrium profit is higher than the profit corresponding to any other contract. That is

$$(1) \quad A_A = a_{A1}; \qquad (2) \quad B_A = b_{A1}; \qquad (3) \quad J_A = \pi_{A1}; \qquad (4) \quad \pi_{A1} > \pi_{A2}, \pi_{A3}.$$

Appendix C determines the no-quitting APE equilibrium summarized in the following proposition:

Proposition 4.2. *If every firm uses the APE scheme and the hiring cost H is higher than*

$$\underline{H}_A = \max\left(\frac{P_H^X E(Y^2)[b_{A3}X_H^2 - B_A E(X^2)]}{4cP_L^X}, \quad \frac{b_{A2}E(Y^2X^2 - P_L^Y P_L^X Y_L^2 X_L^2) - B_A(1 - P_L^Y P_L^X)E(Y^2X^2)}{4cP_L^Y P_L^X}\right),$$

the no-quitting equilibrium incentive scheme consists of a fixed cash compensation A_A and the equity sharing B_A as

$$A_A = \frac{1}{\gamma}\ln\left(\frac{U_0 - pU_0 E\left[\exp\left(\gamma B_A^2(Y_L^2X_L^2 - Y^2X^2)/4c\right)\right]}{1 - p + D_m}\right) - \frac{1}{4c}\left(B_A Y_L X_L\right)^2 \qquad \text{and} \qquad B_A = \frac{E(X^2Y^2)}{2E(X^2Y^2) - X_L^2 Y_L^2}.$$

The equilibrium expected profit J_A for a principal and the reservation utility $\underline{V}_A$ for a CEO are

$$J_A = \frac{B_A E(X^2Y^2)}{4c} - \frac{1}{\gamma}\ln\left(\frac{U_0 1-pE\left[\exp\left(\gamma B_A^2(Y_L^2X_L^2-Y^2X^2)/4c\right)\right]}{1-p+D_m}\right)$$

and

$$\underline{V}_A = \frac{p-p(1+D_m)E\left[\exp\left(\gamma B_A^2(Y_L^2X_L^2-Y^2X^2)/4c\right)\right]}{1-pE\left[\exp\left(-\gamma B_A^2(Y^2X^2-Y_L^2X_L^2)/4c\right)\right]}.$$

Comparative static analyses show that the equilibrium cash compensation A_A increases with the disutility of job changing and the effort aversion parameter, decreases with the expected value and variance of the aggregate state and firm-specific shocks. In addition, it decreases with the low states of the aggregate and firm-specific shocks. The equilibrium equity sharing B_A decreases with the expected value and the variance of the aggregate and firm-specific shocks and increases with the low states of the aggregate and firm-specific shocks.

Proposition 4.1 also indicates that the equilibrium reservation utility $\underline{V}_A$ depends on the aggregate state. Specifically, a high expected aggregate state or a high volatility will induce a high reservation utility. The equilibrium reservation utility $\underline{V}_A$ depends on firm-specific shock in a similar fashion. In addition, the equilibrium reservation utility $\underline{V}_A$ for a CEO is higher when the disutility of job change increases, or the CEO is more risk averse. The equilibrium reservation utility $\underline{V}_A$ decreases when the CEO becomes more effort averse. The dependence of the equilibrium reservation utility on the aggregate state obtained in this paper provides a theoretical support to the conjecture made by Himmerberg and Hubbard (2000).

4.2. Equilibrium Compensation when Only the RPE scheme is Used

First, I need to determine the reservation utility of firm i's CEO. Suppose that all firms other than firm i offer a RPE scheme $W_j = A_R + B_R\frac{S_j}{Y}$, where $j \neq i$. A potential CEO working for firm j will choose his effort $e_j^{R^*}$ to maximize his utility.

$$\max_{e_j}\left\{1 - U_0\exp[-\gamma(A_R + B_R\frac{S(e_i^*, X_i, Y)}{Y} - ce_j^2)]\right\}.$$

In this case, the optimal effort level is $e_j^{R*} = \frac{B_R X_j}{2c}$ and the CEO's utility level is $U_j^R = 1 - U_0 \exp(-\gamma A_R - \gamma B_R X_j^2/4c)$.

The expected reservation utility of firm i's CEO is

$$\underline{V}_R = pE(U_j^R) = p - pU_0 E\left[\exp(-\gamma A_R - \gamma B_R X_j^2/4c)\right]. \tag{4.2}$$

It is worth noting that, under the RPE scheme, the optimal effort and reservation utility do not depend on the aggregate state. The reason is that the RPE scheme has already filtered out the aggregate performance. In this case, his optimal effort and reservation utility depend positively on the dollar amount of equity sharing and the firm's own state and depends negatively on the effort-aversion parameter. I define the no-quitting RPE equilibrium as follows:

Definition 4.3. *When every firm uses only the RPE scheme, the no-quitting RPE equilibrium consists of a pair (A_R, B_R) which ensures that no CEO quits his current job and each principal obtains the highest profit.*

The above definition entails similar restrictions on firm i's compensation scheme and the average firm's. That is,

(1) $A_R = a_{R1}$; (2) $B_R = b_{R1}$; (3) $J_R = \pi_{R1}$; (4) $\pi_{R1} > \pi_{R3}$.

Under these equilibrium conditions, the no-quitting RPE equilibrium is characterized in the following proposition (see Appendix D for a proof):

Proposition 4.4. *If every firm uses the RPE scheme and the hiring cost H is higher than $\underline{H}_R = \frac{P_H^Y}{P_L^X}\frac{\mu_y^2}{4c}\left(X_H^2 - EX^2\frac{EX^2}{2EX^2 - X_L^2}\right)$, then the no-quitting RPE equilibrium incentive scheme consists* *of a fixed cash compensation A_R and the dollar amount of equity sharing B_R as*

$$A_R = \frac{1}{\gamma}\ln\left(\frac{U_0\left[1-pE[\exp(-\gamma B_R^2(X^2-X_L^2)/4c)]\right]}{1-p+D_m}\right) - \frac{1}{4c}\left(B_R X_L\right)^2 \qquad \text{and} \qquad B_R = \mu_y \frac{E(X^2)}{2E(X^2)-X_L^2}.$$

The equilibrium profit J_R for a principal and the reservation utility $\underline{V}_R$ for a CEO are

$$J_R = \frac{B_R \mu_y E(X^2)}{4c} - \frac{1}{\gamma}\ln\left(\frac{U_0 - pU_0 E[\exp(-\gamma B_R^2(X^2-X_L^2)/4c)]}{1-p+D_m}\right)$$

and

$$\underline{V}_R = \frac{p - p(1+D_m)E[\exp(-\gamma B_R^2(X^2-X_L^2)/4c)]}{1-pE[\exp(-\gamma B_R^2(X^2-X_L^2)/4c)]}.$$

Evidently, the equilibrium RPE incentive contract does not depend on the aggregate state of the economy. Comparative static analyses show that the equilibrium fixed cash payment A_R increases with the disutility of job changing and the effort aversion parameter, decreases with the expected value and variance of firm-specific shocks. In addition, it also decreases with the low states of the firm-specific shocks. The equilibrium equity sharing rule B_R becomes smaller when the expected value and the variance of the firm-specific shocks increases, and is larger when the expected aggregate state is higher.

Under the RPE scheme, the equilibrium reservation utility $\underline{V}_R$ does not depend on the aggregate state, or on the expected aggregate state through the equity sharing. Specifically, a high expected aggregate state induces a high reservation utility. The equilibrium reservation utility $\underline{V}_R$ primarily depends on firm-specific shock. A high expected firm-specific shock or a high volatility induces a high reservation utility. In addition, the equilibrium reservation utility $\underline{V}_R$ for a CEO is higher when the disutility of job change increases, or the CEO is more risk averse. The equilibrium reservation utility $\underline{V}_R$ decreases when the CEO becomes more effort averse.

4.3. Market Equilibrium under No-Quitting

To show that the equilibria described in the previous two sections are indeed the market equilibria, I need to show that an individual firm has no incentive to use the alternative compensation scheme. For example, when all other firms use the APE schemes, I need to show that a firm has no incentive to use the RPE contract. Under different economic conditions, either APE or RPE can be the equilibrium outcome (see Appendix E). To facilitate the presentation, I denote $\alpha_x = \frac{E(X^2)-X_L^2}{2E(X^2)-X_L^2}$.

Proposition 4.5. *When both the APE and RPE schemes are offered in the market, the APE scheme becomes the market equilibrium when* $\left(\frac{\sigma_y}{\mu_y}\right)^2 > \sqrt{(\alpha_x-1)^2+(2\alpha_x-1)\left(\frac{Y_L}{\mu_y}\right)^2}-\alpha_x$ *and no firm has incentive to deviate to a RPE contract. When* $\left(\frac{\sigma_y}{\mu_y}\right)^2 < \sqrt{(\alpha_x-1)^2+(2\alpha_x-1)\left(\frac{Y_L}{\mu_y}\right)^2}-\alpha_x$*, the RPE scheme becomes the market equilibrium. When* $\left(\frac{\sigma_y}{\mu_y}\right)^2 = \sqrt{(\alpha_x-1)^2+(2\alpha_x-1)\left(\frac{Y_L}{\mu_y}\right)^2}-\alpha_x$*, both types of incentive contracts coexist.*

The above proposition shows that the RPE scheme is not always the dominating scheme. This suggests that the lack of using RPE may not be a puzzle in a competitive market with many firms and CEOs and the current practice with APE schemes is consistent with the equilibrium analysis.

The economic condition indicates that firms prefer to use the APE equilibrium scheme when the aggregate economy is more likely to be in the low state (i.e., a high P_L^Y) and more volatile (i.e., a high σ_y). If the aggregate economy is less likely to be in the low state, and if firms are more likely to be in good state (i.e., a high P_H^X) or its volatility is high (i.e., a high σ_x), they also prefer the APE compensation. These results are intuitive. When a low state of the aggregate economy is anticipated, CEOs are likely to use low effort under the APE scheme. For firms with bad shocks, firms' values are likely to be very low. In this case, the principals would prefer the APE scheme. For firms with good shocks, their firms' values are likely to be higher than with bad shock. In this case, principals are better off because they can avoid a higher payment to

CEOs under the RPE schemes if firms' own states are good.[4]

On the other hand, firms prefer the RPE scheme when the economy is likely to be in a high state (i.e., a high P_H^Y) and less volatile (i.e., a low σ_y). If the aggregate economy is more likely to be in the low state, and if firms are more likely to be in good state (i.e., a high P_H^X) or its volatility is low (i.e., a low σ_x), they also prefer the RPE compensation. To exploit the intuition, consider the case where the aggregate economy is in the high state. For firms with good shocks, their CEOs would work harder. Therefore, firms' values are likely to be very high. In this case, principals would prefer the RPE scheme since they pay less to their CEOs relative to the APE scheme.

Under the APE equilibrium, the equilibrium optimal effort is $e_A^* = \frac{E(X^2Y^2)}{2E(X^2Y^2)-X_L^2Y_L^2}\frac{YX}{2c}$. Clearly, the effort level depends on both the aggregate state and firm-specific state. In contrast, the equilibrium optimal effort under the RPE equilibrium is $e_R^* = \frac{E(X^2)}{2E(X^2)-X_L^2}\frac{\mu_y X}{2c}$. This effort level depends on the firm-specific state and the expected aggregate economy. It does not depend on the actual realization of the aggregate state.

The RPE scheme does not induce higher effort in all states than the APE scheme does. In particular, when the economy is in the low state, the RPE contract induces high effort level, i.e., $e_R^* > e_A^*$. The intuition is that a motivated CEO prefers the RPE to the APE scheme because his high effort will be rewarded more under the RPE scheme than under the APE scheme when the aggregate state is low. By contrast, when the economy is in the high state and the APE scheme is the equilibrium outcome, the optimal effort level under the APE is higher than that under the RPE scheme.

[4] It is very likely that the existing use of APE plans is induced by the economic conditions. An empirical investigation would help to verify this observation. I plan to carry out the empirical analysis in a follow-up study.

5. Extension: Quitting Behavior

It may be argued that a market equilibrium with possible quitting behavior is more realistic. In this subsection, I allow for possible quitting. Again, I first consider the symmetric situation where all firms use the same type of compensation scheme and then examine the two possible deviations to show that the described equilibria are indeed the market equilibria. To make the quitting behavior consistent under the two deviation cases, I only consider Scenario 3. Similar to the no-quitting equilibrium, the quitting equilibrium under the APE scheme requires that

$$(1) \quad A_A^q = a_{A3}; \qquad (2) \quad B_A^q = b_{A3}; \qquad (3) \quad J_A^q = \pi_{A3}; \qquad (4) \quad \pi_{A3} > \pi_{A1}, \pi_{A2};$$

and that the RPE scheme requires that

$$(1) \quad A_R^q = a_{R3}; \qquad (2) \quad B_R^q = b_{R3}; \qquad (3) \quad J_R^q = \pi_{R3}; \qquad (4) \quad \pi_{R3} > \pi_{R1}.$$

Given these definitions, I first consider the situation where every firm uses the APE scheme, then I analyze the situation where every firm uses the RPE scheme. For brevity, I omit the proof and present the market equilibrium with quitting in the following proposition:

Proposition 5.1. *When both the APE and RPE schemes are used in the market, the APE scheme becomes the equilibrium contract when* $\left(\frac{\sigma_y}{\mu_y}\right)^4 > 1 - \left(\frac{Y_L}{\mu_y}\right)^2$ *and no firm has incentive to deviate to a RPE scheme. When* $\left(\frac{\sigma_y}{\mu_y}\right)^4 < 1 - \left(\frac{Y_L}{\mu_y}\right)^2$*, the RPE becomes the equilibrium outcome. When* $\left(\frac{\sigma_y}{\mu_y}\right)^4 = 1 - \left(\frac{Y_L}{\mu_y}\right)^2$*, both types of contracts can coexist.*

Table 4 presents the market equilibrium incentive contracts under these economic conditions. With quitting behavior, the economic condition ensuring one type of equilibrium scheme only depends the properties of the aggregate economy. Clearly, if the volatility of the aggregate economy is higher than its expected state, then the APE equilibrium with quitting is guaranteed. The above equilibrium results have the same interpretations as those for the no quitting equilibrium.

Table 4: Market Equilibrium with Quitting

	APE as the Equilibrium scheme	RPE as the Equilibrium scheme
Economic Condition	$\left(\frac{\sigma_y}{\mu_y}\right)^4 > 1 - \left(\frac{Y_L}{\mu_y}\right)^2$	$\left(\frac{\sigma_y}{\mu_y}\right)^4 < 1 - \left(\frac{Y_L}{\mu_y}\right)^2$
Hiring Cost	$0 < H < \underline{H}_A$	$0 < H < \underline{H}_R$
Fixed Salary	$A_A^q = \frac{1}{\gamma}\ln\left(\frac{U_0}{1+D_m-\underline{V}_A^q}\right) - \frac{1}{4c}(B_A^q Y_L X_H)^2$	$A_R^q = \frac{1}{\gamma}\ln\left(\frac{U_0}{1+D_m-\underline{V}_R^q}\right) - \frac{1}{4c}(B_R^q X_H)^2$
Equity Reward	$B_A^q = \frac{E(Y^2)}{2E(Y^2)-Y_L^2}$	$B_R^q = \mu_y$
Profit	$J_A^q = \frac{B_A^q X_H^2 E(Y^2)}{4c} - \frac{P_L^X}{P_H^X}H - \frac{1}{\gamma}\ln\left(\frac{U_0}{1+D_m-\underline{V}_A^q}\right)$	$J_R^q = \frac{\mu_y^2 X_H^2}{4c} - \frac{P_L^X}{P_H^X}H - \frac{1}{\gamma}\ln\left(\frac{U_0}{1+D_m-\underline{V}_R^q}\right)$

Reservation Utility under the APE Equilibrium: $\underline{V}_A^q = \frac{p-p(1+D_m)E\left(\exp(-\gamma B_A^{q^2}(Y^2X^2-Y_L^2X_H^2)/4c)\right)}{1-pE\left(\exp(-\gamma B_A^{q^2}(Y^2X^2-Y_L^2X_H^2)/4c)\right)}$

Reservation Utility under the RPE Equilibrium: $\underline{V}_R^q = \frac{p-p(1+D_m)E\left(\exp(-\gamma\mu_y^2(X^2-X_H^2)/4c)\right)}{1-pE\left(\exp(-\gamma\mu_y^2(X^2-X_H^2)/4c)\right)}$

6. A Mixed Compensation scheme - a Better Incentive Scheme

The above market equilibrium analysis shows that neither the APE nor the RPE is the dominating reward scheme under all circumstance. An APE scheme allows the equilibrium reservation utility to depend on aggregate states, and it can be competitive in a multi-firm-multi-agent environment. However, it cannot properly reward a performance which is better than the benchmark. In contrast, a RPE scheme can reward a CEO for a better-than-benchmark performance, but it may not necessarily be competitive in the market. This reasoning motivates a mixed compensation scheme which is anticipated to overcome the shortcomings of the standing alone APE or RPE schemes. The mixed compensation scheme for an average firm can take the following form:

$$W_{mix} = A_{mix} + B_{1mix}S + B_{2mix}\frac{S}{Y},$$

where A_{mix} is the fixed salary, B_{1mix} is the percentage of equity sharing based the firm's value and B_{2mix} is a dollar amount of reward for a better-than-benchmark performance.

The market equilibrium with the mixed contract can be obtained in the same way as the APE (or RPE) scheme. For simplicity, I only present the no-quitting equilibrium here since the quitting equilibrium yields the same qualitative conclusions. Also, I omit the proof for brevity. Table 5 summarizes the no-quitting equilibrium under the mixed scheme.

In this case, the equilibrium profit for a principal is

$$J_{mix} = \frac{E(X^2Y^2)}{8c} - \frac{1}{\gamma}\ln\frac{U_0}{1 + D_m - \underline{V}_{mix}} + \frac{X_L^2Y_L^2}{8c}\left[\frac{2X_L^2E(Y-Y_L)^2 - \sigma_y^2E(X^2)}{X_L^2E(Y-Y_L)^2 - 2\sigma_y^2E(X^2)}\right]$$

and the expected optimal effort level devoted by a CEO is

$$E\left(e^*_{mix}\right) = (B_{1mix}\mu_y + B_{2mix})\frac{\mu_x}{2c}.$$

Table 5: No-Quitting Equilibrium with the Mixed Compensation scheme

Fixed Salary	$A_{mix} = \frac{1}{\gamma}\ln\frac{U_0}{1+D_m-\underline{V}_{mix}} - \frac{X_L^2}{4c}\left(B_{1mix}Y_L + B_{2mix}\right)^2,$
Equity Reward	$B_{1mix} = \frac{1}{2} - \frac{1}{2}X_L^2Y_L\frac{(\mu_y - Y_L)}{X_L^2E(Y-Y_L)^2 - 2\sigma_y^2E(X^2)},$
Relative Performance Reward	$B_{2mix} = \frac{1}{2}X_L^2Y_L\frac{\mu_y(\mu_y-Y_L)+\sigma_y^2}{X_L^2E(Y-Y_L)^2-2\sigma_y^2E(X^2)},$
Reservation Utility	$\underline{V}_{mix} = \frac{1}{\gamma}\ln\frac{U_0\left\{1-pE\left[\exp\left(\gamma(B_{1mix}X_LY_L+B_{2mix}X_L)^2/4c-\gamma(B_{1mix}XY+B_{2mix}X)^2/4c\right)\right]\right\}}{1-p+D_m}$ $-\frac{X_L^2}{4c}\left(B_{1mix}Y_L + B_{2mix}\right)^2.$

It is easy to show that the mixed scheme yields the highest profit J_{mix} to a firm and induces the highest expected effort from a CEO. The intuition behind these results are as follows. The mixed scheme provides a better competitive reward scheme than the RPE scheme and at the same time it motivates the CEO better than the APE scheme for a better-than-benchmark performance. More importantly, the competitive feature and motivation are achieved at a lower compensation cost to the firm. Therefore, the firm can earn a higher expected profit than that under either of the two pure reward schemes discussed earlier. This conclusion provides a very important implication to corporate compensation specialists. To enhance the current compensation practice and to better motivate and rewards their CEOs, firms should use mixed compensation policies consisting of both the APE and RPE reward mechanisms.

7. Conclusion

Contrary to the prediction of the existing economic theories on managerial compensation, very few firms use the relative performance evaluation (RPE) scheme to reward their CEOs. Researchers have dubbed this a puzzle. This paper attempts to shed some light on this puzzle. I demonstrate that the absolute performance evaluation (APE) scheme could be optimal too. To this end, I construct an equilibrium model to analyze the optimal incentive contract in a market with many firms and CEOs. The interactions among firms are modelled through CEOs' reservation utilities which are endogenized in the equilibrium.

I consider two types of equilibria: One features CEOs' quitting decisions and the other does not. Under both equilibria, APE or RPE can constitute the equilibrium incentive scheme, depending on the economic condition. In particular, the APE scheme prevails when the economy is more volatile and more likely to be in a low state; the RPE scheme prevails when the economy is less volatile and more likely to be in a good state. Therefore the RPE scheme is not necessarily always a better reward mechanism in a competitive market, hence the solution to the puzzle.

I carry out the analysis further and demonstrate that a mixture of APE and RPE schemes provides a better motivation to CEOs and yields higher profits to firms. Intuitively, in the mixed scheme, the APE reward feature allows the compensation to be more competitive since the CEOs' reservation utilities depend on the aggregate state. Once the compensation package is competitive, the RPE feature kicks in to better induce CEOs' efforts. Therefore, the mixed scheme is more effective than either the APE or the RPE scheme alone. More important is the fact that the enhanced effectiveness is achieved at a lower total compensation cost.

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Appendix

A. Proof of Table 2

The results in Table 2 are derived in two steps. The first step solves for CEO i's optimal effort level while taking firm i's APE compensation and his reservation utility $\underline{V}_A$ in equation (4.1) as given. Formally, the problem is

$$\max_{e} \left[1 - U_0 \exp\left(-\gamma(a_A + b_A S(e, X_i, Y) - ce^2)\right)\right]$$

$$s.t.$$

$$1 - U_0 \exp\left(-\gamma(a_A + b_A S(e_i^*, X_i, Y) - c{e_i^*}^2)\right) \geq \underline{V}_A - D_m$$

It is easy to show that the optimal solution is as follows: when $Y^2 X_i^2 \geq \frac{4c}{b_A}\left(\frac{1}{\gamma}\ln\frac{U_0}{1+D_m-\underline{V}_A} - a_A\right)$, CEO i works for firm i with his optimal effort $e_i^* = \frac{b_A Y X_i}{2c}$; otherwise, he looks for another job.

The second step solves for the optimal compensation scheme for principal i. As indicated in the paper, there are only three realistic scenarios to be considered. Given the distributions for the aggregate shock Y and firm i's shock X_i, in order to satisfy the no-quitting condition $Y^2 X_i^2 \geq \frac{4c}{b_A}\left(\frac{1}{\gamma}\ln\frac{U_0}{1+D_m-\underline{V}_A} - a_A\right)$ under Scenario 3, I require $Y_L X_H > Y_H X_L$.

Under Scenario 1, the participation constraint is satisfied in all states. That is, the no-quitting condition becomes $Y_L^2 X_L^2 \geq \frac{4c}{b_A}\left(\frac{1}{\gamma}\ln\frac{U_0}{1+D_m-\underline{V}_A} - a_A\right)$ and CEO i stays with firm i with certainty. Principal i's problem simply becomes

$$\max_{a_{A1}, b_{A1}} \pi_{A1} = E\left[S(e_i^*, X_i, Y) - a_{A1} - b_{A1} S(e_i^*, X_i, Y)\right]$$

$$s.t.$$

$$Y_L^2 X_L^2 \geq \frac{4c}{b_{A1}}\left(\frac{1}{\gamma}\ln\frac{U_0}{1+D_m-\underline{V}_A} - a_{A1}\right) \qquad and \qquad e_i^* = \frac{a_{A1} Y X_i}{2c}.$$

Replacing the optimal effort level into the objective function, the above problem becomes:

$$\max_{a_{A1}, b_{A1}} \pi_{A1} = \frac{b_{A1}(1-b_{A1})}{2c} E(Y^2 X^2) - a_{A1}$$

$$s.t.$$

$$Y_L^2 X_L^2 \geq \frac{4c}{b_{A1}}\left(\frac{1}{\gamma}\ln\frac{U_0}{1+D_m-\underline{V}_A} - a_{A1}\right).$$

Tedious exercise yields optimal solutions for a_{A1}, b_{A1} and π_{A1} as in Table 2.

Under Scenario 2, the participation constraint is satisfied in three states. The no-quitting condition becomes $Y_H^2 X_L^2 \geq \frac{4c}{b_A}\left(\frac{1}{\gamma}\ln\frac{U_0}{1+D_m-\underline{V}_A} - a_A\right) > Y_L^2 X_L^2$ under $X_H Y_L > X_L Y_H$. CEO i's optimal action is to quit only in state (Y_L, X_L) and stay in all other possible states. If the current CEO quits, principal i hires a replacement with a hiring cost H. In this case, the expected profit for principal i in states (Y_L, X_L) is $P_L^Y P_L^X (J_A - H)$ where J_A is the equilibrium profit earned by an average firm. In all other states $\{\ (Y_H, X_L), (Y_L, X_H), (Y_H, X_H)\ \}$, the CEO works for firm i at an effort level $e_i^* = \frac{a_A Y X_i}{2c}$. Precisely, the problem for principal i is

$$\max_{a_{A2}, b_{A2}} \pi_{A2} = \frac{b_{A2}(1-b_{A2})}{2c} E(Y^2X^2 - P_L^Y P_L^X Y_L^2 X_L^2) - (1 - P_L^Y P_L^X) a_{A2} + P_L^Y P_L^X (J_A - H)$$

s.t.

$$Y_H^2 X_L^2 \geq \frac{4c}{b_{A2}}\left(\frac{1}{\gamma}\ln\frac{U_0}{1+D_m-\underline{V}_A} - a_{A2}\right) > Y_L^2 X_L^2.$$

Under Scenario 3, the participation constraint is satisfied only in two states $\{(Y_L, X_H), (Y_H, X_H)\}$. The no-quitting condition becomes $Y_L^2 X_H^2 \geq \frac{4c}{b_A}\left(\frac{1}{\gamma}\ln\frac{U_0}{1+D_m-\underline{V}_A} - a_A\right) > Y_H^2 X_L^2$. CEO i quits when firm i is in the bad state, regardless of the aggregate state, and stays when firm i is in good state. In this case, principal i's problem becomes

$$\max_{a_{A3}, b_{A3}} \pi_{A3} = \frac{b_{A3}(1-b_{A3})}{2c} P_H^X X_H^2 E(Y^2) - P_H^X a_{A3} + (1 - P_H^X)(J_A - H)$$

s.t.

$$Y_L^2 X_H^2 \geq \frac{4c}{b_{A3}}\left(\frac{1}{\gamma}\ln\frac{U_0}{1+D_m-\underline{V}_A} - a_{A3}\right) > Y_H^2 X_L^2.$$

Tedious exercises yield the optimal solutions for Scenarios 2 and 3 as in Table 2.

B. Proof of Table 3

Similarly, the results in Table 3 are derived in two steps. The first step solves for CEO i's optimal effort level while taking firm i's RPE compensation as given. The precise problem is

$$\max_{e} \left[1 - U_0 \exp\left(-\gamma(a_R + b_R S(e, X_i, Y) - ce^2)\right)\right]$$

s.t.

$$1 - U_0 \exp\left(-\gamma(a_A + b_A S(e_i^*, X_i, Y) - c{e_i^*}^2)\right) \geq \underline{V}_R - D_m.$$

The solution to the problem is as follows: when $X_i^2 \geq \frac{4c}{b_R}\left(\frac{1}{\gamma}\ln\frac{U_0}{1+D_m-\underline{V}_R} - a_R\right)$, CEO i works for firm i with his optimal effort $e_i^* = \frac{b_R Y X_i}{2c}$; otherwise, he looks for another job.

The second step solves the optimal compensation scheme for principal i. There are only two cases to be considered, given the distribution of firm i's shock X_i:

Scenario 1: when $X_L^2 \geq \frac{4c}{b_R}\left(\frac{1}{\gamma}\ln\frac{U_0}{1+D_m-\underline{V}_R} - a_R\right)$,	CEO i works for firm i with his optimal effort $e_i^* = \frac{b_R Y X_i}{2c}$;
Scenario 3: when $X_H^2 \geq \frac{4c}{b_R}\left(\frac{1}{\gamma}\ln\frac{U_0}{1+D_m-\underline{V}_R} - a_R\right) > X_L^2$,	CEO i works for firm i when $X = X_H$ with his optimal effort $e_i^* = \frac{b_R Y X_H}{2c}$;

Under Scenario 1, the participation constraint is satisfied in all states. The no-quitting condition becomes $X_L^2 \geq \frac{4c}{b_R}\left(\frac{1}{\gamma}\ln\frac{U_0}{1+D_m-\underline{V}_R} - a_R\right)$. The CEO works for firm i with certainty. Principal i's problem becomes

$$\max_{a_{R1}, b_{R1}} \quad \pi_{R1} = E\left[S(e_i^*, X_i, Y) - a_{R1} - b_{R1} S(e_i^*, X_i, Y)\right]$$

$$s.t. \qquad X_L^2 \geq \frac{4c}{b_{R1}}\left(\frac{1}{\gamma}\ln\frac{U_0}{1+D_m-\underline{V}_R} - a_{R1}\right) \qquad and \qquad e_i^* = \frac{b_R Y X_i}{2c}.$$

Under Scenario 3, the participation constraint is satisfied in two states $\{(Y_H, X_H), (Y_L, X_H)\}$ and the no-quitting condition becomes $X_X^2 \geq \frac{4c}{b_R}\left(\frac{1}{\gamma}\ln\frac{U_0}{1+D_m-\underline{V}_R} - a_R\right) > X_L^2$. The CEO works for firm i in its good state and quits in its bad state. When the current CEO quits, principal i hires a replacement with a hiring cost H and obtains the expected profit $P_L^X(J_R - H)$, where J_R is the equilibrium profit an average firm earns with a RPE scheme. When $X = X_H$, CEO i works for firm i at an effort level $e_i^* = \frac{b_R Y X_H}{2c}$. Now, principal i's problem becomes

$$\max_{a_{R3}, b_{R3}} \quad \pi_{R3} = \frac{b_{R3} P_H^X X_H^2}{2c} E(Y - b_{R3}) - P_H^X a_{R3} + P_L^X (J_R - H)$$

$$s.t. \qquad X_H^2 \geq \frac{4c}{b_{R3}}\left(\frac{1}{\gamma}\ln\frac{U_0}{1+D_m-\underline{V}_R} - a_{R3}\right) > X_L^2.$$

Tedious exercise yields optimal solutions for Scenarios 1 and 3 as in Table 3.

C. Proof of Proposition 4.2

The no-quitting APE equilibrium definition requires

$$\begin{aligned}
(1) \quad A_A &= \left[\ln U_0 - \ln(1 + D_m - \underline{V}_A)\right]/\gamma - \left(B_A Y_L X_L\right)^2/4c; \\
(2) \quad B_A &= \frac{E(X^2Y^2)}{2E(X^2Y^2) - X_L^2 Y_L^2}; \\
(3) \quad J_A &= B_A E(X^2Y^2)/4c - \frac{1}{\gamma}\left[\ln U_0 - \ln(1 + D_m - \underline{V}_A)\right].
\end{aligned}$$

Given $\underline{V}_A = p - pU_0 e^{-\gamma A_A} E\left[e^{-\gamma B_A Y^2 X^2/4c}\right]$, the explicit solutions to the equilibrium cash payment A_A, the CEO's reservation utility $\underline{V}_A$ and the equilibrium profit for a firm J_A can be obtained by substituting $A_A = \left[\ln U_0 - \ln(1 + D_m - \underline{V}_A)\right]/\gamma - \left(B_A Y_L X_L\right)^2/4c$ into the expression for $\underline{V}_A$. To ensure the equilibrium profit being higher than profits under Scenarios 2 to 3, I impose the following condition

$$(4) \quad J_A = \pi_{A1} > \pi_{A2}, \pi_{A3},$$

which transforms the requirement into a condition on the hiring cost. Precisely, the hiring cost has to be higher than $\underline{H}_A$ in order to ensure the no-quitting APE equilibrium.

D. Proof of Proposition 4.4

Similarly, the no-quitting RPE equilibrium definition requires

$$\begin{aligned}
(1) \quad A_R &= \left[\ln U_0 - \ln(1 + D_m - \underline{V}_R)\right]/\gamma - \left(B_R X_L\right)^2/4c; \\
(2) \quad B_R &= \mu_y \frac{E(X^2)}{2E(X^2) - X_L^2}; \\
(3) \quad J_R &= B_R \mu_y^2 E(X^2)/4c - \frac{1}{\gamma}\left[\ln U_0 - \ln(1 + D_m - \underline{V}_R)\right].
\end{aligned}$$

Given $\underline{V}_R = p - pU_0 e^{-\gamma A_R} E\left[e^{-\gamma B_R X^2/4c}\right]$, the explicit solutions for the equilibrium cash payment A_R, the CEO's reservation utility $\underline{V}_R$ and the equilibrium profit for a firm J_R are obtained by substituting $A_R = \left[\ln U_0 - \ln(1 + D_m - \underline{V}_R)\right]/\gamma - \left(B_R X_L\right)^2/4c$ into the expression for $\underline{V}_R$. Again, to ensure the equilibrium profit being higher than profits under Scenario 3, I impose the following condition

$$(4) \quad J_R = \pi_{R1} > \pi_{R3},$$

which, in turn, yields a condition on the hiring cost. That is, the no-quitting RPE equilibrium can be obtained when the hiring cost is higher than $\underline{H}_R$.

E. Proof of Proposition 4.5

To show that the two equilibria presented in Propositions 4.2 and 4.4 are indeed the market equilibria, I consider two types of deviations. The first type deals with a firm which deviates to the RPE scheme while all other firms use the APE scheme. For this firm, its compensation strategy is

$$A_R^d = \tfrac{1}{\gamma}\left[\ln U_0 - \ln(1 + D_m - \underline{V}_A)\right] - \tfrac{\left(B_R^d X_L\right)^2}{4c} \qquad \textit{and} \qquad B_R^d = \mu_y \tfrac{E(X^2)}{2E(X^2) - X_L^2}.$$

Its profit is

$$\pi_R^d = J_A - \frac{E(X^2)}{4c}\left[\frac{E(X^2Y^2)}{2E(X^2) - \frac{Y_L^2}{E(Y^2)}X_L^2} - \frac{E(X^2)\mu_y^2}{2E(X^2) - X_L^2}\right].$$

If

$$\frac{E(Y^2)}{2E(X^2) - \frac{Y_L^2}{E(Y^2)}X_L^2} > \frac{\mu_y^2}{2E(X^2) - X_L^2},$$

then the firm is worse off if it deviates to the RPE scheme. The above condition is equivalent to

$$\left(\frac{\sigma_y}{\mu_y}\right)^2 > \sqrt{(\alpha_x - 1)^2 + (2\alpha_x - 1)\left(\frac{Y_L}{\mu_y}\right)^2} - \alpha_x,$$

which implies that the APE scheme is the equilibrium outcome. Otherwise, every firm has the incentive to deviate to the RPE and then the RPE scheme becomes the equilibrium outcome.

Consider the second type of deviation where a firm uses the APE scheme while all other firms use the RPE scheme. For this firm, its compensation strategy is

$$A_A^d = \frac{1}{\gamma}\left[\ln U_0 - \ln(1 + D_m - \underline{V}_R)\right] - \frac{\left(B_A^d X_L Y_L\right)^2}{4c} \qquad and \qquad B_A^d = \frac{E(X^2Y^2)}{2E(X^2Y^2) - X_L^2 Y_L^2}.$$

Its profit is

$$\pi_A^d = J_R - \frac{E(X^2)}{4c}\left[\frac{E(X^2)\mu_y^2}{2E(X^2) - X_L^2} - \frac{E(X^2Y^2)}{2E(X^2) - \frac{Y_L^2}{E(Y^2)}X_L^2}\right].$$

If

$$\frac{\mu_y^2}{2E(X^2) - X_L^2} > \frac{E(Y^2)}{2E(X^2) - \frac{Y_L^2}{E(Y^2)}X_L^2},$$

then the firm is worse off if it deviates to the APE scheme. This condition yields the same results as those of the first type of deviation.