Coordination, matching, and wages

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Abstract. We analyse the coordination problem in the labour market by endogenizing the matching function and the wage share. Each firm posts a wage to maximize the expected profit, anticipating how the wage affects the expected number of applicants. In equilibrium workers apply to firms with mixed strategies, which generate coordination failure and persistent unemployment. We show how the wage share, unemployment, and the welfare loss from the coordination failure depend on the market tightness and the market size. The welfare loss from the coordination failure is as high as 7.5 per cent of potential output. JEL Classification: C78, J64

Coordination, arrimage et salaires. Les auteurs analysent le problème de la coordination dans le marché du travail en endogénéisant la fonction d'arrimage et la part des revenus qui va aux salaires. Chaque entreprise définit le niveau de salaire qui maximise ses profits anticipés, en tenant compte de l'effet de ce niveau de salaire sur le nombre des applications qu'elle peut anticiper. De même, les travailleurs font application auprès d'une entreprise à un salaire donné en tenant compte d'une certaine relation d'équivalence entre niveau de salaire et probabilité d'obtenir l'emploi. Voilà qui engendre incoordination et chômage persistant. On montre que la part des revenus qui revient aux salaires, le niveau de chômage, et les pertes de bien-être attribuables au manque de coordination dépendent de la taille du marché et du degré de rareté de la main d'oeuvre. Les pertes de bien-être attribuables au manque de coordination correspondent à quelques 7,5 pour-cent de la production potentielle.

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1. Introduction

Unemployment and under-utilization of machines are two examples of problems that can be caused by coordination failure among agents in the economy. When productive factors are not fully utilized, the society suffers a loss. How large is this welfare loss? How is the wage share affected by the coordination failure? Does more coordination make everyone better off? These are the questions posited in this paper. They are important for many policies. For example, policymakers may pass regulations to increase the average skill of workers or to integrate segregated local markets into a larger one. Although these policies have obvious benefits, they also make the labour force more mobile and increase the coordination problem among firms and workers. To evaluate the policies, regulators should know how large the welfare loss is from the increased coordination failure and how the policies affect wages and redistribute the match surplus among agents.

To answer these questions the standard Walrasian theory is of little help, because it stipulates that all markets be cleared. Researchers have analysed persistent unemployment using the search theory pioneered by Diamond (1982), Mortensen (1982), and Pissarides (1990). This theory uses an exogenous matching function to determine the number of matches between workers and firms. Not all potential matches are exhausted, and so wages do not clear the labour market. Instead, wages split the match surplus between the worker and the firm according to some exogenous rules, such as the Nash bargaining solution.

Although the search theory improves upon the Walrasian theory, it is inadequate for answering the above questions. First, the exogenous matching function makes it difficult to analyse how policies affect the extent of matches. The moment one specifies a matching function, he/she has already determined the extent of matches exogenously. Second, the exogenous wage rule precludes agents' attempt to affect the number of matches by setting prices (wages). With the Nash bargaining rule, in particular, workers' share of the match surplus is exogenous and independent of the market tightness. To answer the above questions in a satisfactory way, we need to treat the coordination failure as an outcome of agents' actions rather than an exogenous characteristic. This entails a theory that uses agents' actions to determine the matching function and the wage share.

To achieve this objective we adapt the framework of Peters (1991) and Montgomery (1991), in which firms post wages to direct workers' search decisions. By posting a particular wage, a firm anticipates how likely it is that each worker will apply to the wage, thus making a trade-off between the ex post profit and the likelihood of obtaining such a profit. Similarly, by applying to a particular wage, a worker makes a trade-off between the wage and the likelihood of obtaining it. These decisions by firms and workers induce a matching function.[1]

1 Carlton (1978) seems to be the first one to formally analyse the trade-off in the goods market between price and service probability. Rather than generating this relationship endogenously by agents' strategic behaviour, he exogenously assumes that each buyer has a smooth preference ordering over the pair of price and service probability.

We illustrate the framework with an example in section 2 and extend it to a market with any finite number of agents in section 3. The main result in these sections is that an increase in the coordination failure does not always make everyone worse off, although it always reduces the overall welfare level. In particular, an increase in the market size exacerbates the coordination problem, but when firms are on the shorter side of the market, wages increase and workers can be better off. The reason is that a larger market reduces the relative monopoly power of the shorter side, reduces its factor price, and hence transfers surpluses to agents on the longer side. Such changes in the wage share occur even when the market tightness is unchanged.[2]

In section 4 we extend the analysis to an infinite horizon with infinitely many firms and workers. Several interesting results emerge. First, despite the coordination failure, the wage-posting model approximates the Walrasian economy well. When there is a large disparity between supply and demand, the wage is close to the Walrasian wage; when supply and demand are close to each other, a relative increase in supply reduces the wage in large magnitudes. Second, an increase in the recruiting cost has little effect on the overall worker/job ratio, but it greatly increases the relative number of unemployed workers to vacancies and hence significantly changes the unemployment rate. Finally, with reasonable job separation rates, the welfare loss from the coordination failure is as high as 7.5 per cent of potential output.

The basic structure of this paper is taken from Peters (1991) and Montgomery (1991). We have made two contributions to this structure. First, in section 3 we characterize the equilibrium with finite numbers of workers and firms; this allows us to show how the market size affects the coordination failure and the wage share. In contrast, Peters focused on a market with infinitely many agents and Montgomery made an approximation for the finite economy. Second, in section 4 we examine the steady state of a large economy. In contrast, Peters focused on the non-steady state and Montgomery examined only a one-period problem. By examining the steady state we can calculate the welfare loss from persistent unemployment.

Two other papers, Burdett, Shi, and Wright (1996) and Julien, Kennes, and King (forthcoming), are also closely related to the current paper. Using a price-posting model, Burdett, Shi, and Wright (1996) analysed firms' choices on capacity and numerically illustrated how the price depends on the numbers of agents. But they do not examine the steady state or analytically establish a link between the wage and the market size. Julien, Kennes, and King (forthcoming) used an auction model to make the matching function endogenous. As argued in subsection 3.3, below, both the wage-posting model and the auction model capture agents' trade-off between a wage mechanism and the matching probability. In this regard, the two models share some qualitative results. However, the details are different in the two models. In particular, the auction model generates wage dispersion between identical

2 Whether one side is shorter than the other side is not determined by simply counting the numbers of workers and jobs. Workers are on the shorter side, even when there are more workers than jobs, as long as the worker/job ratio is greater than 0.83 (see section 3, below, for an explanation).

workers, which makes it more difficult to characterize the steady state than in our model.[3]

2. An example (example 1)

We use this example to illustrate the wage-posting framework and introduce the issues, making assumptions that will be discussed in subsection 3.3, below. Consider a labour market with two workers and two firms. Workers and firms are both risk neutral and living for one period. Workers are indexed by i and firms are indexed by j. The workers, termed worker 1 and worker 2, are identical in all aspects, each wanting one job and working for an indivisible amount of time. The utility cost of time is normalized to zero. The firms, termed firm A and firm B, also are identical, each having one job to offer. Each worker-firm pair produces one unit of output.

The labour market is frictional, in the sense that each worker can apply to at most one job at a time. Also, agents cannot coordinate. To create matches, each firm posts a wage to attract workers, taking the other firm's wage offer as given, and each worker decides which firm to apply to after observing all posted wages. More precisely, the agents' actions are as follows. At the beginning of the period, each firm j posts a wage w_j $(j = A, B)$, taking the other's wage as given. Observing all wages, each worker i $(= 1,2)$ chooses a probability α_{Ai} to apply to firm A and a probability $\alpha_{Bi} = 1 - \alpha_{Ai}$ to apply to firm B, taking the other worker's strategy as given. If both workers end up with the same firm, the firm selects one applicant and pays the posted wage, each applicant being selected with probability 1/2.

Examine, first, the second-stage game between the two workers who have observed the two wages, $(w_A, w_B) \equiv W$. If worker i applies to firm j, he/she gets the job unless the other worker i' $(\neq i)$ also applies to the same firm and is chosen by the firm. The latter joint event occurs with probability $\alpha_{ji'}/2$, and so worker i's expected utility from applying to firm j is $(1 - \alpha_{ji'}/2)w_j$. Worker i's strategy is

$$\alpha_{Ai}(W) \begin{cases} = 1, & \text{if } \left(1 - \dfrac{\alpha_{Ai'}}{2}\right) w_A > \left(1 - \dfrac{\alpha_{Bi'}}{2}\right) w_B \\ = 0, & \text{if } \left(1 - \dfrac{\alpha_{Ai'}}{2}\right) w_A < \left(1 - \dfrac{\alpha_{Bi'}}{2}\right) w_B \\ \in [0,1], & \text{if } \left(1 - \dfrac{\alpha_{Ai'}}{2}\right) w_A = \left(1 - \dfrac{\alpha_{Bi'}}{2}\right) w_B. \end{cases} \tag{1}$$

Now examine the first-stage game between the two firms. For firm j $(= A, B)$, it fills the vacancy if it receives at least one applicant, that is, if both workers do not apply to the other firm j' $(\neq j)$. Firm j's expected profit is

3 Our paper is also generally related to the optimal search literature surveyed by McMillan and Rothschild (1994). The difference is that firms in our model can direct workers' search by posting wages, and so wages have the ex ante allocative role.

$$F_j = (1 - \alpha_{j'1}\alpha_{j'2})(1 - w_j), \qquad j = A, B; j' \neq j.$$

An equilibrium consists of wages $W = (w_A, w_B)$ and workers' strategies $(\alpha_{j1}(W), \alpha_{j2}(W))_{j=A,B}$, with $\alpha_{j'i}(W) = 1 - \alpha_{ji}(W)$, such that (i) given wages and the other worker's strategy, each worker i's strategy $(\alpha_{ji}(W))_{j=A,B}$ maximizes his/her expected utility; and (ii) given $(\alpha_{j1}(W), \alpha_{j2}(W))_{j=A,B}$ and the other firm's wage, each firm posts a wage to maximize his/her expected profit. The important feature of the equilibrium is that each firm can choose a wage to influence workers' strategies and hence changes the expected number of matches he/she gets.

Let us examine the equilibrium where both workers use mixed strategies; that is, $\alpha_{ji}(W) \in (0,1)$ for $j = A, B$ and $i = 1,2$, leaving the discussion on pure strategies to the end of this subsection. The following lemma can be established.

LEMMA 1. *If $\alpha_{ji}(W) \in (0,1)$ for $j = A, B$ and $i = 1,2$, then $w_A > 0$, $w_B > 0$, $\alpha_{A1} = \alpha_{A2}$ and $\alpha_{B1} = \alpha_{B2}$.*

Proof First, let us show $w_A > 0$ and $w_B > 0$. Suppose, to the contrary, that at least one firm posts the bottom wage $w = 0$. Let this firm be firm B. If $w_A > 0$, both workers will choose firm A with probability one, since applying to firm B obtains zero surplus, while applying to firm A obtains an expected surplus no less than $w_A/2 > 0$. In this case, workers will not mix between the two firms, contradicting the assumption of mixed strategies. If $w_A = 0$, it is profitable for firm A to increase the wage to $w_A = \varepsilon$, where ε is a sufficiently small positive number. Since $w_B = 0$, the wage increase will induce both workers to apply to firm A with probability one, and so firm A's expected profit is $1 - \varepsilon$. The expected profit before the wage increase is $1 - \alpha_{A1}\alpha_{A2}$. By choosing $\varepsilon < \alpha_{A1}\alpha_{A2}$, which can be done, since $\alpha_{A1}, \alpha_{A2} > 0$, firm A increases his expected profit. A contradiction.

Now that $w_A > 0$ and $w_B > 0$, we show $\alpha_{A1} = \alpha_{A2}$. Substituting $\alpha_{Bi}(W) = 1 - \alpha_{Ai}(W)$ in (1) we get

$$\left(1 - \frac{\alpha_{A2}}{2}\right) w_A = \frac{1 + \alpha_{A2}}{2} w_B;$$

$$\left(1 - \frac{\alpha_{A1}}{2}\right) w_A = \frac{1 + \alpha_{A1}}{2} w_B.$$

Subtracting the two equations, we have $(\alpha_{A1} - \alpha_{A2})(w_A + w_B) = 0$. Since $w_A + w_B > 0$, $\alpha_{A1} = \alpha_{A2}$, and so $\alpha_{B1} = \alpha_{B2}$. QED

The above lemma shows that, if both workers mix between the two firms, then the two workers must use the same strategy and the two firms must post wages above the bottom wage. Denote $\alpha_A = \alpha_{A1} = \alpha_{A2}$. Then, $\alpha_{B1} = \alpha_{B2} = 1 - \alpha_A$ and (1) yields

$$\alpha_A(W) = \frac{2w_A - w_B}{w_A + w_B}. \tag{2}$$

Taking w_B as given, firm A solves

$$\max_{w_A} [1 - (1 - \alpha_A(W))^2](1 - w_A).$$

Firm B's maximization problem can be formulated similarly.

Let us denote a firm's expected profit by F, a worker's expected surplus by U, and the social welfare level by V. Social welfare is measured by giving the same weight to every agent; that is, $V = (F_A + F_B + U_1 + U_2)/4$. Then we can solve the firms' maximization problems and establish the following proposition.

PROPOSITION 1. *In the market with two workers and two firms, there is a unique mixed-strategy equilibrium. In this equilibrium agents' strategies and expected surpluses are given as follows:*

$$w_A = w_B = \tfrac{1}{2}, \ \alpha_{A1} = \alpha_{A2} = \alpha_{B1} = \alpha_{B2} = \tfrac{1}{2}$$

$$F_A = F_B = \tfrac{3}{8}; \quad U_1 = U_2 = \tfrac{3}{8}; \quad V = \tfrac{3}{8}. \tag{3}$$

This equilibrium has an important feature that workers use mixed strategies to apply for the jobs. The mixed strategies generate uncertainty. Each worker becomes unemployed and each job becomes unfilled with probability 1/4, although there are enough jobs for all workers at the aggregate level. Thus, the market does not realize its full potential: if the two workers were both employed, the social welfare level would be 1/2. The welfare loss in the equilibrium with mixed strategies is 1/8, which is 1/4 of potential output.

This welfare loss arises from agents' failure to coordinate. If agents can coordinate their decisions, they can achieve the efficient outcome with pure strategies. In particular, worker 1 can apply to firm A with probability 1, worker 2 can apply to firm B with probability 1, and each firm can post a wage $w = 0$. This is an equilibrium. No firm has any incentive to deviate from the wage $w = 0$, since each gets the highest expected profit from posting such a wage. No worker has incentive to deviate either, since a worker gets zero surplus everywhere and deviations do not make any gain.[4]

We do not believe that agents can easily coordinate their actions in a large market to play the pure strategies. Even for very small markets, experiments by Ochs (1990) have shown significant failure in coordination. For example, when there are only four locations and nine buyers with nine units of goods as the total supply, a buyer can fail to get a good with a probability as high as 2/9. Ochs has found that buyers mix among all locations, even when some firms have many more units of goods than do other firms and when prices vary. When all firms have the same stock

4 Using the mixed-strategy equilibrium as a punishment, one can support a host of other pure-strategy equilibria (see Burdett, Shi, and Wright 1996). Also, the welfare loss from the coordination failure can be eliminated by separating the market into two submarkets, each having one worker and one firm.

of goods, buyers buy from each firm with roughly the same probability, a result consistent with symmetric mixed strategies. To reflect this reality, we will restrict attention to the equilibrium where all workers in the market mix among all available firms and we will interpret the welfare loss in this equilibrium as the cost of the coordination failure.

The above example shows that the coordination failure does not necessarily make everyone worse off: Workers are better off without coordination, since the wage increases from 0 to 1/2 when workers switch from complete coordination to no coordination. It is not clear how general this result is. In the next section we extend the analysis to a large market and allow the number of workers to be different from the number of firms.

3. A large market

3.1. Equilibrium

Now consider an economy with N workers and M firms, where $N \geq 2$ and $M \geq 2$. Each firm wants to fill one vacancy and each worker wants one job. The numbers N and M are not necessarily equal to each other and the worker/job ratio is $r = N/M$. Another useful notation is $x \equiv 1/M$. In equilibrium x is also the probability with which a worker applies to each firm. Since $M \geq 2$ and $N \geq 2$, we have

$$x \leq \bar{x} \equiv \tfrac{1}{2} \min\{1, r\}. \tag{4}$$

The equilibrium wage is $w(x)$. To find the equilibrium wage, let us consider a single firm's deviation to a wage $w^d > 0$, while every other firm continues to post w. Observing the deviation and other wages, each worker applies to the deviator with probability α and applies to each of the non-deviators with probability $\hat{\alpha} = (1-\alpha)/(M-1)$. If a worker applies to the deviator, he/she will be chosen with the following probability:[5]

$$\sum_{t=0}^{N-1} \frac{1}{t+1} C_{N-1}^{t} (\alpha)^t (1-\alpha)^{N-1-t} = [1-(1-\alpha)^N]/(N\alpha).$$

5 To compute the sum, define

$$A(y) = \sum_{t=0}^{N-1} \frac{1}{t+1} C_{N-1}^{t} (y\alpha)^t (1-\alpha)^{N-1-t}.$$

Clearly, $A(0) = 0$ and the probability to be computed is $A(1)$. Since

$$\frac{d}{dy}[yA(y)] = \sum_{t=0}^{N-1} C_{N-1}^{t} (y\alpha)^t 1 - \alpha^{N-1-t} = (y\alpha + 1 - \alpha)^{N-1},$$

integration yields

$$A(1) = \int_0^1 (y\alpha + 1 - \alpha)^{N-1} dy = \frac{1-(1-\alpha)^N}{N\alpha}.$$

If a worker applies to a non-deviator, he gets the job with probability $[1-(1-\hat{\alpha})^N]/(N\hat{\alpha})$. For the worker to be indifferent between the deviator and non-deviators, the expected wage must be the same; that is,

$$\frac{1-(1-\alpha)^N}{N\alpha}\cdot w^d = \frac{1-\left(1-\dfrac{1-\alpha}{M-1}\right)^N}{N(1-\alpha)/(M-1)}\cdot w. \tag{5}$$

This defines a smooth function $\alpha = \alpha(w^d, w)$, where α is an increasing function of w^d. A marginal wage increase will not attract all workers: if workers applied to the deviator with probability one, each would be chosen with a very low probability.

Note that the right-hand side of (5) is an increasing function of α. Since α is an increasing function of w^d, the right-hand side of (5) is an increasing function of w^d. That is, a wage increase by the deviator raises the expected payoff to workers who apply to non-deviators. This is because the wage increase attracts more workers to the deviator, reduces the congestion of workers applying to the non-deviators, and so increases the probability with which each applicant to a non-deviator is selected. This is an indirect cost to the deviator, in addition to the higher wage, because the firm must match up with the increased workers' surplus from elsewhere.[6]

When each worker applies to the deviator with probability α, the deviator successfully hires a worker with probability $1-(1-\alpha)^N$. Taking other firms' wages w as given, the deviator chooses w^d to solve:

$$\max_{w^d}(1-w^d)[1-(1-\alpha)^N], \qquad \text{s.t. } \alpha = \alpha(w^d, w).$$

In equilibrium the deviation cannot be profitable and so $w^d = w(x)$ solves the above maximization problem, which in turn implies $\alpha = x$. Substituting $(w^d, \alpha) = (w, x)$ and $N = r/x$ into the first-order condition of the maximization problem, we obtain

$$w(x) = \left[1 + \frac{(1-x)^{-r/x}-1}{r} - \frac{1}{1-x}\right]^{-1}. \tag{6}$$

The expected number of matches per firm is $H(x) = 1-(1-x)^{r/x}$. Each firm's expected profit, each worker's expected surplus, and the social welfare level are functions of x:

$$F(x) = (1-w)[1-(1-\alpha)^N] = [1-w(x)][1-(1-x)^{r/x}] \tag{7}$$

$$U(x) = w\cdot\frac{1-(1-\alpha)^N}{N\alpha} = w(x)\cdot\frac{1-(1-x)^{r/x}}{r} \tag{8}$$

$$V(x) = \frac{M\cdot F(x) + N\cdot U(x)}{M+N} = \frac{1-(1-x)^{r/x}}{1+r}. \tag{9}$$

6 Montgomery (1991) assumes that a worker's expected payoff from the market is exogenous to each firm. This is true only when there are infinitely many agents on both sides of the market.

3.2. Features

We first examine the welfare cost of coordination, which is measured as a percentage of potential output. Potential output is realized when at least one side of the market is fully employed, which is $\min\{M, N\}$. Potential output per capita is $\min\{M, N\}/(M + N) = \min\{1, r\}/(1 + r)$. The welfare loss from the coordination failure is

$$L(x) \equiv 1 - \frac{V(x)}{\min\{1, r\}/(1 + r)} = 1 - \frac{1 - (1 - x)^{r/x}}{\min\{1, r\}}. \tag{10}$$

This is simply the percentage of failed matches. We have the following proposition.

PROPOSITION 2. *For any fixed x, the welfare loss increases with the worker/job ratio r if and only if $r < 1$. The welfare loss increases with the market size for any fixed r; that is, $L'(x) < 0$.*

This proposition states two ways in which the welfare loss from the coordination failure can increase. One is through the worker/job ratio, r. For any given number of jobs, the welfare loss is the largest when the number of workers is equal to the number of jobs. The other way is through the market size. For any given worker/job ratio, a larger market makes coordination more difficult and increases the welfare loss. These features are intuitive. In particular, when there are as many jobs as workers, it is most likely that both experience low utilization. In contrast, when one side is much shorter than the other side, the shorter side will be utilized with a large probability, and hence the loss from the coordination failure is small.

These features are not unique to the wage-posting setup but rather general to any model where the matching process is probabilistic and non-cooperative. Such a process generates a matching function that exhibits decreasing returns to scale, as in our model. In contrast, matching functions in standard search models of unemployment typically have constant returns to scale, for example, Mortensen (1982) and Pissarides (1990). With constant-returns-to-scale matching functions, the welfare loss from the coordination failure is maximized at $r = 1$, as in our model, but the loss is independent of the market size.

The most important feature of the wage-posting set-up is not the form of the matching function but rather the wage share. In contrast to an exogenous wage share in the standard search model, the wage share in our model endogenously responds to changes in r and x. To describe the wage responses, define

$$f(x) \equiv 2x - \frac{1}{g(x)} \ln g'(x), \tag{11}$$

where $g(x) = -x^{-1} \ln(1 - x)$. Appendix B provides a proof for the following proposition:

PROPOSITION 3. *The wage share, w, is a decreasing function of r for any market size. The wage share increases with the market size, that is,* $w'(x) < 0$*, if and only if* $x < f^{-1}(r)$*. In particular, for all* $x \in (0, \bar{x}]$, $w'(x) > 0$ *if* $r < \ln 2 \approx 0.693$ *and* $w'(x) < 0$ *if* $r \geq f(f(\frac{1}{2})/2) \approx 0.83$.

The wage share decreases as the number of workers increases relative to the number of jobs. This is intuitive, since competition among workers presses wage down. The wage response to the market size, however, is not obvious. When the market becomes larger, the wage share increases only when the worker/job ratio is high. To understand this result, consider a market with only one worker and two firms. In this market the wage rate is pushed up to one by Bertrand competition between the two firms. If the market size increases to two workers and four firms, wages can only be lower. More generally, when there are many more jobs than workers, workers have a strong market power that supports high wages. Increasing the market size allows each firm to have access to more workers. Although the larger market size also allows each worker to have access to more firms, such a benefit to workers is relatively small at the margin, since workers started with an already strong market power. In this case, wages decrease with the market size, even though the worker/job ratio does not change.

Since the worker/job ratio is held constant in the calculation of $w'(x)$, the wage response arises entirely from changes in the coordination cost. When the coordination cost increases with the market size, the two sides unevenly share the increased cost, with the 'longer' side of the market sharing less of it. The factor price of the 'longer' side thus increases with the coordination cost. However, the division between the 'long' side and the 'short' side is not at $r = 1$. Workers are on the longer side even when there are fewer workers than firms, as long as $r \geq 0.83$. This is because of the asymmetric treatment of workers and firms in our model – firms can set wages to exploit the market but workers can only respond to the wages. The asymmetry gives firms a relatively higher market power even when $r = 1$, which is reduced by the increase in the market size.

The wage response also implies that an increase in the coordination failure, brought about by an increase in the market size, does not always create Pareto inferior outcomes. When $r < \ln 2$, a marginal increase in the coordination failure makes workers worse off but may make firms better off. When $r > 0.83$, a marginal increase in the coordination failure makes firms worse off but may make workers better off. Nevertheless, the social welfare level always decreases with the coordination cost as a result of a greater matching difficulty. The following examples illustrate the responses of wages, workers' expected surpluses and firms' expected profits.

Example 2. $r = 1.5$ (Figure 1). As the coordination cost increases with the market size (i.e., as x decreases), wages increase, and so firms' expected profit decreases. The higher wages also make workers better off.

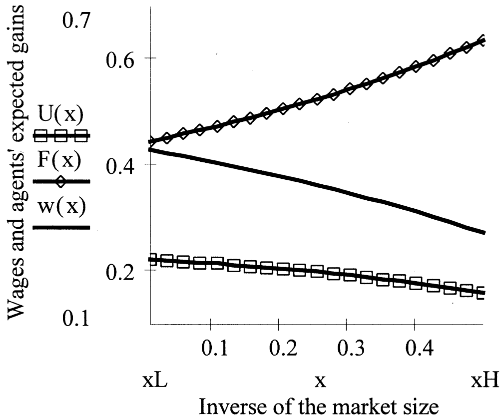

FIGURE 1 Effects of decreasing the market size: the case $r = 1.5$

Example 3. $r = 0.5$ (Figure 2). This case is opposite to example 2. As the coordination cost increases with the market size, wages fall, firms are better off, and workers are worse off.

Example 4. $r = 1$ (Figure 3). As in example 2, the increase in the coordination cost increases wages and makes firms worse off. In contrast to example 2, workers are better off only when x is large. When x is low, workers are worse off as the market size increases. (This non-monotonic pattern is almost indiscernible in figure 1.) Workers' expected surplus responds to the market size non-monotonically because the wage response diminishes with the market size. When the market is small, an increase in the market size increases wages sufficiently to make workers better off. When the market is large already, further increases in the market size bring very small additional increases in wages and so the matching difficulty dominates.

3.3. Discussions on modelling assumptions

Our model relies on two realistic assumptions: (i) agents cannot successfully coordinate, and (ii) some agents can use wages to direct other agents' search decisions. Assumption (i) distinguishes the market from a Walrasian one and makes the coordination problem an interesting issue; Assumption (ii) distinguishes the model from a typical search model of unemployment (e.g., Mortensen 1982) and enables us to examine how agents can mitigate the coordination problem by organizing matches non-cooperatively.

Maintaining these assumptions, we now argue that other auxiliary assumptions are not necessary for the qualitative results, although they kept our analysis tractable. First, we restricted each firm to have only one vacancy at a time. In appendix C we allow for multiple vacancies per firm and obtain similar results.

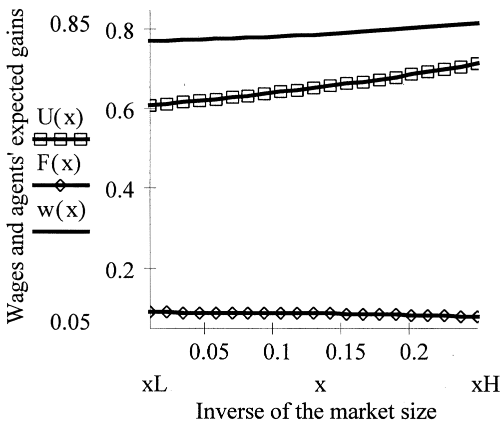

FIGURE 2 Effects of decreasing the market size: the case $r = 0.5$

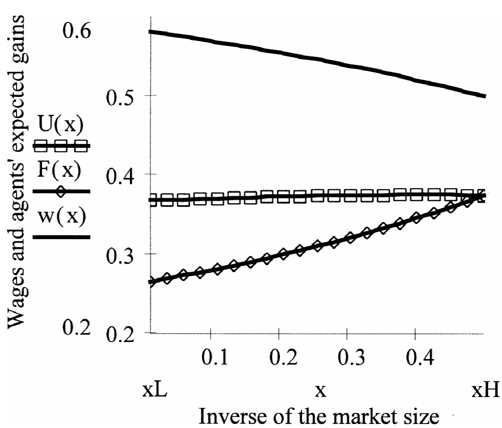

FIGURE 3 Effects of decreasing the market size: the case $r = 1$

Second, workers observe all posted wages before applying to one. Although this is unrealistic, for the essential results we need assume only that each worker observes two wages randomly drawn from the posted wages. This provides incentive for firms to use wages to entice workers, as in the above model, since any worker who observes a particular firm's wage also observes a different firm's wage (see Acemoglu and Shimer 1998 for this set-up). Third, each worker applies to only one job at a time. To some extent this is just semantics – we can always make a period short enough to support this assumption (see section 4, below, for an infinite horizon model). Even without this assumption our results will likely go through, since workers and firms must still make a trade-off between wages and the matching probability.[7]

Fourth, firms are the ones that post wages in the our model but, in some specific labour market, workers might post wages (see Julien, Kennes, and King (forthcoming)). In this alternative set-up, the differences are quantitative rather than qualitative. The equilibrium wage is given by (6), with w being replaced by $1 - w$, x by $1/N$, and r by $1/r$. Wages respond to the market size in the same qualitative way as in proposition 3. However, the critical levels of r are different. For all $x \in (0, \bar{x}]$, $w'(x) > 0$ if $r < 1/f(f(\frac{1}{2})/2) \approx 1.206$ and $w'(x) < 0$ if $r > 1/\ln 2 \approx 1.443$. A value of r larger than 1 is required for the wage to increase with the market size, because workers have higher market power than firms in this alternative set-up when $r = 1$.

Even the wage-posting framework seems unnecessary. What is necessary is an ex ante mechanism that can induce participants to balance the benefit of participation and the likelihood of being selected. For example, firms can commit to an auction mechanism that specifies a reserve wage and allows applicants to bid (see Julien, Kennes, and King (forthcoming)). In this alternative framework, the reserve wage serves very much the same role as the actual wage in our model, since it entices workers to participate in the firm's mechanism.

4. Market with an infinite horizon

So far agents play the game for only one period. The short horizon is restrictive because it does not allow unfilled vacancies and unemployed workers to get matched in future search. Consequently, the unemployment rate and the welfare loss from the coordination failure may be far off the realistic mark. To obtain reasonable numbers, we extend the labour market to an infinite horizon.

4.1. Wage determination

There are N number of workers and M_t number of workers/jobs in period t, where N is exogenous but M_t may be endogenous. The worker/job ratio in period t is $r_t = N/M_t$. Not all firms and workers participate in the matching process in all periods. Rather, in any period, only unemployed workers and vacancies participate.[8] Let u_t be the unemployment rate at the beginning of period t. The number of unemployed workers at the beginning of t is $u_t N$ and the number of vacancies is $M_t - N(1 - u_t)$. The labour market tightness is

$$T_t = \frac{u_t N}{M_t - N(1 - u_t)} = \frac{u_t}{r_t^{-1} - 1 + u_t}. \tag{12}$$

7 The exercise is complicated, since there is a non-degenerate wage distribution. Each firm uses mixed strategies to select the wage offer (see Lang 1991).

8 This is an equilibrium feature in the current environment, since the present value of staying in a match is higher for both the worker and the firm than that of dissolving the match.

If a worker and a firm are matched, they produce one unit of output each period until the match is separated. To emphasize matching rather than job separation, we assume that separation is exogenous. At the end of every period, a fraction $\sigma \in (0,1)$ of matched pairs separate and each pair has the same probability (σ) to separate. Once separated, a worker joints the unemployment pool at the beginning of the next period. A separated firm can decide whether to maintain the vacancy in the next period. The cost of maintaining a vacancy for one period is c. Both workers and firms discount future with a discount factor $\beta \in (0,1)$.

The infinite horizon creates two difficulties for characterizing an equilibrium with finite N and M. First, the number of matches is uncertain and hence the aggregate state of the future economy, such as the future unemployment rate, has a non-degenerate distribution. It is difficult to characterize this distribution. Second, each firm's action affects the aggregate state of the future economy. This makes it difficult to compute how a single firm's deviation affects the payoffs.

To avoid these difficulties, we take the numbers of workers and firms to infinity, with a finite ratio between the two. In the limit, the matching rate for each worker (and firm) and the future aggregate state are deterministic. Denote $x_t = 1/[M_t - N(1-u_t)]$. The matching rate for a vacancy in period t is $1-(1-x_t)^{u_t N} \to 1-e^{-T_t}$. Similarly, the matching rate for an unemployed worker in t is

$$\phi_t = \frac{1-(1-x_t)^{u_t N}}{x_t u_t N} \to \frac{1-e^{-T_t}}{T_t}.$$

Since there are $(1-\phi_t)u_t N$ unemployed workers after hiring in t, the number of workers employed in period t is $N-(1-\phi_t)u_t N$. With a separation rate σ, the unemployment rate at the beginning of period $t+1$ is

$$u_{t+1} = \frac{1}{N}\{(1-\phi_t)u_t N + \sigma[N-(1-\phi_t)u_t N]\} = \sigma + (1-\sigma)(1-\phi_t)u_t. \tag{13}$$

Now we can describe firms' wage decisions. Let the equilibrium wage in t be w_t and the wage path from t onward be $W_t = \{w_{t+\tau}\}_{\tau\geq 0}$. Consider a single firm that deviates to a wage path $W_t^d = \{w_{t+\tau}^d\}_{\tau\geq 0}$. This deviation affects the firm's tightness, denoted as T_t^d. If the firm gets a match, the worker is paid according to the new wage path until the match separates. If the firm does not get a match, it reverts to the equilibrium wage path in the next period, W_{t+1}. Let $J_{ft}(W_t^d)$ be the present value to the deviator when it successfully recruits a worker, and let $J_{vt}(W_t^d)$ be the present value to the deviator from maintaining a vacancy. For other recruiting firms, let $J_{vt}(W_t)$ be the present value of maintaining a vacancy. Then

$$J_{ft}(W_t^d) = 1 - w_t^d + \sigma\beta J_{vt+1}(W_{t+1}) + (1-\sigma)\beta J_{ft+1}(W_{t+1}^d) \tag{14}$$

$$J_{vt+1}(W_t^d) = -c + (1-e^{-T_t^d})J_{ft}(W_t^d) + e^{-T_t^d}\beta J_{vt+1}(W_{t+1}). \tag{15}$$

The value function $J_{vt}(W_t)$ is given by (15), with W_t replacing W_t^d and T_t replacing T_t^d.

Let us explain these equations. When the deviator successfully hires a worker, the firm gets a surplus $1 - w_t^d$ in period t. At the end of period t, the match separates with probability σ, in which case the firm reverts to the equilibrium wage path W_{t+1}, and so the discounted future value is $\beta J_{vt+1}(W_{t+1})$. If the match survives, the firm continues to pay wages according to the path W_{t+1}^d, and so the discounted future value is $\beta J_{ft+1}(W_{t+1}^d)$. The right-hand side of (14) sums up the firm's current surplus and expected future surpluses. Equation (15) can be explained similarly. After paying a cost c to post a vacancy, the firm gets a match with probability $1 - e^{-T_t^d}$, in which case the expected surplus is $J_{ft}(W_t^d)$. When the vacancy is not filled, the discounted future value is $\beta J_{vt+1}(W_{t+1})$.

The value functions for workers can be obtained similarly. Let $U_{et}(W_t^d)$ be the present value to a worker who is hired by the deviator and $U_{ut}(W_t^d)$ be the present value to a worker who applies to the deviator. If a worker applies to other firms, the expected value is $U_{ut}(W_t)$. Then

$$U_{et}(W_t^d) = w_t^d + \sigma\beta U_{ut+1}(W_{t+1}) + (1-\sigma)\beta U_{et+1}(W_{t+1}^d) \tag{16}$$

$$U_{ut}(W_t^d) = \phi(T_t^d)U_{et}(W_t^d) + (1-\phi(T_t^d))\beta U_{et+1}(W_{t+1}^d), \tag{17}$$

where $\phi(T_t^d) = (1 - e^{-T_t^d})/T_t^d$. The value function $U_{ut}(W_t)$ is given by (17), with W_t replacing W_t^d and T_t replacing T_t^d.

The best deviation is W_t^d, which solves the following problem:

$$\max(1 - e^{-T_t^d})[J_{ft}(W_t^d) - \beta J_{vt+1}(W_{t+1})],$$

subject to

$$\phi(T_t^d)[U_{et}(W_t^d) - \beta U_{ut+1}(W_{t+1})] \geq \text{ES}_t.$$

The objective function is the deviator's expected surplus, where the discounted value of leaving the vacancy unfilled is $\beta J_{vt+1}(W_{t+1})$. The constraint requires that an applicant to the deviator gets an expected surplus that is as large as he/she can get elsewhere, ES_t. In the limit with $N, M \to \infty$, ES_t, $J_{vt}(W_t)$, and $U_{ut}(W_t)$ all are unaffected by the firm's deviation.

In equilibrium the deviation cannot be profitable, and so $W_t^d = W_t$ solves the above problem. Setting $W_t^d = W_t$ and $T_t^d = T_t$ in the first-order condition of the above problem, we have

$$U_{et}(W_t) - \beta U_{ut+1}(W_{t+1}) = \frac{T_t}{e^{T_t} - 1 - T_t}[J_{ft}(W_t) - \beta J_{vt+1}(W_{t+1})]. \tag{18}$$

That is, the worker's surplus is a share $T_t/(e^{T_t} - 1)$ of the total match surplus. Intuitively, this share is lower if there are more unemployed workers per vacancy.

Together with the value functions for (J_f, J_v, U_e, U_u), the above equation determines the wage sequence in equilibrium.

It is difficult to decipher from (18) the wage share in any particular period. To be specific, we focus on the steady state. In the steady state, the value functions and the wage are as follows:

$$J_f = \frac{(1-w)(1-\beta e^{-T}) - \sigma\beta c}{(1-\beta)[1-\beta(1-\sigma)e^{-T}]}$$

$$J_v = \frac{(1-w)(1-e^{-T}) - [1-\beta(1-\sigma)]c}{(1-\beta)[1-\beta(1-\sigma)e^{-T}]}$$

$$U_e = \frac{w}{1-\beta} \cdot \frac{1-\beta[1-\phi(T)]}{1-\beta(1-\sigma)[1-\phi(T)]}$$

$$U_u = \frac{w}{1-\beta} \cdot \frac{\phi(T)}{1-\beta(1-\sigma)[1-\phi(T)]}$$

$$w = [1+\beta(1-\sigma)c]\, \frac{1-\beta(1-\sigma)[1-\phi(T)]}{e^{T}\phi(T) - \beta(1-\sigma)(1-e^{-T})}.$$

We can calculate these value functions and the wage once we determine the market tightness in the steady state. This we will do for both the case where the number of firms is fixed and the case where the number of firms is endogenously determined.

4.2. Equilibrium without entry

In this subsection, we fix the number of firms and conduct a comparative statistic analysis with respect to the worker/job ratio. Since the number of workers is always fixed, the worker/job ratio r is exogenous. The market tightness in the steady state is $T(u) = u/(r^{-1} - 1 + u)$ by (12). Equation (13) implies that the unemployment rate in the steady state solves the following equation:

$$u = \frac{\sigma}{\sigma + (1-\sigma)\phi(T(u))}. \tag{19}$$

After solving for the unemployment rate in the steady state, we can recover the values of other variables in the steady state.

To calculate the welfare loss from the coordination failure, we assume that the planner must incur the vacancy cost c in the first period in order to match workers with firms. This assumption ensures that the welfare loss in equilibrium comes from the coordination failure rather than from the difference in the first-period recruiting cost. In subsequent periods, the planner does not incur any further vacancy cost, because the shorter side of the market is all matched in the first period in the efficient allocation. The present value of potential output is

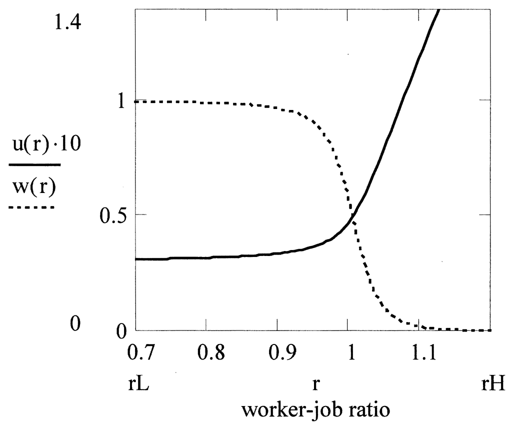

FIGURE 4 Dependence of the unemployment rate and the wage share on the worker/job ratio

$(1-\beta)^{-1}\min\{M,N\} - Mc$. The present value of output in the equilibrium is the weighted average of agents' value functions. Thus, the welfare loss is the following percentage of potential output:

$$L = 1 - \frac{uU_u + (1-u)(U_e + J_f) + (r^{-1} - 1 + u)J_v}{(1-\beta)^{-1}\min\{1, r^{-1}\} - cr^{-1}}.$$

To know more about the features of the steady state, let us parameterize the model. We interpret a period as a month and set the discount factor to $\beta = 1.04^{-1/12}$ to produce an annual real interest rate 4 per cent. The monthly job separation rate is $\sigma = 0.03$. This is realistic for the United States, since the quarterly job separation rate is 6 per cent in the manufacturing sector (see Davis and Haltiwanger 1998), which is lower than that in non-manufacturing sectors. We set $c = 0$ to obtain a lower bound on the welfare loss. With these parameters, in Figures 4 through 6 we depict the comparative statics with respect to changes in the overall worker/job ratio, r.

There are several noteworthy features. First, the unemployment rate is insensitive to changes in the worker/job ratio when $r < 1$, but when r exceeds 1, the unemployment rate responds very sensitively to further increases in r (in figure 4 the unemployment rate is multiplied by 10). For example, when r increases from 1 to 1.1, the unemployment rate is more than doubled. This is because the unemployment rate is directly related to the market tightness rather than to the overall worker/job ratio (see (19)), and the market tightness does not change proportionally with the overall worker/job ratio. As shown in figure 5, the market tightness increases rapidly with r when $r > 1$, since most excess workers end up unemployed in this case.

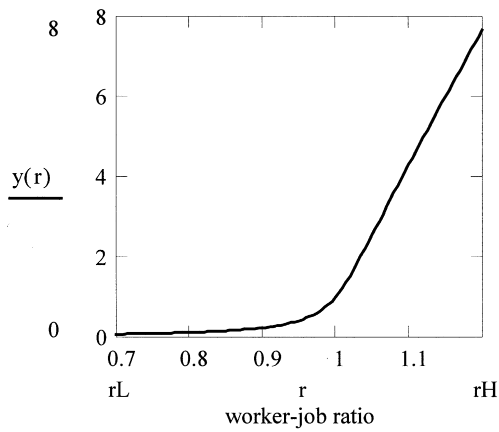

FIGURE 5 Dependence of the market tightness on the worker/job ratio

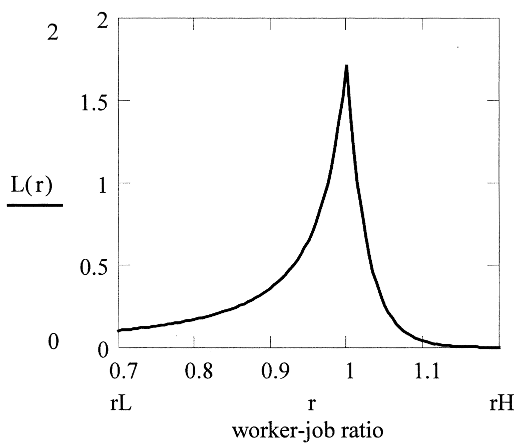

FIGURE 6 Dependence of the welfare loss on the worker/job ratio

Second, the wage approximates the Walrasian outcome well (figure 4). When one side is sufficiently longer than the other side, the corresponding price approaches zero. For example, the wage remains close to 1 for all $r < 0.9$ and remains close to 0 for all $r > 1.1$. Moreover, in the neighbourhood of $r = 1$ the wage responds to changes in r in large magnitudes. For example, when r increases from 0.95 to 1.05, the wage falls from 0.908 to 0.104. In contrast to the Walrasian outcome, the wage is a continuous function of r when r crosses the level 1.

Third, the welfare loss from the coordination failure increases with r when $r < 1$ and decreases with r when $r > 1$. The maximum is achieved at $r = 1$ and it is

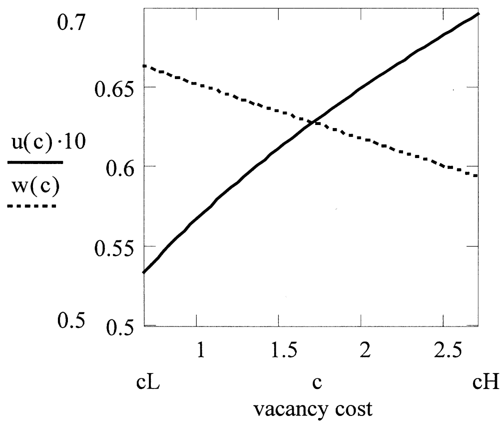

FIGURE 7 Dependence of the unemployment rate and the wage share on the vacancy cost

about 1.8 per cent of potential output (figure 6). This is a significant number, given that we have abstracted from all other frictions in the economy and have set $c = 0$.

4.3. Equilibrium with firms' entry

Now firms can enter the market by paying an entry (set-up) cost $K > 0$. The net gain from entry must be zero, and so $J_v = K$. This condition determines the overall worker/job ratio, r. Other steady-state variables can be determined in the same way as in the last subsection.

To calculate the welfare loss, we assume that the planner must incur both the vacancy cost c and the set-up cost K in the first period for every firm. The present value of potential output is $(1-\beta)^{-1}\min\{M,N\} - M(c+K)$. The present value of output in equilibrium is the weighted average of the value added by each type of agents. Since $J_v = K$, the value added by a vacancy is zero. The welfare loss is the following percentage of potential output:

$$L = 1 - \frac{uU_u + (1-u)(U_e + J_f - K)}{(1-\beta)^{-1}\min\{1, r^{-1}\} - (c+K)r^{-1}}.$$

As in the last subsection, we interpret a period as a month and use the parameter values $\beta = 1.04^{-1/12}$ and $\sigma = 0.03$. To identify c and K, we set the unemployment rate to 6 per cent and the wage/output ratio to 0.64. Together with steady-state formulas for u and w, these two conditions identify $c = c_0 = 1.354$ and $K = K_0 = 92.7$. Then we examine the steady-state responses to changes of c in the range $[0.5c_0, 2c_0]$ (the responses to changes in K are similar and hence omitted). Figures 7 through 9 depict the comparative statistics.

The qualitative results are predictable. As the vacancy cost rises, firms have less desire to maintain vacancies. The market tightness increases, the unemployment

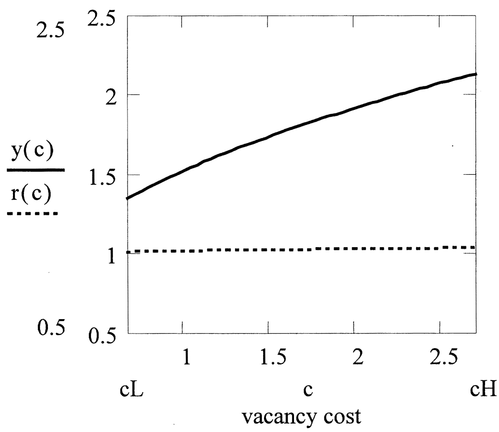

FIGURE 8 Dependence of the market tightness and the worker/job ratio on the vacancy cost

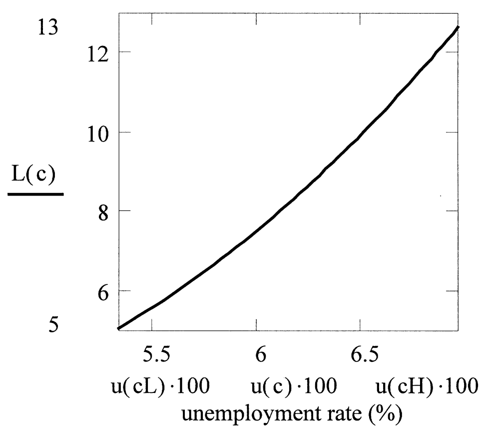

FIGURE 9 Dependence of the welfare loss on the vacancy cost through the unemployment rate

rate increases, the wage share falls, and the welfare loss increases (in figure 7 the unemployment rate is multiplied by 10).

The quantitative results are interesting. First, the overall worker/job ratio barely changes with c, but the unemployment/vacancy ratio (i.e., the market tightness) increases with c sufficiently to cause large increases in the unemployment rate. This indicates that the burden of a higher vacancy cost is largely shifted to workers. It also indicates that the flows, rather than the stocks, of workers and jobs are important for the labour market equilibrium. Second, the welfare cost is significantly higher than in the last subsection without entry because of the vacancy cost and the set-up cost. When $c = c_0$, the welfare loss from the coordination failure is about 7.5 per cent of potential output. The loss increases with c. To see the response of the welfare loss in a different way, we picture it against the unemployment rate in figure 9. An increase in the unemployment rate, caused by an increase in the vacancy cost, generates a large increase in the welfare loss. For example, when the unemployment rate increases from 6 to 6.5 per cent, the loss increases from 7.5 to 9.3 per cent of potential output.

5. Conclusion

We have analysed the coordination problem in the labour market. In the absence of a Walrasian auctioneer, firms try to organize matches by setting wages. Each firm posts a wage to maximize the expected profit, anticipating how the wage affects the expected number of applicants. Each worker maximizes the expected wage by making a trade-off between a wage and the probability of obtaining it. In equilibrium workers apply to firms with mixed strategies. This coordination failure generates unemployment that persists even in the steady state. The welfare loss from the coordination failure is as high as 7.5 per cent of potential output.

The welfare loss varies from market to market and is not evenly shared by the two sides of the market. For any given market size, the welfare loss from the coordination failure is the highest when the numbers of firms and numbers of jobs are equal. For any worker/job ratio, a larger market produces a higher welfare loss from the coordination failure. It is surprising that such a higher loss does not necessarily make everyone worse off. The longer side of the market may benefit as their factor price increases.

In order to focus on the coordination failure, we have abstracted from many realistic features of the market. For example, workers (jobs) are homogeneous in our model, which precludes any discussion on how the coordination failure interacts with match qualities. For this reason, we caution readers to interpret the effects of the market size carefully. Although a larger market exacerbates the coordination problem, it also improves match qualities in reality. This benefit must outweigh the increased coordination loss in order to be socially desirable (For models of directed search with heterogeneous firms and workers, see Shi 1997, 1998).

Appendix A

Proof of Proposition 2

First, let us fix $x \in (0,1)$ and examine the dependence of L on r. For $r > 1$, $L = (1 - x)^{r/x}$, which is clearly a decreasing function of r. For $r < 1$, $L = 1 - [1 - (1 - x)^{r/x}]/r$. The derivative of this function to r has the same sign as that of $(a \ln a + 1 - a)$, where $a = (1 - x)^{r/x} \in (0,1)$. Since the function $(a \ln a + 1 - a)$ is a decreasing function for all $a \in (0,1)$ and has a value 0 when $a = 1$, it is positive for all $a \in (0,1)$. Thus L increases with r when $r < 1$.

Now we show $L'(x) < 0$ for any fixed r. For this, it suffices to show $H'(x) > 0$. Define

$$g(x) = -\frac{1}{x}\ln(1-x). \tag{A1}$$

Then $H(x) = 1 - e^{-rg(x)}$, and so $H'(x) > 0$ is equivalent to $g'(x) > 0$. Compute

$$g'(x) = \frac{1}{x}\left(\frac{1}{1-x} - g(x)\right) = \frac{1}{x^2}\left[\frac{x}{1-x} + \ln(1-x)\right].$$

The function $[x/(1-x)] + \ln(1-x)$ has a value 0 at $x = 0$, a positive derivative for all $x \in (0,1)$ and so it is positive for all $x > 0$, yielding $g'(x) > 0$. QED

Proof of proposition 3
To show that w is a decreasing function of r for any fixed $x \in (0, \bar{x}]$, compute the derivative of w with respect to r. The sign of the derivative is the same as that of $(a - 1 - a\ln a)$, where $a = (1-x)^{-r/x} > 1$. Since the function $(a - 1 - a\ln a)$ is a decreasing function for all $a > 1$ and has a value 0 when $a = 1$, it is negative for all $a > 1$.

To find the sign of $w'(x)$, differentiate (6) with respect to x. Then we can show that $w'(x) > 0$ if and only if $r < f(x)$. The function $f(x)$ is an increasing function for all $x \in (0, \bar{x}]$, as shown later. Thus, $w'(x) > 0$ if and only if $x > f^{-1}(r)$. With the L'Hopital's rule, we can compute $g(0) = 1$, $g'(0) = 1/2$, and so $f(0) = \ln 2$. Thus, if $r < \ln 2$, then $r < f(x)$ and $w'(x) > 0$ for all $x \in (0, \bar{x}]$. To show that $w'(x) < 0$ for all $x \in (0, \bar{x}]$ if $r \geq f(f(\frac{1}{2})/2)$, note, first, that $\bar{x} \leq 1/2$ and $f(f(\frac{1}{2})/2) < f(\frac{1}{2})$. If $r \geq f(\frac{1}{2})$, then $r \geq f(\bar{x})$, and so $w'(x) < 0$ for all $x \leq \bar{x}$. If $r < f(\frac{1}{2})$, then $\bar{x} \leq r/2 < f(\frac{1}{2})/2$. In this case $r \geq f(f(\frac{1}{2})/2)$ implies $r \geq f(\bar{x})$ and again $w'(x) < 0$ for all $x \leq \bar{x}$.

Let us now show $f'(x) > 0$ for $x \in (0, \bar{x}]$. Calculate $f'(x) = g'(x)f1(x)/[g(x)]^2$, where

$$f1(x) = \frac{2(g(x))^2}{g'(x)} + \ln(g'(x)) - \frac{g(x)g''(x)}{(g'(x))^2}.$$

Since $g(x) > 0$ and $g'(x) > 0$ for all $x \in (0,1)$ (see the proof of proposition 2), $f'(x) > 0$ iff $f1(x) > 0$. It can be computed that $f1(\frac{1}{2}) > 0$. Since $\bar{x} \leq 1/2$, $f1(x) > 0$ for all $x \in (0, \bar{x}]$ if $f1'(x) < 0$ in this range. Compute

$$g'(x) = \frac{1}{x}\left(\frac{1}{1-x} - g(x)\right); \quad g''(x) = \frac{1}{x^2}\left[\frac{3x-2}{(1-x)^2} + 2g(x)\right]$$

$$g'''(x) = \frac{1}{x^3}\left[\frac{11x^2 - 15x + 6}{(1-x)^3} - 6g(x)\right].$$

Then

$$f1'(x) = \frac{g(x)}{x^4(1-x)^2[g'(x)]^3}\left[\frac{2-3x}{1-x} - (4-x)g(x) + 2[g(x)]^2\right].$$

Thus, $f1'(x) < 0$ iff $g(x) \in (g_1(x), g_2(x))$, where

$$g_1(x) = 1 - \frac{x}{4}\left[1 + \sqrt{\frac{9-x}{1-x}}\right], \quad g_2(x) = 1 - \frac{x}{4}\left[1 - \sqrt{\frac{9-x}{1-x}}\right].$$

It is easy to show that the function $[xg(x) - xg_1(x)]$ is an increasing function for $x \in (0,1)$ and has a value 0 at $x = 0$. Thus $g(x) > g_1(x)$. To show $g(x) < g_2(x)$, consider the function $f2(x) \equiv xg_2(x) - xg(x)$. Then $f2(0) = 0$ and

$$f2'(x) \sim (1-x)(9-x) + 2x^2 - (3-x)\sqrt{(1-x)(9-x)}.$$

It can be verified that the expression on the right-hand side is negative for all $0 < x \leq 1/2$. Thus $f2'(x) < 0$, and so $f2(x) \geq f2(\frac{1}{2}) > 0$ for all $0 < x \leq 1/2$. This shows $g(x) < g_2(x)$, and so $f1(x) > 0$ for all $0 < x \leq 1/2$, yielding $f'(x) > 0$. QED

Appendix C

The case when each firm has multiple vacancies

Let us extend the model in section 3 by allowing each firm to have $b \geq 2$ vacancies. Restrict $b < N$, so that a single firm cannot satisfy the entire market. In equilibrium all firms post a wage $w \in (0,1)$, and each worker applies to each firm with probability $1/M$. If a firm gets b or fewer workers, each worker gets a job with probability one; if the firm gets $t > b$ applicants, only b applicants will be chosen randomly, and so each applicant will be chosen with probability b/t.

To determine w, consider a single firm's deviation to a wage $w^d \in (0,1)$. Observing the deviation, each worker applies to the deviator with probability α and applies to each of the non-deviators with probability $\hat{\alpha} = (1-\alpha)/(M-1)$. If a worker applies to the deviator, the probability that he gets a job is

$$q(\alpha) \equiv \sum_{t=0}^{b-1} C_{N-1}^t \alpha^t (1-\alpha)^{N-1-t} + \sum_{t=b}^{N-1} \frac{b}{t+1} C_{N-1}^t \alpha^t (1-\alpha)^{N-1-t}.$$

The first summation deals with cases where the firm has at most $(b-1)$ other applicants; the second summation deals with cases where the firm has at least b other applicants. The probability $q(\alpha)$ can be rewritten as

$$q(\alpha) = b \cdot \frac{1-(1-\alpha)^N}{N\alpha} - \sum_{t=0}^{b-2}\left(\frac{b}{t+1} - 1\right) C_{N-1}^t \alpha^t (1-\alpha)^{N-1-t}.$$

Similarly, when a worker applies to a non-deviator, the probability that he gets a job is $q(\hat{\alpha})$. For the worker to be indifferent between the two firms, we must have

$$w^d \cdot q(\alpha) = w \cdot q\left(\frac{1-\alpha}{M-1}\right). \tag{C1}$$

Again, this defines a relationship $\alpha = \alpha(w^d, w)$.

The deviator chooses w^d to maximize the expected profit, taking w as given and facing the constraint $\alpha = \alpha(w^d, w)$. The deviator's expected profit is

$$(1-w^d)\left[\sum_{t=1}^{b-1} tC_N^t \alpha^t (1-\alpha)^{N-t} + b \cdot \sum_{j=b}^{N} C_N^t \alpha^t (1-\alpha)^{N-t}\right].$$

The expression in $[\cdot]$ can be shown to be $N\alpha q(\alpha)$. If we use (C1) to eliminate w^d, the deviator's expected profit is

$$N\alpha\left[q(\alpha) - wq\left(\frac{1-\alpha}{M-1}\right)\right].$$

Deriving the first-order condition for α and setting $\alpha = 1/M$, we obtain

$$w = \frac{q\left(\frac{1}{M}\right) - \frac{\delta}{M}}{q\left(\frac{1}{M}\right) + \frac{\delta}{M(M-1)}},$$

where $\delta = -q'(\alpha)|_{\alpha=1/M}$.

Let H now be the probability that a firm successfully fills each vacancy and F be a firm's expected profit per vacancy. Define (U, V) accordingly. When the market expands from $(M, N) = (4, 20)$ to $(20, 100)$, the equilibrium has the following changes:

$$\Delta w \approx 0.047,\ \Delta H \approx -0.019,\ \Delta V \approx -0.016,\ \Delta F \approx -0.049,\ \Delta U \approx 0.030.$$

In this example, market integration increases wages, increases workers' surplus, but reduces firms' surplus and the social welfare level. Workers can be worse off if $b = 4$. For example, if the market expands from $(M, N) = (4, 16)$ to $(20, 100)$, the equilibrium changes as follows:

$$\Delta w \approx 0.013,\ \Delta H \approx -0.017,\ \Delta V \approx -0.017,\ \Delta F \approx -0.015,\ \Delta U \approx -0.003.$$

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