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Screening, Bidding, and the Loan Market Tightness ⋆

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Abstract. Bank loans are more available and cheaper for new and small businesses in the U.S. in concentrated banking areas than in competitive banking areas. We explain this anomaly by analyzing banks' decisions to screen projects and their competition in loan provisions. It is shown that, by exacerbating the winner's curse, an increase in the number of banks can reduce banks' screening probability by so much that the number of banks that actively compete in loan provisions falls and the expected loan rate rises. This is the case when the screening cost is low, which induces all active bidders to be informed. The opposite outcome occurs when the screening cost is high, in which case there are sufficiently many uninformed banks in bidding to attenuate the winner's curse. We also examine the social optimum.

Keywords: loans, screening, bidding, informational externality.

JEL classifications: G21, D44, L15

1. Introduction

In an influential paper Petersen and Rajan (1995) have documented an anomaly in U.S. loan markets: Bank loans to new and small businesses are more available and cheaper in areas with concentrated banks than in areas with competitive banks.[1]

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[1] The measure of the loan market concentration in Petersen and Rajan (1995) is the Herfindahl index, which is the sum of the banks' shares (squared) of deposits in the area. The loan availability is

We explain this anomaly by constructing a theory of loan market competition. The analysis is important not only for theory but also for policy making. Because small businesses are often viewed as the backbone of economic growth and because bank loans account for a dominant fraction of small business financing, the analysis can help design policies to enhance small business financing and encourage growth.

One explanation for the anomaly, provided by Petersen and Rajan (1995), is that banks in concentrated markets use their monopoly power to extract future surpluses from the firm to subsidize the higher loan availability and the lower loan rate at the beginning of the relationship. Although this important insight has empirical support, the cross-subsidization does not work well between young and not-so-young (but not-old) firms. If a bank in a concentrated banking area offers a higher loan availability and a lower loan rate to young firms with the intention to exchange for future surpluses, it has even greater incentive to do so to firms that are not so young and yet not old, because such firms possess greater future surpluses than young firms whose survivorships are highly uncertain. That is, the monopoly-power story by itself suggests that the difference in the loan availability between a concentrated market and a competitive market should be more pronounced for the not-so-young firms than for young firms. This is at odds with the evidence in Petersen and Rajan (1995, p. 424).[2]

To construct an alternative theory for why the observed differences between differently concentrated markets can be consistent with rational choices, we abstract from any intertemporal trade-off by restricting attention to a one-period financing problem. The alternative theory stresses the informational problem that banks encounter in screening a project and consequently bidding on loan provision. In particular, there is one entrepreneur with a project whose quality is unknown to the banks. Each bank decides the probability with which to screen the project at a cost. The screening activity yields an informative but inaccurate signal about the true quality of the project. Without knowing other banks' signals, each bank (including those that did not screen) decides whether to submit a sealed bid and how much to bid on the loan rate. The entrepreneur takes the lowest bid and carries out the project to reveal the outcome.

The loan availability can decrease with the number of banks. The culprit for this "perverse" result is a negative informational externality. As a form of the *winner's curse*, this externality arises because a bank's chance of winning the bidding game is greater when the project's quality is bad than when the quality is good; thus winning increases the expectation that the quality is bad and reduces the winning bank's expected profit. This externality is stronger when there are more banks, because winning against more banks that could have received a good signal (but

the frequency with which a firm utilizes the discount offered by trading partners on early repayments to trade credits, which measures the opportunity cost of not having (or having not enough) bank loans.

[2] Petersen and Rajan (1995) avoided this unpleasant implication of their model by restricting attention to a two-period model, which does not distinguish a not-so-young firm from an old firm.

did not) greatly increases the expectation that the project is bad. When the number of banks increases, each bank faces not only more potential competitors but also a worse negative informational externality. To cover the screening cost, each bank reduces the screening probability greatly and this negative intensive adjustment can outweigh the extensive increase in the number of banks, making loans less available and the average loan rate higher.

For the above perverse result to occur, the screening cost must be low. A low screening cost induces each bank to screen with a high probability, in which case there are sufficiently many informed banks in bidding and an increase in the number of banks exacerbates the winner's curse greatly. The equilibrium screening probability is inefficiently low. In contrast, when the screening cost is very high, most active bidders are uninformed. Since winning against uninformed opponents does not yield any new information, the negative informational externality to an informed winner is weak and increases only slightly with the number of banks; the extensive effect of the increase in the number of banks dominates the intensive effect and so the loan availability rises. In this case the equilibrium can be efficient.

1.1. CONTRIBUTIONS TO THE LITERATURE

The main contribution of this paper is to formally establish the result that the negative informational externality in loan competition can be strong enough to generate a negative response of the loan availability to an increase in the number of banks. This explanation for the observed loan market behavior is complementary to the one provided by Petersen and Rajan (1995). The emphasis on the fact that loan market competition reveals private information, albeit imperfectly, separates this paper from standard screening models, such as Stiglitz and Weiss (1981) and Wang and Williamson (1998), which have ignored the informational externality.

We are not the first to articulate the importance of the winner's curse in the loan market. Broecker (1990) has examined how an increase in the number of banks affects each bank's willingness to participate in loan provision when banks exogenously receive signals about the project's quality. Riordan (1993) uses a setup very similar to Broecker's but models the participation choice differently.[3] Thakor (1996) endogenizes banks' screening decisions and examines the effect of capital requirements on the loan market. In contrast to our analysis, these papers have not formally established that an increase in the number of banks can reduce the loan availability. What they have established is that an increase in the number of banks reduces each bank's probability to provide a loan. This intensive adjustment does not necessarily translate into a reduction in the loan availability at the aggregate level since the number of banks also increases extensively. For the intensive effect

[3] Shaffer (1998) makes a further assumption that the loan rate is fixed and examines how loan provision decisions affect the informational content in subsequent provisions to firms that have not been funded. Rajan (1992) also applies an auction model to loan provisions, but he focuses on a firm's choice between borrowing from the incumbent lender and the arm's length debt.

to dominate the extensive effect, the presence of more banks must sufficiently increase the negative informational externality through the winner's curse. We show formally that this can occur when the screening cost is low.

Another contribution of our paper is to allow uninformed banks to bid, while the other papers set the signal acquisition as a precondition for bidding. Such a requirement is unrealistic for the loan market. First, it is difficult to verify the amount of effort a bank puts into screening. Since uninformed banks are generally ones that have spent less effort in screening and obtained less precise signals, it is difficult to bar these banks from bidding. Second, in a dynamic extension of the environment studied here, uninformed banks can learn from the market and partially free-ride on informed banks' actions. Having not acquired a signal costly does not mean that a bank does not want to bid. Even in the one-period environment here, we show that uninformed banks do want to bid when the screening cost is high.

We also generate novel results by allowing uninformed banks to bid. First, the relationship between the loan availability and the number of banks depends on the mix of informed and uninformed banks in the market. The relationship is positive when uninformed banks bid as well and so the pattern of the loan availability observed by Petersen and Rajan (1995) might be specific to their data set. Second, since the differential in the loan availability between areas depends on the screening cost, it may be cyclical and industry-specific (see Section 6).

Finally, our analysis is related to the auction literature surveyed by McAfee and McMillan (1987). The informational structure here is similar to that in Wang (1991). In contrast to Wang's work, where bidders' information is exogenous and bidders always participate in bidding, we examine banks' decisions on whether to obtain a signal and whether to participate in bidding. Some other common-value auction models, such as Harstad (1990) and Levin and Smith (1994), also examine agents' choices of obtaining a costly signal. They allow only those who receive a signal to bid, use a continuous signal space and do not focus on the loan market.

2. The Environment

In this section we describe the model, delaying the discussions on modelling assumptions to Section 7. There are $n \geq 2$ banks, indexed by i, and one entrepreneur. Both the banks and the entrepreneur are risk-neutral, living for one period. The entrepreneur has one project to be financed with an investment normalized to one unit of goods but does not have any internal funds and must resort to outside financing from the banks. If financed, the project yields output at the end of period that can be consumed immediately. Output is publicly observable and depends on the quality of the project. The quality, denoted q, is either good ($q = g$) or bad ($q = b$). Output is $a(q)y$, where $a(q) = 1$ if $q = g$ and 0 if $q = b$.

Banks do not know the project's true quality but have the following common prior:

$$q = \begin{cases} g, & \text{with prob. } \alpha \in (0, 1); \\ b, & \text{with prob. } 1 - \alpha. \end{cases}$$

The entrepreneur may or may not know the true quality of the project. This is not important in the current setting. Since the entrepreneur has no internal funds, under limited liability the entrepreneur always likes to go ahead with the project if financing is obtained. The entrepreneur applies to all banks in the market separately for the fund. There is no application fee.[4]

Banks can choose whether to screen the project. Denote the screening probability by p. It costs $c > 0$ to screen, which yields a signal about the quality of the project. The signal, denoted s, can be either g (good) or b (bad). Conditional on the true quality of the project, different screening banks' signals are independent draws from the same distribution:

$$s|q = \begin{cases} q, & \text{with prob. } \gamma \in (1/2, 1); \\ q' \neq q, & \text{with prob. } 1 - \gamma. \end{cases}$$

That is, the signal is right with probability γ and wrong with probability $1 - \gamma$. The information is thus not accurate but, since $\gamma > 1/2$, the signal is more likely to be right than wrong. The conditional distribution of the signal has the monotone likelihood ratio property or, according to Milgrom and Weber (1982), the signal and the true quality are affiliated.[5]

Without knowing other banks' screening decisions or outcomes, each bank decides whether to participate in loan competition. If a bank participates, it submits a sealed bid to the entrepreneur at no cost. A bid specifies the percentage of output, denoted r, that the entrepreneur gives to the bank if the project is successful. This is equivalent to a loan rate $(ry - 1)$ when the project is successful. For brevity, we often refer to r as the loan rate. Because of limited liability, the entrepreneur gives the bank nothing if the project is not successful. Similarly, a bank cannot ask for more than what the project yields and so a bid is *feasible* if and only if $r \leq 1$. A winning bid r generates a profit $a(q)yr - 1$ to the bank and $(1 - r)a(q)y$ to the entrepreneur.[6] Clearly, the entrepreneur will choose the lowest bid (the most aggressive bid). If there are two or more identical lowest bids, the entrepreneur chooses one randomly with equal probability. Once financed, the entrepreneur produces immediately and repays the loan if the project is good.

[4] One interpretation of the entrepreneur's strategy is that he/she applies to all banks simultaneously. Another interpretation, which we prefer, is that different banks may receive the application at different times but each bank must make the loan decision without knowing other banks' decisions. See Section 7 for more discussions on the entrepreneur's application decisions.

[5] A density f has the monotone likelihood ratio property if for all $s' > s$ and $q' > q$, $f(s'|q')/f(s|q') > f(s'|q)/f(s|q)$. In the current case, the ranking is $g > b$ and so the property is equivalent to $\gamma > 1/2$.

[6] Throughout this paper the profit from bidding is the expected profit that has not deducted the screening cost. When the screening cost is deducted, we use the phrase "net profit".

The screening effort is unobservable by outsiders. This is realistic since an entrepreneur is likely to contact each bank separately for funds. The assumption also simplifies the analysis. When choosing the bidding strategy, each bank faces the same probability distribution of other banks' signals. In this case, it is possible to focus on a symmetric equilibrium in which all banks screen with the same probability and each bank's bidding strategy depends only on the bank's own signal and the equilibrium screening probability. This is not possible in the alternative environment where each bank observes other banks' screening outcomes before bidding. In such an environment, bids depend on all banks' screening outcomes. To solve for an equilibrium in this environment, we must solve for all n banks' bids for each combination of the screening outcomes. This is cumbersome when n is large.

Screening is *not* a precondition for bidding, because an uninformed bank can always pretend to be informed and bid. Thus, the screening cost is different from the participation cost in Harstad (1990) and Levin and Smith (1994).

The above loan competition is a common-value, first-price auction with sealed bids, where "first price" means that the lowest (most aggressive) loan rate wins. As in a typical common-value auction model, the information structure has two features. First, a bank with a higher signal has a higher expected value of the project than a bank with a lower signal. In particular, the unconditional (marginal) distribution of a signal is

$$\Pr(S=s)=\sum_{q=g,b}\Pr(S=s|q)\cdot\Pr(q)=\begin{cases}\gamma\alpha+(1-\gamma)(1-\alpha), & \text{if } s=g;\\ (1-\gamma)\alpha+\gamma(1-\alpha), & \text{if } s=b.\end{cases}$$

By Bayes' rule, the posterior for the project's success after observing s alone is

$$\Pr(q=g|s)=\begin{cases}\dfrac{\gamma\alpha}{\gamma\alpha+(1-\gamma)(1-\alpha)}, & \text{if } s=g;\\[2ex] \dfrac{(1-\gamma)\alpha}{(1-\gamma)\alpha+\gamma(1-\alpha)}, & \text{if } s=b.\end{cases} \tag{2.1}$$

The posterior for $q=b$ can be calculated similarly. Because $\gamma>1/2$, $\Pr(q=g|s=g)>\alpha>\Pr(q=g|s=b)$ indeed. As verified later, this implies that a bank with signal g bids lower than an uninformed bank that in turn bids lower than a bank with signal b.

Second, winning conveys information (Wilson, 1977) and the winner's curse arises. If a bank wins the auction, it obtains information about other banks' signals and changes its assessment of the project. This is because banks' signals are dependent on each other unconditionally, although they are independent conditional on the true quality of the project. For example, if a bank receives signal b and bids according to this signal alone, the bid that makes a zero expected profit is r_1 such that $r_1 y\cdot\Pr(q=g|s=b)=1$. If this bid wins, then all other banks must have received signal b as well, since the bank would not have won if there were any bank with signal g. Thus, the winner's information is $I = \{s_1 = \cdot = s_n = b\}$ and the expectation on the project's success is much lower than the estimate based on the bank's own signal alone. That is, $\Pr(q = g|I) < \Pr(q = g|s = b)$. The bank's expected profit conditional on I is negative with r_1.

Anticipating the informational content of winning, a bank calculates the project's success probability conditioning on both the bank's own signal and the outcome that its bid r wins. Denoted $m_s(r)$, the expected profit from bidding r with a signal s is

$$m_s(r) = W(r|s) \cdot [ry \cdot \Pr(q = g|s; \text{bid } r \text{ wins}) - 1], \tag{2.2}$$

where $W(r|A)$ is the winning probability for a bid r conditional on event A. Rewrite (2.2) as:[7]

$$m_s(r) = \boxed{\Pr(q = g|s)}(yr - 1)W(r|q = g) - \boxed{\Pr(q = b|s)}W(r|q = b). \tag{2.3}$$

The boxes highlight the terms where $m_s(r)$ differs between different levels of s for a given r.

In contrast to many common-value auction models, the signals here are distributed in a discrete space. The discrete signal space reflects the reality that information is usually coarse. Wang (1991) has analyzed this type of auction for the case where the number of bidders is known. One result is that there cannot be a symmetric equilibrium where all banks bid with pure strategies on the loan rate. The reason is simple. Banks with signal g have incentive to under-bid each other. If each uses a pure strategy, the competition drives their expected profit to zero. But if the expected profit is indeed zero then a bank with signal g can bid slightly higher than other type g banks but lower than other types of banks. This bid guarantees winning when all other banks have no signal or have signal b. Since the latter event occurs with a strictly positive probability, the bid makes a positive profit.

We will examine only symmetric equilibria where all banks screen with the same probability and all banks that have the same information use the same

[7] To derive the result, note that for any events A, B and C,

$$\Pr(C|B \cap A)\Pr(B|A) = \Pr(C \cap B|A).$$

Then for $q^* \in \{g, b\}$,

$$\Pr(q = q^*|s;\ \text{bid } r \text{ wins})W(r|s) = \Pr(q = q^*;\ r\text{wins }|s) = W(r|q = q^*; s)\Pr(q = q^*|s).$$

Finally, since signals are independent conditional on the true quality, the probability that a bid r wins conditional on both the true quality and the bank's own signal is the same as that conditional on only the true quality. That is, $W(r|q, s) = W(r|q)$.

strategies in participation and bidding. A symmetric equilibrium will be such that each participating bank bids with a mixed strategy over a range of bids. We characterize such equilibrium strategies below.

3. Strategies and Equilibrium

3.1. DEFINITION AND DESCRIPTION

With symmetry, all banks choose a probability p to screen. After screening, there are three different types of banks. Let us refer to a bank as a bank G if it receives signal g, a bank B if it receives signal b and a bank U (uninformed) if it did not screen. We extend the notation s to include $s = u$ to indicate that the bank did not screen, with $\Pr(q|s = u) = \Pr(q)$. For a bank s $(= g, u, b)$, we also use s to denote the probability with which the bank participates in bidding. If a bank s bids, let F_s be the cumulative distribution function from which the bank draws the bid and $[r_{sL}, r_{sH}]$ be the support of the distribution. As shown later, banks G always bid in equilibrium and so we set their participation probability to one.

DEFINITION 3.1. A symmetric equilibrium consists of the screening probability p, participation probabilities (u, b), and bid distributions (F_g, F_u, F_b) such that the following conditions hold: (i) Given p, the participation and bidding strategies maximize each bank's expected profit conditional on the bank's signal; (ii) The screening probability maximizes the expected profit from screening, given that every other bank screens with probability p; and (iii) If $0 < p < 1$, the expected profit from screening equals the screening cost.

ASSUMPTION 1. The smallest number of banks considered, n_L, is at least 3 and the following condition holds:

$$\frac{1}{\alpha} < y < 1 + \frac{1-\alpha}{\alpha}\left(\frac{\gamma}{1-\gamma}\right)^{n_L-2}.$$

The part $y > 1/\alpha$ requires that, if all banks are uninformed, there are feasible loan rates to finance a project and yield a non-negative profit. The upper bound on y enables us to focus on the most interesting case where the screening probability is less than one. This upper bound is likely to be non-binding for typical loan markets because the bound increases exponentially with n_L. In any case this bound is only a convenient restriction and we analyzed the case where it is violated in an earlier version of this paper (Cao and Shi, 1999). The restriction $n_L \geq 3$ ensures that the interval for y in the above assumption is non-empty for all $n \geq n_L$.[8]

[8] We can lower the restriction to $n_L \geq 2$ and change the upper bound for y to $1+\frac{1-\alpha}{\alpha}(\frac{\gamma}{1-\gamma})^{n_L-1}$. This introduces an additional case $n = 2$ that can be analyzed similarly but would require careful handling in algebra.

We analyze the bidding strategies first. Any of the bid distributions F_s ($s = g, u, b$) should not have a mass point within its support. To see this, suppose that F_g has a mass point at $r_m \in [r_{gL}, r_{gH}]$. Given that all other banks G bid according to F_g, consider an individual bank G that deviates from F_g by shifting the mass at r_m to $r_m - \varepsilon$, where ε is an arbitrarily small positive number. The deviation increases the bank's winning probability by a discrete amount. Because ε is arbitrarily small, the bank's assessment of the project's quality after the bid $r_m - \varepsilon$ wins is arbitrarily close to that when the bid r_m wins. Thus, the deviation increases the bank's expected profit by a discrete amount, contradicting the fact that F_g is an equilibrium bid distribution.

To find more about the bidding strategies, we calculate the bank's expected payoff from (2.3) without additional knowledge of the relative location of the supports of the three bid distributions. For $s = g, b$, denote

$$Em_s(r) = m_s(r)\,\Pr(s); \quad EM_s = \int_{r_{sL}}^{r_{sH}} Em_s(r)\mathrm{d}F_s(r). \tag{3.1}$$

For $s = u$, denote $Em_u(r) = m_u(r)$.

To calculate $m_s(r)$, we calculate $W(r|q = g)$. This is the probability with which a bid r wins when the true quality of the project is g, without knowing the number of other banks participating in bidding or whether they have screened. Suppose that the project's true quality is good. A bid $r \in [0, 1]$ loses to an arbitrarily chosen competitor in three cases: (i) The competitor screened, received a signal g and bid below r, the probability of which is $\gamma p F_g(r)$; (ii) The competitor did not screen, participated in bidding and bid below r, the probability of which is $(1 - p)uF_u(r)$; (iii) The competitor screened, received signal b, participated in bidding and bid below r, the probability of which is $(1 - \gamma)pbF_b(r)$. Thus, a bid r loses to an arbitrary competitor with probability $\gamma p F_g(r) + (1-p)uF_u(r) + (1-\gamma)pbF_b(r)$. For the bid to win against $n - 1$ potential competitors, the probability is

$$W(r|q = g) \equiv [1 - \gamma p F_g(r) - (1 - p)uF_u(r) - (1 - \gamma)pbF_b(r)]^{n-1}. \tag{3.2}$$

Similarly, when the true quality of the project is b, the winning probability of a bid r is

$$W(r|q = b) \equiv [1 - (1 - \gamma)pF_g(r) - (1 - p)uF_u(r) - \gamma pbF_b(r)]^{n-1}. \tag{3.3}$$

Substituting these probabilities into (2.3), we obtain the expected payoff $m_s(r)$.

An important feature of the payoff function is that the ratio $\Pr(q = g|s)/\Pr(q = b|s)$ is increasing in s, where s is ranked according to $g > u > b$. That is, the assessment of the project's success is higher for a bank G than for a bank U, which in turn is higher than for a bank B. If there are bids for a bank U to make a non-negative profit, a bank G can under-bid slightly, win the bidding and obtain a greater expected profit. The similar comparison holds between a bank B and a bank U. Thus, a bank G's bids are higher than a bank U's, which in turn are higher than a bank B's. More precisely, we have (see Appendix A for a proof):

LEMMA 3.2. $r_{gH} = r_{uL}$ if $u > 0$; $r_{uH} = r_{bL}$ if $b > 0$.

The lemma implies $F_g(r) > F_b(r)$ for any bid r below the highest bid. From (3.2) and (3.3) one can immediately verify the following result:

LEMMA 3.3. $W(r|q = b) > W(r|q = g)$ for any bid r below the highest bid, and the ratio $W(r|q = b)/W(r|q = g)$ is an increasing function of n for given (p, u, b, F_s).

This lemma highlights the negative information externality in loan competition. First, it is more likely for a bid to win when the true quality is bad than when the true quality is good. Thus, a bank's assessment of the project decreases when it wins. Anticipating this winner's curse, each bank has lower incentive to screen. We refer to this as an externality because it depends on other banks' screening and bidding decisions that are taken as given by each individual bank. More important, the externality is exacerbated by the increase in the number of banks in the market. When there are more potential competitors, the chance for a bank to win the bidding when the quality of the project is good relative to that when the quality of the project is bad is much smaller and so, if the bank wins, the bank must greatly reduce its expectation that the project is of good quality. As shown later, this negative informational externality induces a large reduction in the screening probability in response to an increase in the number of banks.

3.2. BANKS B'S AND U'S STRATEGIES

With Lemma 3.2, we can show that banks B will not bid. To see why, note that a bank B can possibly win only when all other banks are banks B as well, i.e., when all other banks have screened and received signal B. If such a bank makes a non-negative profit, an uninformed bank can make a positive expected profit and hence will bid with probability one, i.e., $u = 1$. But if banks G and U always bid, then the probability for a bank B to win in the bad state relative to that in the good state, $W(r|q = b)/W(r|q = g)$, is too high for any feasible bid to make a non-negative expected profit under Assumption 1.

Similarly, it is not feasible for a bank U to bid with probability one. Uninformed banks participate in bidding with a positive probability when the screening probability p is small enough and, when they participate, the expected profit is zero. Formally, define

$$p_A(n) = \left[\gamma + \frac{2\gamma - 1}{\left(\frac{\alpha(y-1)}{1-\alpha}\right)^{1/(n-1)} - 1} \right]^{-1} \in (0, 1). \tag{3.4}$$

Appendix B contains a proof for the following proposition:

PROPOSITION 3.4. Assume $p > 0$. Banks B do not bid. Uninformed banks bid if and only if $p < p_A(n)$, in which case they bid with probability $u(p, n) < 1$ and bid according to the cdf F_u over the support $[r_{uL}, 1]$, where

$$u(p, n) = \frac{1 - (p/p_A)}{1 - p}, \tag{3.5}$$

$$r_{uL}(p, n) = \frac{1}{y} \left[1 + \frac{1-\alpha}{\alpha} \left(\frac{1 - (1-\gamma)p}{1 - \gamma p} \right)^{n-1} \right]. \tag{3.6}$$

The inverse of F_u, $H_u(F)$, is given by

$$r = H_u(F) = \frac{1}{y} \left[1 + \frac{1-\alpha}{\alpha} \left(\frac{1 - (1-\gamma)p - (1-p)uF}{1 - \gamma p - (1-p)uF} \right)^{n-1} \right]. \tag{3.7}$$

Uninformed banks make zero expected profit. Their participation probability decreases with p and their lowest bid increases with p.

The reason why a bank U's participation rate decreases with and their lowest bid increases with other banks' screening probability is as follows. When each bank screens with a higher probability, it is more likely that at least one bank receives signal g and hence less likely that an uninformed bank wins. The expected profit of banks U from bidding is lower for any given participation rate. If an uninformed bank wins despite the low likelihood, the bank's expectation of the project's success must be low. This further reduces the expected profit for a bank U. To break even, such a bank bids with a lower probability and bids more pessimistically.

The density $F'_u(r)$ is a decreasing function, which can be verified from (3.7) by showing $H''_u(F) > 0$. That is, for the same signal a bank is more likely to bid low loan rates than high loan rates. This is because a higher loan rate makes a higher profit if it wins the bidding and so it must be less likely to win in order for the mixing strategy to be rational. This is achieved by having more banks bid at low loan rates.

3.3. BANKS G'S STRATEGIES

Since $b = 0$ and $F_u(r_{gH}) = 0$, the expected payoff to a bank G from bidding $r \in [r_{gL}, r_{gH}]$ is $m_g(r) = EM_g/\Pr(s = g)$, where

$$\begin{aligned} EM_g &= (yr - 1)\alpha\gamma[1 - \gamma p F_g(r)]^{n-1} - (1-\alpha)(1-\gamma)[1 - (1-\gamma)pF_g(r)]^{n-1} \\ &= (yr_{gH} - 1)\alpha\gamma(1-\gamma p)^{n-1} - (1-\alpha)(1-\gamma)[1-(1-\gamma)p]^{n-1}. \end{aligned}$$

Using the two equalities we can solve for the inverse of the bid distribution F_g. Also, the payoff is an increasing function of r_{gH}. Thus, $r_{gH} = 1$ if banks U do not bid ($p \geq p_A(n)$) and $r_{gH} = r_{uL}$ if banks U bid ($p < p_A(n)$). For a bank G to bid, as we have maintained so far, the above profit must be non-negative, which requires $p \leq p_H(n)$ where

$$p_H(n) = \left[\gamma + \frac{2\gamma - 1}{\left(\dfrac{\alpha\gamma(y-1)}{(1-\alpha)(1-\gamma)}\right)^{1/(n-1)} - 1}\right]^{-1}. \tag{3.8}$$

Assumption 1 ensures $p_H(n) \in (0, 1)$ and $p_H(n) > p_A(n)$. Similar to Proposition 3.4, we can prove the following proposition (the proof is omitted):

PROPOSITION 3.5. Suppose that all other banks screen with probability $p \in [0, p_H(n))$. A bank G bids with probability 1. Its expected profit from bidding is $m_g = EM_g/\Pr(s = g)$, the bid distribution is F_g with an inverse H_g, and the support is $[r_{gL}, r_{gH}]$, where

$$EM_g(p,n) = \begin{cases} (1-\alpha)(2\gamma-1)[1-(1-\gamma)p]^{n-1}, & \text{if } 0 < p < p_A(n) \\ (y-1)\alpha\gamma(1-\gamma p)^{n-1} \\ \quad -(1-\alpha)(1-\gamma)[1-(1-\gamma)p]^{n-1}, & \text{if } p_A(n) \leq p \leq p_H(n), \end{cases} \tag{3.9}$$

$$r_{gH}(p,n) = \begin{cases} r_{uL}(p,n), & \text{if } 0 < p < p_A(n), \\ 1, & \text{if } p_A(n) \leq p \leq p_H(n). \end{cases} \tag{3.10}$$

$$r_{gL}(p,n) = \frac{1}{y}\left\{1 + \frac{(1-\alpha)(1-\gamma) + EM_g(p,n)}{\alpha\gamma}\right\}, \tag{3.11}$$

$$r = H_g(F) = \frac{1}{y}\left[1 + \frac{EM_g(p,n)}{\alpha\gamma(1-\gamma pF)^{n-1}} + \frac{(1-\alpha)(1-\gamma)}{\alpha\gamma}\left(\frac{1-(1-\gamma)pF}{1-\gamma pF}\right)^{n-1}\right]. \tag{3.12}$$

The expected profit of a bank G is positive for all $p < p_H(n)$ and is a decreasing function of p.

There are a number of notable features. First, in contrast to a bank U, a bank G makes a positive expected profit conditional on its signal, even when the bid is arbitrarily close to but lower than a bank U's lowest bid. This is simply because a bank G's assessment on the project's success is higher than a bank U's and the expected profit is conditional on each bank's own signal. When the signal becomes uninformative (i.e., when $\gamma \rightarrow 1/2$), the difference between the assessments by the two types of banks vanishes and a bank G's expected profit goes to zero.

Second, the lowest bid by a bank G, r_{gL}, is higher than what a bank G would bid if it knows that other banks all had signal g. This is because a bank does not know other banks' signals and other banks receive signal b with a positive probability. Even bidding at r_{gL} makes a positive expected profit m_g. In fact, r_{gL} is the supremum among such bids that bidding below them, which guarantees winning, will make an expected profit less than m_g.

Third, like F_u', the density $F_g'(r)$ is a decreasing function and so a lower bid is compensated by a higher chance of winning to generate the same expected profit as that from a higher bid.

Finally, a bank G's expected profit is a decreasing function of the screening probability used by other banks. This is intuitive. As every other bank increases the screening probability, there are more banks that receive signal g and the ensuing competition drives down the profit from bidding. Similarly, for any given screening probability, a bank G's expected profit is a decreasing function of the number of banks in the market.

3.4. EQUILIBRIUM

Now we determine the screening probability. If a bank does not screen, its expected profit is 0 regardless of whether it participates in bidding (see Proposition 3.4). If the bank screens, its expected profit is m_g when it receives a good signal and 0 when it receives a bad signal. Before knowing the signal, a screening bank's unconditionally expected profit is $\Pr(s = g) \cdot m_g = EM_g$. The net expected profit from screening is $EM_g - c$. Given that every other bank screens with probability p, an individual bank's screening probability, denoted p^*, is as follows

$$p^*(p,n) \begin{cases} = 1, & \text{if } EM_g(p,n) > c \\ = 0, & \text{if } EM_g(p,n) < c \\ \in [0,1], & \text{if } EM_g(p,n) = c. \end{cases} \tag{3.13}$$

The equilibrium screening probability is such that $p^*(p,n) = p$. Since $EM_g(p,n)$ is a decreasing function of p, the screening probability is unique, as shown in Figure 1 by point E.

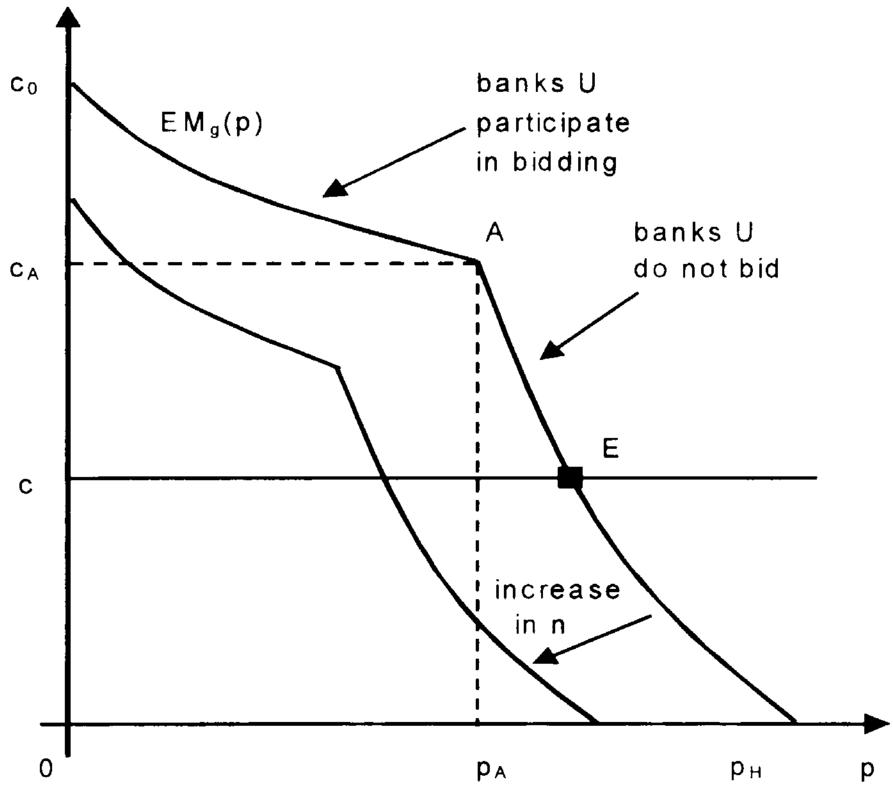

Figure 1. p: other banks' screening probability; c: screening cost; $EM_g(p)$: expected profit from screening.

Define $c_A = EM_g(p_A(n), n)$ and $c_0 = EM_g(0, n) = (1 - \alpha)(2\gamma - 1)$. Note that $p < p_A(n)$ if and only if $c > c_A$. The following proposition becomes evident.

PROPOSITION 3.6. Under Assumption 1, a unique equilibrium exists as follows:
(i) If $c \geq c_0$, then $p = 0$ and all banks bid $1/(\alpha y)$, making zero expected profit; (ii) If $0 < c < c_0$, then $p \in (0, p_H(n))$ and p satisfies $EM_g(p, n) = c$. The screening probability is a decreasing function of the screening cost. Banks G always bid and make positive expected profit from bidding. Uninformed banks bid if and only if $c > c_A$, in which case they bid with probability $u(p, n) > 0$ and make zero expected profit. Banks B do not bid. The bids are characterized in Propositions 3.4 and 3.5.

Remark 1. Even when $c = 0$, each bank screens with a probability p_H that is strictly less than one. This is because of the limited liability and because a bank's expected profit from bidding decreases sufficiently in other banks' screening probability as a result of the winner's curse.

4. Loan Rates and Loan Market Tightness

The effects of the screening cost and the number of banks on loans depend on whether the project is good or bad. We focus on the case $q = g$ in the remainder of this paper, because the distortion created by the informational problem on the financing of a good project is the one that is socially costly.[9] Also, assume $0 < c < c_0$ and so $p > 0$.

4.1. DEFINITIONS

We measure the loan market tightness for a good project by the probability with which the project fails to get financed. Denoted T_g, the tightness is the probability with which no bid is submitted when the quality is good (but unknown to the banks). Conditional on $q = g$, there are two cases where a randomly selected bank bids: when it screens and receives signal g, the probability of which is γp, or when it does not screen and chooses to bid, the probability of which is $(1 - p)u$. Thus, a randomly selected bank will bid with probability $\gamma p + (1 - p)u$. For every bank not to bid, the probability is:

$$T_g = [1 - \gamma p - (1 - p)u]^n. \tag{4.1}$$

The effects of an increase in c or n on loan rates are complicated. The change directly affects loan rates as well as indirectly through the effect on the screening probability. There is very little hope that such an change will in general affect loan rates monotonically in the sense of the first-order stochastic dominance. To obtain concrete results, we focus on the expected loan rate. Let R_g be the expected loan rate that the entrepreneur gets when the project's quality is good.

We calculate R_g as follows. Consider the case $p < p_A(n)$ and calculate the joint probability with which the project receives at least one bid and the winning bid does not exceed a level $r_1 \in [r_{gL}, r_{gH}]$. Given $q = g$, a randomly selected bank participates in bidding and its bid is lower than or equal to r_1 with probability $\gamma p F_g(r_1)$. Then $[1 - \gamma p F_g(r_1)]^n$ is the probability with which no bid below or equal to r_1 is received and $1 - [1 - \gamma p F_g(r_1)]^n$ is the probability with which the project receives at least one bid below or equal to r_1. Since the project may not receive any bid at all, which occurs with probability T_g, the observed winning bid does not exceed $r_1 \in [r_{gL}, r_{gH}]$ with the following probability:

$$FW_1(r_1) = \frac{1}{1 - T_g}\{1 - [1 - \gamma p F_g(r_1)]^n\}.$$

The similar probability for the observed winning bid not to exceed $r_1 \in [r_{uL}, 1]$ is

$$FW_2(r_1) = \frac{1}{1 - T_g}\left\{1 - [1 - \gamma p - (1 - p)uF_u(r_1)]^n\right\}.$$

[9] The case $q = b$ can be analyzed similarly, but some integrals do not admit reduced form solutions.

Express the loan rate as the value of $(yr - 1)$ rather than r. Using FW_1 and FW_2 as the cumulative distribution functions of the winning bids, we have

$$R_g = -1 + y\left[\int_{r_{gL}}^{r_{uL}} r\mathrm{d}FW_1(r) + \int_{r_{uL}}^{1} r\mathrm{d}FW_2(r)\right].$$

From Propositions 3.4 and 3.5 we substitute $r = H_g(F)$ into the first integral and $r = H_u(F)$ into the second integral. Changing the integration variable from r to F, we can compute

$$R_g = \frac{cnp + (1-\alpha)\{1 - [1 - (1-\gamma)p - (1-p)u]^n\}}{\alpha\{1 - [1 - \gamma p - (1-p)u]^n\}}. \tag{4.2}$$

If $p \geq p_A(n)$, a similar derivation shows that R_g obeys (4.2) with $u = 0$.

4.2. EFFECTS OF A HIGHER SCREENING COST

The expected loan rate and the loan market tightness depend on the screening cost as follows (see Appendix C for a proof):

PROPOSITION 4.1. *The loan market tightness is an increasing function of the screening cost when $c < c_A$ and a decreasing function of the screening cost when $c > c_A$. The expected loan rate R_g increases with the screening cost when $c < c^*$ and decreases with the screening cost when c is close to c_0, where $c^* \in (c_A, c_0)$ is defined in Appendix C.*

The loan market tightness has a hump-shaped dependence on the screening cost. When $c < c_A$, only banks G bid. As the screening cost increases, the expected number of bidding banks falls because each bank screens with a lower probability. This generates a tighter loan market. When c passes the level c_A, further increases in the screening cost induce banks U to bid as well and so loans become more available.

The dependence of the expected loan rate on the screening cost has a similar hump shape, although the peak of the effect occurs at a higher level $c^* > c_A$. When $c < c_A$, the expected loan rate increases with c because only banks G bid and a higher loan rate is required to cover the higher screening cost. In this case, the higher screening cost increases the winning bid in the sense of the first-order stochastic dominance. Banks G's bids continue to increase with c when $c > c_A$ but the response of the average loan rate is complicated by the participation of banks U. When $c_A < c < c^*$, there are not many banks U in bidding and, since their bids are higher than those by banks G, their participation makes the expected loan rate increase even faster than in the case $c < c_A$. When $c > c^*$, most active banks are uninformed. Further increases in c increase competition among these banks and hence reduce the average loan rate.

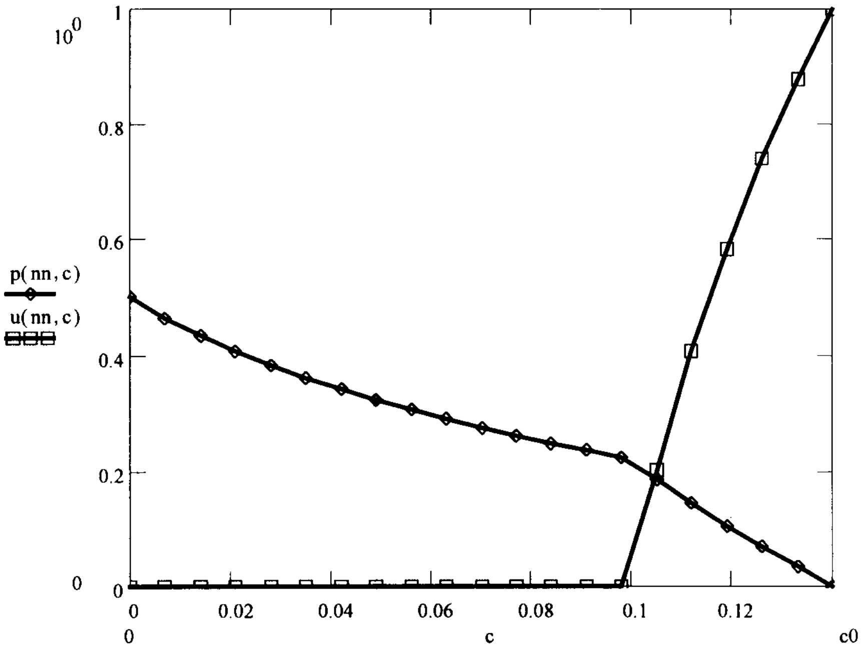

Figure 2a. $nn = 6$: a fixed number of banks; $p(nn, c)$: screening probability as a function of the screening cost, c; $u(nn, c)$: uniformed bank's participating probability as a function of the screening cost, c.

EXAMPLE 4.2. $\alpha = 0.65$, $\gamma = 0.7$, $y = 1.89$, $n = 6$. These parameters satisfy Assumption 1. In this case, the highest screening cost that induces positive screening is $c_0 = 0.14$. Figure 2a depicts the screening probability of each bank and bank U's participation probability; Figure 2b depicts the loan market tightness and the expected loan rate. As discussed above, when c increases, p falls, u increases and the graphs of (T_g, R_g) both have a hump shape.[10]

Two economies can be quite different in the screening cost and yet exhibit similar loan market characteristics such as the average loan rate and the loan market tightness. In one economy, the screening cost is low and the active bidders are all informed. In the other economy, the screening cost is high and most active bidders are uninformed. Although these two economies may have similar market characteristics like the average loan rate, they respond differently to policies that reduce the screening cost. In the economy with a low screening cost, the policy reduces the market tightness and lowers the loan rate by increasing the number of informed bidders. In the economy with a high screening cost, the policy also increases the number of informed bidders but it reduces the number of uninformed bidder by

[10] Although expected loan rates appear large in the figure, they are clearly reasonable if each period is interpreted as, say, 5 years.

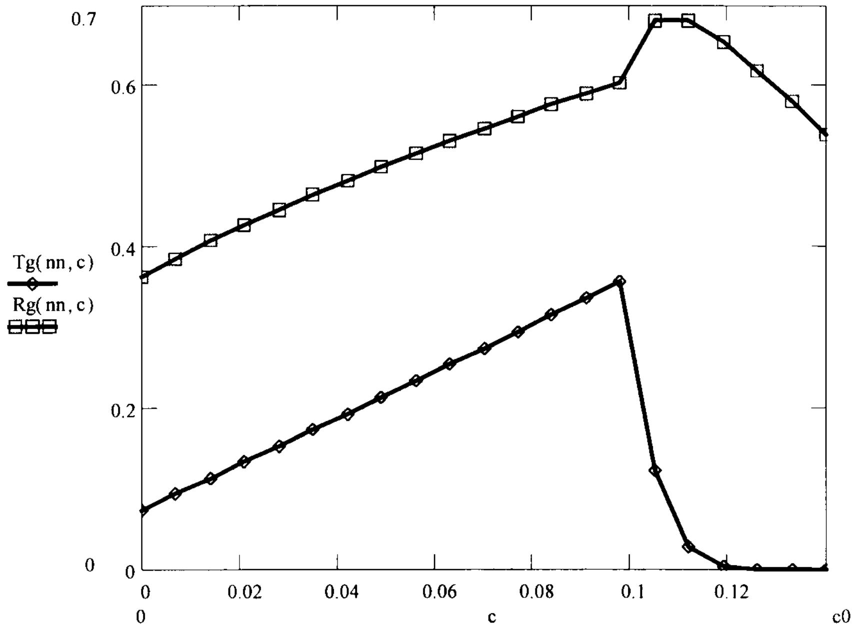

Figure 2b. $nn = 6$: a fixed number of banks; $T_g(nn, c)$: market tightness for a good project, as a function of the screening cost, c; $R_g(nn, c)$: expected loan rate for a good project, as a function of c.

more so that the total number of active bidders falls. Therefore, by reducing the screening cost slightly, a policy may not necessarily make loans more available. A significant reduction in the screening cost, however, can increase loans as it pushes the economy over the hump in Figure 2b.

4.3. EFFECTS OF INCREASING THE NUMBER OF BANKS

We now turn to the influence of the number of banks on the loan rate and the loan market tightness. Appendix D contains a proof for the following Proposition:

PROPOSITION 4.3. For any fixed $c \in (0, c_0)$, the equilibrium screening probability is a decreasing function of the number of banks. The loan market tightness is an increasing function of n if and only if $c < c^{**}$, where $c^{**} \geq c_A$ is given in Appendix D.

The equilibrium screening probability decreases in n because, for any given $p \in [0, p_H]$, the expected screening profit of a bank G is a decreasing function of n. To cover the screening cost, each bank's screening probability must fall when n increases. In Figure 1, the increase in n shifts the $EM_g(p)$ curve down and produces a smaller solution for p.

The most interesting feature is that the loan market can become tighter when there are more banks, as in the case $c < c^{**}$. This seemingly paradoxical result arises because an increase in the number of banks, by reducing the screening probability, reduces the expected number of bidding banks intensively by more than the extensive increase in the number of banks. The dominating intensive effect is a manifestation of the negative informational externality to the winner documented in Lemma 3.3. That is, an increase in the number of banks reduces a bank G's expected profit below the level which conventional competition would produce. This calls for a large reduction in the screening probability that makes the market tighter.

To illustrate, let us consider the case $c < c_A$ ($< c^{**}$) and rewrite

$$EM_g(p, n) = (1 - \gamma p)^{n-1} \cdot \alpha\gamma \left\{ (y - 1) - \frac{(1-\alpha)(1-\gamma)}{\alpha\gamma} \left[\frac{1 - (1-\gamma)p}{1 - \gamma p} \right]^{n-1} \right\}. \tag{4.3}$$

The conventional competition effect of an increased n is captured by the term $(1 - \gamma p)^{n-1}$, which is the probability that the bid r_{gH} wins when the true quality is good. An increase in n reduces this winning probability for any given $p > 0$ and hence calls for a reduction in p to cover the screening cost. The additional effect, i.e., the informational externality, works through the term $[.]^{n-1}$ in (4.3), which is the relative likelihood of winning when the true project is bad as opposed to when the quality is good. For any given $p > 0$, an increase in n increases this relative likelihood and reduces the expected profit. This additional effect calls for a further reduction in the screening probability and increases the loan market tightness.

The negative informational externality increases with banks' screening probability. When no one screens and every bidder is uninformed, winning does not yield any new information and hence the winner's curse is absent. When each bank screens with a higher probability, the expected number of banks with signal g is higher and so, for any given bid, the chance of winning against a randomly selected bank when the true quality is good falls relative to that when the true quality is bad. Winning in this case yields more negative information about the project's quality and the content of this information is more responsive to changes in n. More precisely, the derivative of the term $[.]^{n-1}$ in (4.3) with respect to n is an increasing function of p.

An implication is that the negative informational externality is weaker when uninformed banks bid than when they do not. When $u > 0$, an increase in n does not increase the negative informational externality by as much as when $u = 0$. Moreover, the increase in the number of uninformed bidders itself eases the loan market tightness. Therefore, when $c > c^{**}$, increases in n may not increase the market tightness. In fact, when c is sufficiently large, most bidders are uninformed and so the negative informational externality to an informed winner is dominated by other forces described above. In this case, the loan market becomes less tight when n increases.

Another implication is that the loan market will never approach a standard competitive one:

COROLLARY 4.4. Let $c = 0$. Then $u = 0$ and $p = p_H \in (0, 1)$. Moreover, $\lim_{n\to\infty} T_g \in (0, 1)$ and $\lim_{n\to\infty} R_g > 0$.

The proof is straightforward and hence is omitted. The corollary states that even when there is no screening cost and when there are infinitely many banks, a good project is not always financed and the average loan rate is not zero. This is because, as the number of banks increases, the negative externality increases to reduce each bank's screening probability sufficiently so that the expected number of screening banks in the limit, $\lim_{n\to\infty} np$, is bounded above. This result is similar to the one in Broecker (1990, proposition 2.3).

In comparison to the effect on the market tightness, the effects of an increase in the number of banks on the expected loan rate are more difficult to detail analytically. Numerical exercises illustrate the following general pattern: An increase in the number of banks increases the expected loan rate when the screening cost is low and decreases the expected loan rate when the screening cost is high. We provide one set of numerical examples below to illustrate this pattern. In these examples, (α, γ, y) have the same values as in Example 4.2. The lowest value of n is $n_L = 3$, the highest value is $n_H = 15$, and $c_0 = 0.14$.

EXAMPLE 4.5. Low screening cost: $c = 0.056$. Figure 3a shows the equilibrium screening probability, the market tightness and the expected loan rate. Figure 3b shows the distribution of the winning bids for $n = n_L$ and n_H, denoted $FWg(\cdot, n, c)$ for given (n, c).

In this example, the screening cost is so low that all bidders are informed ones. As the number of banks increases, each bank reduces the screening probability and the loan market gets tighter. The lowest bid r_{gL} does not change with n since it is pinned down by the screening cost (setting $EM_g = c$ in (3.11)). The highest bid does not change with n either since it is 1. However, the density of bids is more concentrated at high bids when n is larger since fewer banks bid. The winning bid distribution with a higher n dominates that with a smaller n (see Figure 3b).

EXAMPLE 4.6. High screening cost: $c = 0.119$. Figure 4a shows the equilibrium screening probability, the participation probability of a bank U, the market tightness and the expected loan rate. Figure 4b shows the distribution of the winning bids for $n = n_L$ and n_H.

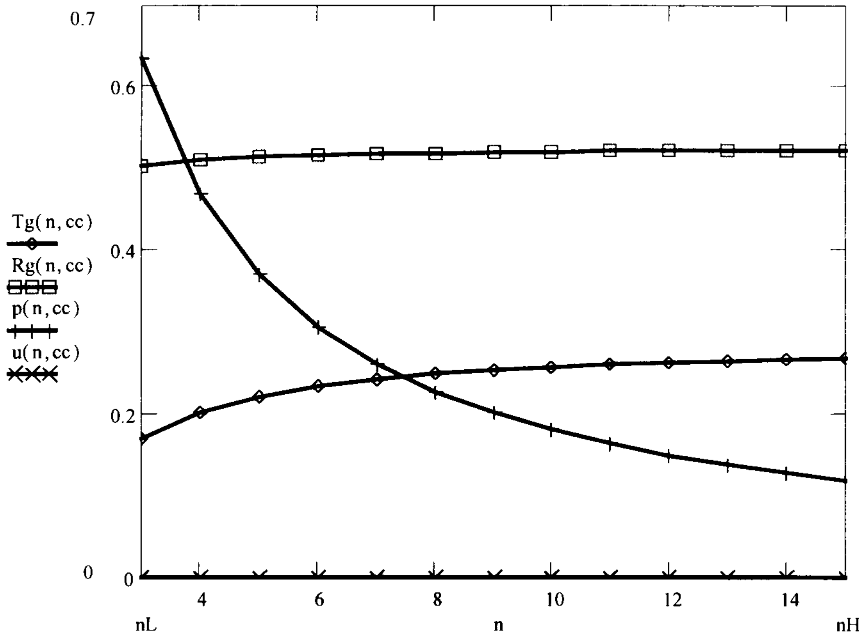

Figure 3a. cc = 0.056: a low screening cost; $T_g(n, cc)$: loan tightness for a good project as a function of the number of banks, n; $R_g(n, cc)$: expected loan rate as a function of n; $p(n, cc)$: screening probability as a function of n; $u(n, cc)$: uninformed bank's participation probability.

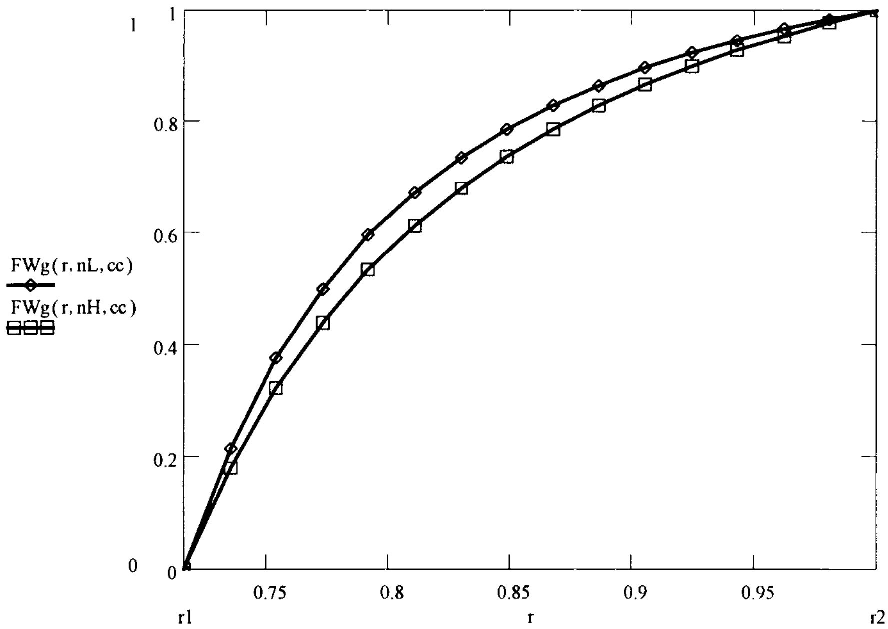

Figure 3b. cc = 0.056: a low screening cost; $nL = 3$, $nH = 15$; a low and a high value for the number of banks; $FWg(r, nL, cc)$, $FWg(r, nH, cc)$: cumulative distribution function of winning bids, r, corresponding to nL and nH number of banks, respectively.

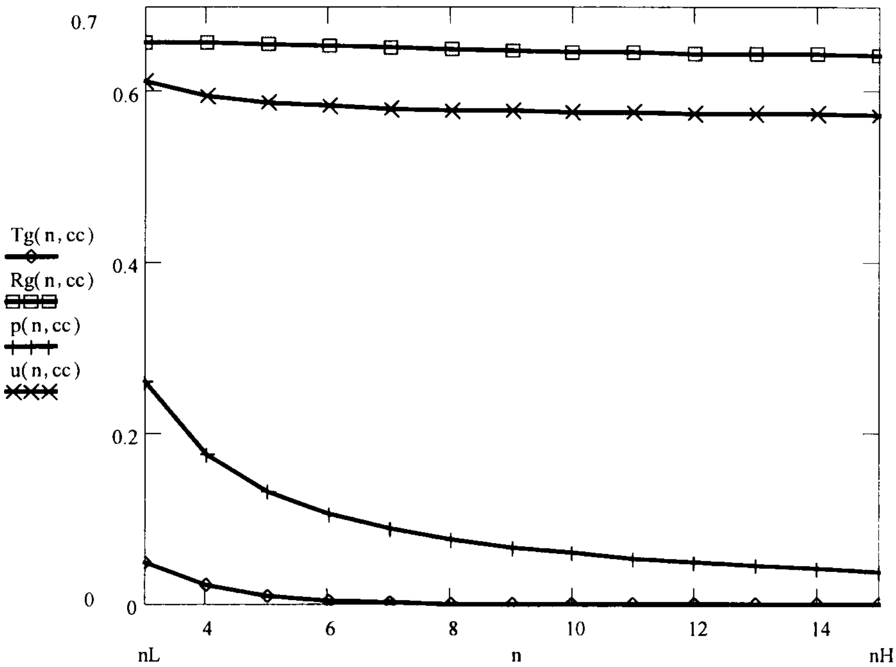

Figure 4a. $cc = 0.119$: a high screening cost; $T_g(n, cc)$: loan tightness for a good project as a function of the number of banks, n; $R_g(n, cc)$: expected loan rate as a function of n; $p(n, cc)$: function of n; $u(n, cc)$: uninformed bank's participation probability.

In this example, the screening cost is sufficiently high that most bidders are uninformed. Again, the increase in the number of banks reduces each bank's screening probability by reducing the expected profit from screening. The increased number of banks also increases competition among uninformed banks and so the participation rate u falls. Despite this reduction in the participation probability, the total number of uninformed banks in bidding increases with n, leading to a less tight loan market.

On loan rates, the increase in the number of banks increases banks G's bids as in the previous example. But banks U's bids decrease. Shown in Figure 4b, an increase in n tilts the distribution of the winning bids submitted by banks G toward higher bids and tilts the distribution of the winning bids submitted by banks U toward lower bids, producing the bulging shape in the middle of the winning bid distribution. The overall effect on the expected loan rate is ambiguous in general but, for the current example, it is negative as shown in Figure 4a. Since the winning bids are more concentrated in the middle with a higher n, the standard deviation of the winning bids is likely to fall.

EXAMPLE 4.7. Moderate screening cost: $c = 0.098$. Figure 5a shows the equilibrium screening probability, the participation probability of a bank U, the market

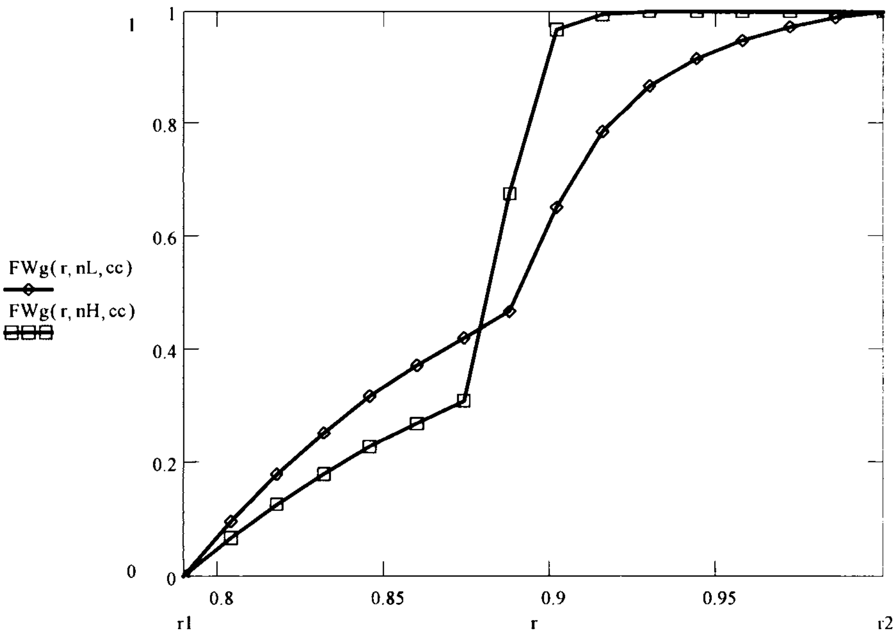

Figure 4b. $cc = 0.119$: a high screening cost; $nL = 3$, $nH = 15$: a low and a high value for the number of banks; $FWg(r, nL, cc)$, $FWg(r, nH, cc)$: cumulative distribution function of winning bids, r, corresponding to nL and nH number of banks, respectively.

tightness and the expected loan rate. Figure 5b shows the distribution of the winning bids for $n = n_L$ and n_H.

In this example, increases in the number of banks change the nature of the equilibrium. For $n < 9$, the screening probability is high, which deters uninformed banks from participating in bidding. In this case, the loan market tightness and the expected loan rate behave very like those in Example 4.5. For $n > 9$, the screening probability is low, uninformed banks bid and, as in Example 4.6, the loan market becomes less tight when n increases. In contrast to Example 4.6, the participation probability by uninformed banks increases with n. Also, the expected loan rate increases with n, because the number of banks U in bidding is not significant and so their reduced bids are not sufficient to outweigh the increased bids by banks G. The distribution of the winning bids when n is higher dominates the one when n is lower (see Figure 5b).

5. Constrained Social Optimum and Efficiency

To examine the inefficiency created by the informational externality, we analyze the following allocation problem faced by a social planner. At the beginning of the period, the social planner chooses the screening probability for each bank. After seeing all screening outcomes, the social planner decides whether to fund

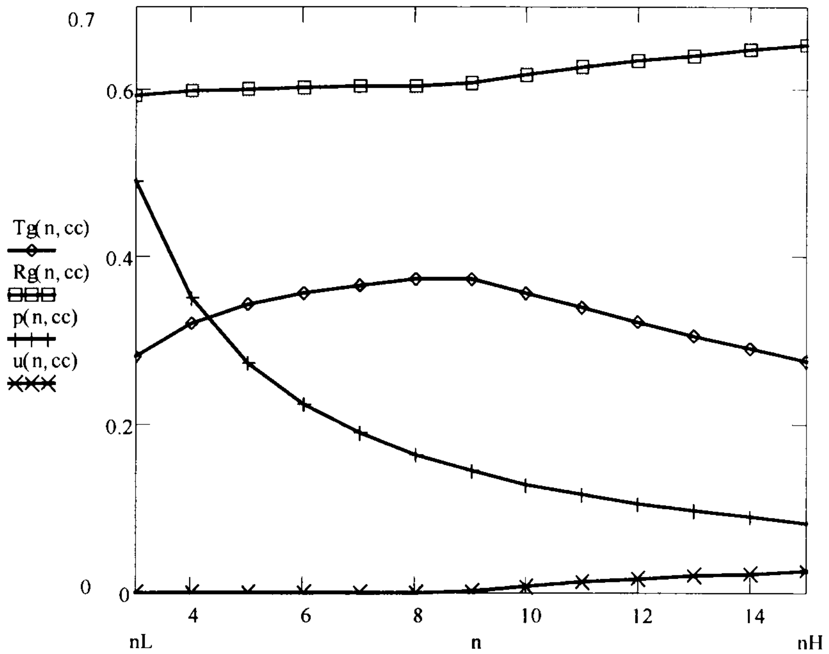

Figure 5a. $cc = 0.098$: a moderate screening cost; $T_g(n, cc)$: loan tightness for a good project as a function of the number of banks, n; $R_g(n, cc)$: expected loan rate as a function of n; $p(n, cc)$: screening probability as a function of n; $u(n, cc)$: uninformed bank's participation probability.

the project. Because the social planner takes into account the effect of all banks' screening on the social surplus, the decision internalizes the informational externality. However, we assume that the social planner, like the banks in the equilibrium, does not know the project's quality. For this reason, the planner's allocation is a constrained optimum.

The social planner asks each bank to screen the project with a probability p_s. Consistent with the equilibrium environment, all screening outcomes are realized simultaneously. Let $s_i \in \{g, u, b\}$ be bank i's screening outcome where $i = 1, 2, \ldots, n$. Because a bank receives a good signal if and only if it screens and the outcome is good, $\Pr(s_i = g|q = g) = p_s\gamma$. Let $(s_1, \ldots, s_n)$ be a particular vector of realizations in which there are exactly n_g banks that received good signals while other banks either received bad signals or did not screen. Then,

$$\Pr(q = g|s_1, \ldots, s_n)$$

$$= \frac{\alpha(p_s\gamma)^{n_g}(1 - p_s\gamma)^{n-n_g}}{\alpha(p_s\gamma)^{n_g}(1 - p_s\gamma)^{n-n_g} + (1 - \alpha)[p_s(1 - \gamma)]^{n_g}[1 - p_s(1 - \gamma)]^{n-n_g}}.$$

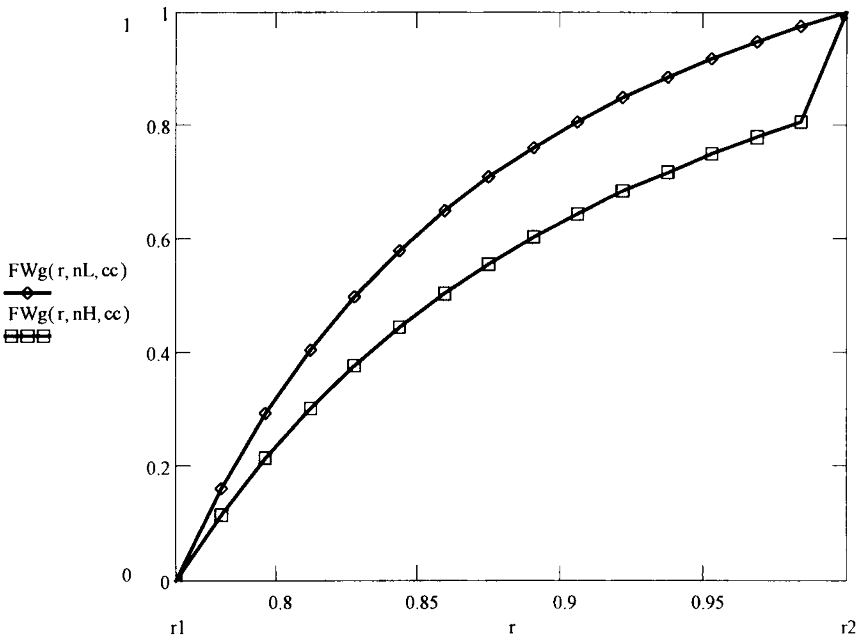

Figure 5b. $cc = 0.098$: a moderate screening cost; $nL = 3$, $nH = 15$: a low and a high value for the number of banks; $FWg(r, nL, cc)$, $FWg(r, nH, cc)$: cumulative distribution function of winning bids, r, corresponding to nL and nH number of banks, respectively.

After observing the particular realizations $(s_1, \ldots, s_n)$, the social planner funds the project if and only if the expected social surplus is non-negative, i.e., iff $y \cdot \Pr(q = g|s_1, \ldots, s_n) - 1 \geq 0$. Thus, the social planner funds the project if and only if

$$\alpha(y-1)\gamma^{n_g}(1-p_s\gamma)^{n-n_g} - (1-\alpha)(1-\gamma)^{n_g}[1-p_s(1-\gamma)]^{n-n_g} \geq 0. \tag{5.1}$$

This condition can be written equivalently as

$$n_g \geq N(p_s) \equiv \text{ceil}\left\{\frac{n\ln\left(\dfrac{1-p_s(1-\gamma)}{1-p_s\gamma}\right) - \ln\left(\dfrac{\alpha(y-1)}{1-\alpha}\right)}{\ln\left(\dfrac{\gamma[1-p_s(1-\gamma)]}{(1-\gamma)(1-p_s\gamma)}\right)}\right\}, \tag{5.2}$$

where ceil(x) is the smallest integer that is greater than or equal to x.

Before observing the screening outcomes, the expected social surplus from the project is

$$\Delta(p_s) \equiv \sum_{n_g=N(p_s)}^{n} \frac{n!}{n_g!(n-n_g)!}\Pr(s_1, ..., s_n) \times \left[y \cdot \Pr(q = g|s_1, ..., s_n) - 1\right]. \tag{5.3}$$

The social planner chooses the screening probability p_s for each bank to maximize the net expected social surplus, $\Delta(p_s) - ncp_s$, where ncp_s is the expected screening cost.

The function $\Delta(p_s)$ is continuous but its derivative has a finite number of jumps in the interval $p \in [0, 1]$. This is because $N(p_s)$ changes discretely from one integer to the next when p_s passes certain levels. Denote

$$\bar{N} = \text{ceil}\left\{\frac{1}{2}\left[n - \ln\left(\frac{\alpha(y-1)}{1-\alpha}\right) \Big/ \ln\left(\frac{\gamma}{1-\gamma}\right)\right]\right\}. \tag{5.4}$$

For integer $i \in \{0, 1, \ldots, \bar{N}\}$, define

$$P(i) \equiv \left\{\gamma + \frac{2\gamma - 1}{\left[\frac{\alpha(y-1)}{1-\alpha}\left(\frac{\gamma}{1-\gamma}\right)^{i}\right]^{1/(n-i)} - 1}\right\}^{-1}. \tag{5.5}$$

We have the following lemma (see Appendix E for a proof):

LEMMA 5.1. (i) $\Delta(p)$ is continuous for all $p \in [0, 1]$; (ii) The derivative $\Delta'(p)$ is continuous except for points $P(i)$, $i \in \{0, 1, \ldots, \bar{N} - 1\}$, where $\lim_{p \uparrow P(i)} \Delta'(p) = 0 < \lim_{p \downarrow P(i)} \Delta'(p)$; (iii) $\Delta'(p) = 0$ for $p \in [0, P(0))$; (iv) For $p \geq P(0)$ and $p \notin \{P(0), \ldots, P(\bar{N} - 1)\}$, $\Delta'(p) > 0$.

Figure 6 depicts $\Delta(p)$ for $\bar{N} = 3$. The segments of $\Delta(p)$ correspond, respectively, to the cases where the social planner funds the project only when the number of good signals is greater than or equal to 0, 1, 2, etc. In particular, the first curved segment (i.e., the one for $p \in (P(0), P(1))$) is the expected social surplus when the social planner funds the project only when there is one or more good signals. In Figure 6 we also depict the expected screening cost for two values of c, where c_1 is sufficiently large and c_2 is sufficiently small. When the screening cost is sufficiently high, the net expected social surplus is maximized at point A, which implies $p_s = 0$. When the screening cost is sufficiently low, the net expected social surplus is maximized at point B, which implies $p_s = 1$. When the screening cost is moderate, the social optimum is given by a point on the curve $\Delta(p)$ where the slope is equal to nc.

To compare the equilibrium solution with the social optimum, notice that $p_H = P(1)$ and $p_A > P(0)$, where p_A is defined in (3.4) and p_H in (3.8). Moreover, $\Delta'(p) = n \cdot EM_g(p, n)$ for $p \in [p_A, P(1))$, where EM_g is the expected profit from screening in equilibrium. Because the equilibrium screening probability satisfies $EM_g(p, n) = c$, it is a point on the first curved segment of $\Delta(p)$ in Figure 6 where the slope of $\Delta(p)$ is equal to nc. Point E depicts this equilibrium solution when the screening cost is c_2. Because the socially optimal screening probability

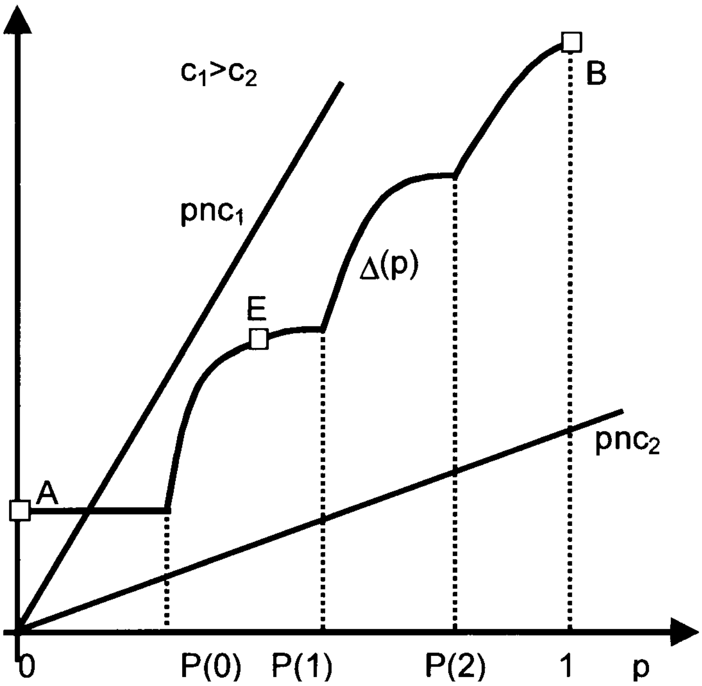

Figure 6. $\Delta(p)$: total expected social surplus from screening (before deducting the screening cost), as a function of the screening probability, p; pnc: total expected cost of screening.

is 1 when $c = c_2$, the equilibrium screening probability is inefficiently low in this case. This is true generally for low screening costs. For moderately high screening costs, the social optimum and the equilibrium both lie on the first curved segment of $\Delta(p)$ where $\Delta'(p) = nc$, and so the equilibrium is efficient. The equilibrium is also efficient when the screening cost is sufficiently high, in which case the social optimum and the equilibrium both yield 0 screening probability. With these arguments and Corollary 4.4, the following proposition is evident.

PROPOSITION 5.2. The equilibrium is efficient if the screening cost is high and inefficient if the screening cost is low. For low screening costs, the socially optimal screening probability exceeds the equilibrium level by at least $1 - p_H$. As the number of banks increases in this case, the difference between the social optimum and the equilibrium in the expected number of banks that screen approaches the limit $\lim_{c \to 0} n(1 - p_H) = \infty$.

This proposition is consistent with our previous discussion on the informational externality. Because the externality is stronger when the screening cost is lower, the equilibrium is more likely to be inefficient for low screening costs than for high screening costs. The amount of inefficiency grows as the number of banks increases. It is worthwhile noting that the equilibrium is inefficient whenever the social planner requires two or more good signals before providing the loan to the entrepreneur, in which case the social optimum lies on the second or higher curved segment of $\Delta(p)$ in Figure 6.

6. Implications

Our model generates the following implications.

Implication 1. It is possible that a market with fewer banks can provide both a lower loan rate and a higher loan availability than a market with more banks.

Implication 2. The average "quality" of projects financed in a concentrated banking area is lower than in a competitive banking area when the screening cost is low (see Proposition 4.3), where the meaning of "quality" differs from that in previous sections and refers to the prior probability of project success, α.

Implication 3. An increase in the number of banks is likely to have a greater effect on the market tightness than on the expected loan rate.

In Proposition 4.3 we established Implication 1, which is consistent with empirical findings by Petersen and Rajan (1995). To verify Implication 2, one can show that the iso-tightness condition, $(1-T_g) =$ constant, induces a positive relationship between n and α when $c < c^{**}$. Because $(1 - T_g)$ is the probability with which a project is financed, the positive relationship says that, for the same number of financed projects, the average quality is lower in the market with a small number of banks than in the market with a large number of banks. Implication 3 comes from Examples 4.5 and 4.6, where the market tightness responds to n significantly but the expected loan rate remains flat. Only in Example 4.7 and only for n large enough does the increase in n have a noticeable effect on the expected loan rate. The response of the expected loan rate is small because, when a bank responds to an increase in n, it changes both the participation probability and the bids; these two have opposite effects on the expected loan rate. This result suggests that the loan availability is a more important channel through which banks compete against each other when information is incomplete.[11]

Implication 4. The differential in the loan availability between a concentrated banking area and a competitive area can be lower for not-so-young firms than for young firms.

As we argued in the introduction, the monopoly-power story by Petersen and Rajan (1995) suggests that the loan availability differential between areas with

[11] Our model also implies that a lower expected loan rate often, but not always, indicates a less tight loan market (see Example 4.7).

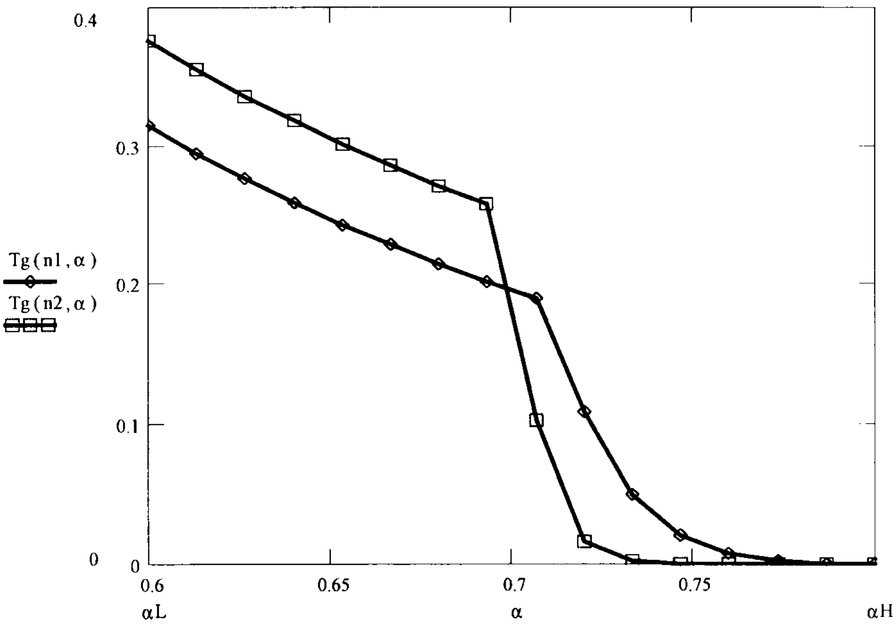

Figure 7a. $n1 = 4$, $n2 = 10$: the number of banks in two markets; α: a larger α is a proxy for an older firm; $T_g(n1, \alpha)$, $T_g(n2, \alpha)$: loan tightness for a good project in the two markets, as functions of α.

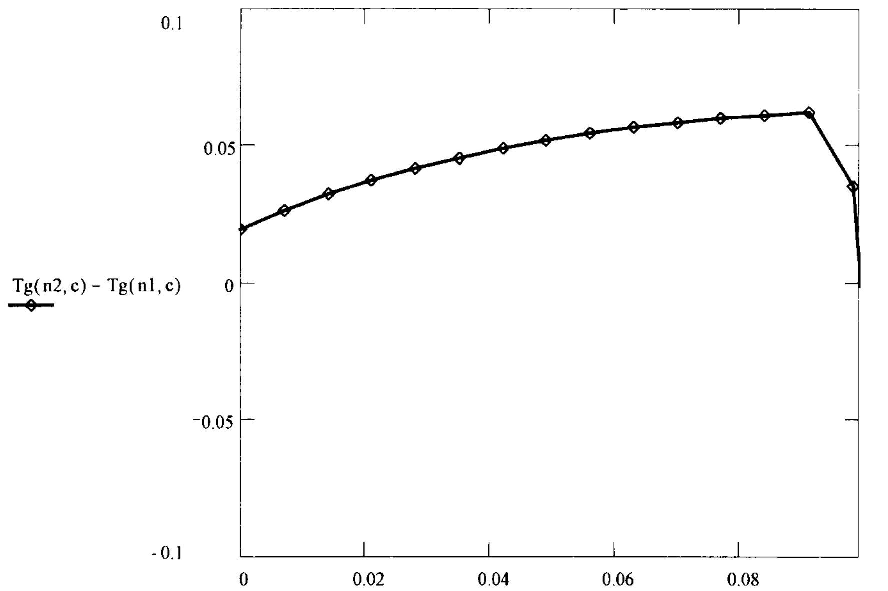

Figure 7b. $n1 = 4$, $n2 = 10$: number of banks in two markets; c: a smaller c is another proxy for an older firm. $T_g(n2, c) - T_g(n1, c)$: difference in loan tightness between the two markets, as a function of c.

different bank concentration should be even greater for not-so-young (but not-old) firms than for young firms. Our model can generate the opposite but realistic pattern, although our model does not have an explicit intertemporal structure. One way to do this is to interpret the parameter α as a proxy for a firm's age, as in the following example, because a firm's good quality is increasingly revealed to the public when the firm's survivorship increases.[12]

EXAMPLE 6.1. Let $\gamma = 0.7$, $y = 1.89$, $c = 0.072$, $n1 = 4$, and $n2 = 10$. We depict the pattern of the market tightness in Figure 7a. As α increases from 0.6 to 0.8, the tightness is first lower with $n1$ firms than with $n2$ firms; then the pattern is reversed. Eventually, the tightness levels in the two markets decrease toward the same level.

One may also argue that the bank's screening cost falls when a firm becomes more established. To see how the differential in the loan availability between different banking areas changes with the firm's age under this interpretation, reconsider Example 4.2 for a competitive banking area ($n = 10$) and a concentrated banking area ($n = 4$). Figure 7b illustrates the difference in the loan market tightness between these two areas for all values of c that induce a tighter loan market for the competitive area (i.e., for $c < 0.099$). The figure shows that this difference narrows when the screening cost decreases, as long as the two areas both start with only informed banks in bidding (i.e., as long as $c < 0.091$). When $0.091 < c < 0.099$, the difference in the tightness increases when c falls, because the competitive area has many uninformed banks in bidding initially which are driven out by the lower screening cost. Thus, unless the two areas differ initially in the nature of the equilibrium, the differential in the loan availability between the two areas is larger for young firms than for not-so-young firms.

Implication 5: The relationship between the loan availability and the number of banks is not invariably negative and the nature of this relationship is industry-specific.

The market tightness increases with n when the screening cost is low and decreases with n when the screening cost is high (Proposition 4.3). Since the screening cost depends on the industry's type, the relationship between the loan availability and the number of banks varies across industries. For example, it might be easier for banks to estimate the expected profit from running a McDonald's franchise than to estimate the profit from developing the next cutting-edge computers. Our model indicates that the loan availability differential between a concentrated area and a competitive area is greater for the first type of industry than for the

[12] Another proxy for a firm's age is the project's productivity y. When a firm becomes established, the productivity increases. This alternative interpretation of age leads to outcomes similar to those caused by an increase in α.

second type. In contrast, the model by Petersen and Rajan (1995) predicts the opposite if the cutting-edge computer industry also has a higher expected future profit than the McDonald's franchise.

The link between the industry type and the screening cost is supported by some evidence in Petersen and Rajan (1995). In their sample, a majority of firms are in retail and service sectors. For example, the most concentrated banking areas have more than 60% of the firms in these sectors. Since the screening cost is lower for such sectors than for other sectors in the sample like construction, manufacturing and transportation, Implication 5 indicates that their sample is biased toward a strong negative relationship between the loan availability and the number of banks. The relationship should be weaker once the industry type is controlled. This is exactly what Petersen and Rajan (1995) found: When they added the two-digit industry classification into their regression, the dependence of the loan availability on bank concentration fell. Of course, to satisfactorily test the role of the industry type, one needs more detailed industry classification than the two-digit one.

Implication 6: If economic upturns make information much cheaper than in downturns, then the loan availability differential between a concentrated banking area and a competitive banking area is pro-cyclical.

This implication again derives from the observation that the loan availability differential between the two areas is negative when the screening cost is high and positive otherwise (Proposition 4.3). Thus, if economic upturns change the economy from one with a high screening cost to another with a low screening cost, then the loan availability differential changes from a negative one to a positive one and hence is pro-cyclical. In this case, economic recoveries benefit small businesses in concentrated banking areas more than those in competitive banking areas.

There is a good reason to believe that economic upturns reduce the information cost. If different firms' productivities have a common unknown component as well as idiosyncratic ones, economic upturns reveal the common component more accurately through high aggregate activities than economic downturns. This argument has been formalized in the literature of informational business cycles, e.g., Chamley and Gale (1994). In contrast, it is not clear how the difference between banks' monopoly powers in different areas would change over business cycles. Unfortunately, the data in Petersen and Rajan (1995) involves a very short time span (1988–1989) and hence cannot be used to check the business cycle features of the loan availability differential.[13]

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[13] Our model also allows for the possibility that, for medium screening cost, loans are less available in the not-so-competitive market than in both the highly concentrated and the highly competitive markets (see Example 4.7). This result, documented in Petersen and Rajan (1995, pp. 428–431), is inconsistent with their story that relies on ex post monopoly power.

7. Modelling Assumptions and Robustness

We can relax some assumptions in our model without affecting the qualitative results. First, there is no cost submitting a bid in our model, but all the results are robust to the introduction of a small bidding cost. Second, the bidding game is a common-value auction. The qualitative results can be valid for more general bidding environments where the signals are affiliated (see Milgrom and Weber, 1982). Third, there is only one screening technology in our model. One can add another screening technology that costs less (say, $c^a < c$) and yields a less precise signal (say, $\gamma^a < \gamma$). If c^a is sufficiently small and γ^a is close to $1/2$, some banks may use this screening technology and their participation choices will be very much like the ones we analyzed for uninformed banks in previous sections. This provides another justification for our earlier assumption that uninformed banks can bid: Those uninformed banks are meant to be banks that choose an inferior but less costly screening technology; as long as the screening effort cannot be verified, they can compete in loan provisions.

Another simplifying assumption in our paper is the one-period environment. We have used a one-period model deliberately to emphasize the informational externality. With multiple periods, a lending bank may gain private information about the project from the relationship and hence may have informational monopoly power in subsequent periods. Sharpe (1990) and von Thadden (1998) have analyzed banks' lending decisions in this environment. In particular, von Thadden (1998) has shown that the informational externality continues to play an important role in loan competition. In fact, by allowing uninformed banks to bid against informed ones, we have already incorporated a similar type of asymmetry between banks at the bidding stage. For this reason, we expect our main results to hold in subsequent periods with asymmetric banks.[14] Nevertheless, such an extension will nicely integrate the information-externality story in this paper and the monopoly-power story in Petersen and Rajan (1995). Also, anticipating the lending bank's ex post monopoly power, the borrower may want to seek alternative sources of financing such as arm's-length debts (Rajan, 1992).

It is more difficult to relax some other assumptions. One of these is the absence of collusion among banks. If banks can collude more easily in a concentrated banking area than in a competitive banking area, then collusion will reduce the differential between average loan rates in these two areas. Although we do not know a tractable way to model collusion with private information, we want to make two remarks. First, the collusion bias is not unique to our model; the monopoly-power story by Petersen and Rajan (1995) has the same bias. Second, although banks might want to collude to raise the loan rate, it is unlikely that they want to change the loan availability in a systematic way. Thus, the relationship between

[14] Like other bidding models discussed in Subsection 1.1, Sharpe (1990) and von Thadden (1998) have assumed that signals arrive costlessly. By endogenizing the screening decision, one can extend our analysis to incorporate banks that acquire different information over time.

the loan availability and the number of banks is likely to survive collusion among banks.

Another important assumption is that the borrower contacts all banks for loans or, more generally, that the number of banks contacted by the borrower before obtaining a loan is an increasing function of the total number of banks in the area. This assumption is in contrast with the result in Thakor (1996), who shows that the optimal number of contacts may be independent of the total number of banks in the area. However, Thakor's model, like the current version of ours, has only one borrower. Our assumption is intended for a realistic market which has many borrowers. When many borrowers compete against each other for loans, each faces the danger of being left out and of being perceived in the subsequent periods as one that has much worse credit worthiness than the pool of borrowers in the first period. Without coordination among borrowers, it is optimal for each borrower to contact as many banks as possible.

Even without the above assumption and with only one borrower, the winner's curse will work similarly if the borrower applies for a loan sequentially. Suppose, for example, that the borrower contacts only one bank at a time and that the project's true quality is good. The first bank that the borrower contacts has incentive not to make an offer immediately. This is because the bank is concerned that its own signal may be too optimistic; if it waits to observe other banks' funding decisions, the bank can obtain additional information about the project. For the same reason, even if the first bank decides to make an offer immediately, it has incentive to offer a very high loan rate. In both cases the borrower is unlikely to get a desirable loan from the first bank it contacts. If there are more banks, the first bank is more reluctant to make an offer immediately; When it makes an offer, it offers a higher loan rate and so the borrower contacts more banks before getting a satisfactory loan. This informational externality, similar to ours, makes the loan availability possibly decrease with the number of banks in the area.

8. Conclusion

The mere presence of more banks in a market does not imply more available and cheaper loans. The active participation of more banks in loan provisions does. Whether a bank participates in loan provisions depends on the expected profit from such participation. When banks can obtain private information about the project's quality by screening, this profit can fall greatly as the number of banks increases, because the presence of more banks greatly increases the negative informational externality through the winner's curse. In a market with many banks, therefore, each bank's screening probability can be much lower and the number of active competitors can be smaller, making loans less available and the expected loan rate higher than in a market with just a few banks. This is the case when the screening cost is low. The equilibrium is inefficient when the screening cost is low and efficient when the screening cost is high.

Although our model rationalizes the empirical finding by Petersen and Rajan (1995), we do not view the presence of more banks as the cause of inadequate financing for small businesses. Rather, other fundamentals of the loan market such as the screening cost are responsible. We have shown that the screening cost affects the relationship between the loan availability and the number of banks. A sufficient reduction in the screening cost can reduce the loan rate and make more good projects financed.

It will be interesting to test the implications listed in Section 6 in future researches and, in particular, to test the implication that the loan availability differential between differently concentrated areas is cyclical and industry-specific. An important theoretical exercise is to extend the model to a market with many borrowers and multiple periods. In this general environment, banks may offer more complicated mechanisms to borrowers than a simple rationing scheme and a loan rate. Although it is challenging to characterize this mechanism-design problem with competing designers (banks) (see Epstein and Peters, 1996), the research in this direction will yield an integrated theory of loan market competition.

Appendix

A. Proof of Lemma 3.2

First, we show that the supports of the three bid distributions, (F_g, F_u, F_b), do not overlap except for the endpoints. Since the proof is similar, we show the result only for the pair (F_g, F_u). Suppose, to the contrary, that $[r_{gL}, r_{gH}] \cap [r_{uL}, r_{uH}] = [r_1, r_2]$, with $r_1 < r_2$. Then $m_s(r) = m_s(r_2)$ for all $r \in [r_1, r_2]$ and $s = g, u$. Using (2.3), we can rewrite these requirements as

$$\begin{bmatrix} \Pr(q = g|s = g), & -\Pr(q = b|s = g) \\ \Pr(q = g|s = u), & -\Pr(q = b|s = u) \end{bmatrix}$$

$$\times \begin{bmatrix} (yr - 1)W(r|q = g) - (yr_2 - 1)W(r_2|q = g) \\ W(r|q = b) - W(r_2|q = b) \end{bmatrix} = 0.$$

The above coefficient matrix is invertible and so $W(r|q = b) = W(r_2|q = b)$ for all $r \in [r_1, r_2]$. But this cannot hold, since $F_g(r)$ or $F_u(r)$ is strictly increasing in r for some $r \in (r_1, r_2)$, implying that $W(r|q = b)$ is strictly decreasing in r for some $r \in (r_1, r_2)$. Thus, F_g and F_u do not have overlapping supports except for the endpoints.

Next, we show that $r_{uL} = r_{gH}$ if $u > 0$. The proof for $r_{bL} = r_{uH}$ when $b > 0$ is similar. Suppose, to the contrary, that $u > 0$ but $r_{uL} \neq r_{gH}$. We have four cases.

Case (i): $r_{uH} = r_{gL} \equiv r^*$. In this case the support of F_b does not cover (r_{uL}, r_{gH}), since the supports of any two bid distributions cannot overlap. That is, $F_b(r)$ is constant (either 0 or 1) for all $r \in (r_{uL}, r_{gH})$ and we use F_b^* to denote it. When $u > 0$, the payoff to a bank U in this case can be obtained from (2.3) as follows:

$$Em_u(r \in [r_{uL}, r^*]) = (yr - 1)\alpha[1 - (1-p)uF_u(r) - (1-\gamma)pbF_b^*]^{n-1}$$
$$-(1-\alpha)[1 - (1-p)uF_u(r) - \gamma pbF_b^*]^{n-1}.$$

Since Em_u is constant over the support, then $Em'_u(r) = 0$ for all $r \in [r_{uL}, r^*]$. In particular, $Em'_u(r^*) = 0$, which yields

$$F'_u(r^*) = \frac{yA_1^{n-1}}{(n-1)(1-p)u}\left[(yr^* - 1)A_1^{n-2} - \frac{1-\alpha}{\alpha}A_2^{n-2}\right]^{-1}, \tag{A.1}$$

where

$$A_1 = 1 - (1-p)u - (1-\gamma)pbF_b^*; \;\; A_2 = 1 - (1-p)u - \gamma pbF_b^*.$$

There is a profitable opportunity for a bank G to deviate. In particular, consider a deviation by a bank G to bidding $r^* - \varepsilon$, where $\varepsilon > 0$ is arbitrarily small. The payoff is

$$Em_g(r^* - \varepsilon) = [y(r^* - \varepsilon) - 1)]\alpha\gamma[1 - (1-p)uF_u(r^* - \varepsilon) - (1-\gamma)pbF_b^*]^{n-1}$$
$$-(1-\alpha)(1-\gamma)[1 - (1-p)uF_u(r^* - \varepsilon) - \gamma pbF_b^*]^{n-1}.$$

For the deviation to be non-profitable, the limit $\lim_{\varepsilon\to 0}[Em_g(r^* - \varepsilon) - Em_g(r^*)]/\varepsilon$ must be non-positive, which requires

$$F'_u(r^*) \le \frac{yA_1^{n-1}}{(n-1)(1-p)u}\left[(yr^* - 1)A_1^{n-2} - \frac{(1-\alpha)(1-\gamma)}{\alpha\gamma}A_2^{n-2}\right]^{-1}.$$

This is violated, given (A.1).

Case (ii): $r_{uH} < r_{gL}$ and the support of F_b is not a subset of $[r_{uH}, r_{gL}]$. In this case, $F_b(r)$ is again constant (either 0 or 1) for all $r \in (r_{uL}, r_{gH})$. The payoff to a bank U from bidding $r \in [r_{uL}, r_{uH}]$ is

$$Em_u = (yr_{uH} - 1)\alpha[1 - (1-p)u - (1-\gamma)pbF_b^*]^{n-1}$$
$$- (1-\alpha)[1 - (1-p)u - \gamma pbF_b^*]^{n-1}.$$

This is an increasing function of r_{uH}. That is, if a bank U deviates from the distribution F_u and bids slightly higher than r_{uH}, the bid has the same winning probability as r_{uH} does and yet receives a higher profit when it wins. Thus, $r_{uH} < r_{gL}$ cannot be an equilibrium in this case.

Case (iii): $r_{uH} < r_{gL}$ and the support of F_b is a subset of $[r_{uH}, r_{gL}]$. In this case, $r_{bH} \le r_{gL}$, but this cannot hold in equilibrium, as the same arguments in Cases (i) and (ii) can be repeated for the pair (F_g, F_b) to yield a contradiction.

Case (iv): $r_{uH} > r_{gL}$. In this case, we must have $r_{uL} \geq r_{gH}$, since the supports of F_g and F_u cannot overlap except for the endpoints. Since $r_{uL} \neq r_{gH}$ by the supposition, then $r_{uL} > r_{gH}$. If the support of F_b is not a subset of $[r_{gH}, r_{uL}]$, then the argument in Case (ii) shows that there is incentive for a bank G to deviate to a bid slightly higher than r_{gH}. If the support of F_b is a subset of $[r_{gH}, r_{uL}]$, then $r_{bH} \leq r_{uL}$, but this cannot be consistent with an equilibrium since the arguments in Cases (i) and (ii) can be repeated for the pair (F_u, F_b) to yield a contradiction.

Therefore, $r_{gH} = r_{uL}$. This completes the proof of the Lemma. ■

B. Proof of Proposition 3.4

We first show $b = 0$. Suppose, to the contrary, $b > 0$. Then the expected profit for a bank B must be non-negative which, by (2.3), requires

$$(yr_{bL} - 1)\alpha[1 - \gamma p - (1 - p)u]^{n-1}$$

$$\geq \frac{\gamma}{1-\gamma} \cdot \frac{1-\alpha}{\alpha}[1 - (1 - \gamma)p - (1 - p)u]^{n-1}.$$

Substituting this into (2.3) for $s = u$ one can show that $m_u(r_{uH}) > 0$ (note $r_{uH} = r_{bL}$). Thus, $u = 1$. But, when $u = 1$ (and $p > 0$), the above non-negative profit condition for a bank B requires

$$r_{bL} \geq \frac{1}{y}\left[1 + \frac{1-\alpha}{\alpha}\left(\frac{\gamma}{1-\gamma}\right)^n\right],$$

which is infeasible under Assumption 1. Thus, $b = 0$.

Next, we can show $u < 1$ for all $p > 0$: If $u = 1$, one can calculate the expected profit for a bank U from (2.3) and show that it is negative for any $p > 0$ under Assumption 1.

Now if a bank U participates in bidding, it must be indifferent between bidding and not bidding (since $u < 1$). The payoff from bidding must be zero. Substituting $b = 0$ into (2.3) for $s = u$ and setting $m_u(r) = 0$ yields (3.7). Setting $F_u(r_{uL}) = 0$ in (3.7) gives (3.6). Also, since banks B do not bid, $r_{uH} = 1$ by the proof of Lemma 3.2 (Case (ii) there). Setting $r = 1$ in (3.7) and solving for u yields (3.5). Then $u > 0$ if and only if $p < p_A(n)$. When $p < p_A(n)$, the cdf F_u defined implicitly by (3.7) has a positive derivative for all $r \in (r_{uL}, r_{uH})$ and so has a positive density. Moreover, it can be verified that $u(p, n)$ decreases with p and $r_{uL}(p, n)$ increases with p.

Finally, we need to show that there is no incentive for a bank U to deviate from the bid distribution F_u, given that other banks U use F_u and that other banks G use F_g described later in proposition 3.5. The proof for this part follows a similar procedure to that used in Case (i) in the proof of Lemma 3.2. ■

C. Proof of Proposition 4.1

We first verify that T_g has the described dependence on c. Recall that $u > 0$ iff $c > c_A$. When $c > c_A$, substituting u from (3.5) into (3.5) yields $T_g = p^n(\frac{1}{p_A} - \gamma)^n$, which is an increasing function of p and hence a decreasing function of c. When $c < c_A$, substituting $u = 0$ into (4.1) yields $T_g = (1 - \gamma p)^n$, which is a decreasing function of p and hence an increasing function of c.

For the effect of c on R_g, examine first the case $c < c_A$. In this case, setting $u = 0$ and substituting $c = EM_g(p, n)$ into (4.2) yields

$$\begin{aligned} R_g \;=\; & \frac{1-\alpha}{\alpha}\left\{1 + \left[\frac{(1+\gamma z)^n - 1}{z}\right]^{-1}\right. \\ & \times\left[n(y-1)\frac{\alpha\gamma}{1-\alpha} - n(1-\gamma)[1+(2\gamma-1)z]^{n-1}\right. \\ & \left.\left. - \frac{[1+(2\gamma-1)z]^n - 1}{z}\right]\right\}, \end{aligned}$$

where $z = 1/(p^{-1} - \gamma)$ is an increasing function of p. It can be verified that $[(1 + x)^n - 1]/x$ is an increasing function of x for any $x > 0$. Then R_g is a decreasing function of z and hence a decreasing function of p. Since p decreases with c, R_g is an increasing function of c.

For the case $c > c_A$, substituting u from (3.5) into (4.2) and replacing c by the expression for $EM_g(p, n)$ yields

$$R_g = \frac{1-\alpha}{\alpha}\left\{1 + \frac{n(2\gamma-1)(x-1+\gamma)^{n-1} + (x_A-\gamma)^n - (x_A-1+\gamma)^n}{x^n - (x_A-\gamma)^n}\right\},$$

where $x = 1/p$ and $x_A = 1/p_A(n)$. The derivative of R_g with respect to x has the same sign as that of $h(x)$ where

$$\begin{aligned} h(x) \;=\; & n(1-\gamma) - x - (n-1)x^{1-n}(x_A-\gamma)^n \\ & + \frac{(x-1+\gamma)^{2-n}}{2\gamma-1}\left[(x_A-1+\gamma)^n - (x_A-\gamma)^n\right]. \end{aligned}$$

This is clearly negative when $x \to \infty$, i.e., $\mathrm{d}R_g/\mathrm{d}c < 0$ when c is close to but lower than c_0. When $c \downarrow c_A$, substitute $x = x_A = \gamma + (2\gamma - 1)/(Y - 1)$, where $Y = [\alpha(y - 1)/(1 - \alpha)]^{1/(n-1)} > 1$:

$$\begin{aligned} h(x) \;=\; & \frac{(2\gamma-1)}{Y-1}\left(\frac{\gamma}{2\gamma-1}Y + 1\right)^{1-n} \\ & \times\left\{\left[\frac{1-\gamma}{2\gamma-1}(n-1)(Y-1)^2 + Y - Y^{2-n}\right]\right. \\ & \left.\times\left(\frac{\gamma}{2\gamma-1}Y + 1\right)^{n-1} - (n-1)(Y-1)\right\}. \end{aligned}$$

The expression in {.} is a decreasing function of γ for any given $Y > 1$ and so

$$h(x) > \frac{(2\gamma - 1)}{Y - 1}\left(\frac{\gamma}{2\gamma - 1}Y + 1\right)^{1-n}\left[(Y - Y^{2-n})(Y + 1)^{n-1} - (n - 1)(Y - 1)\right]$$

The expression in [.] is an increasing function of Y and has value 0 when $Y = 1$. Thus, $h(x) > 0$, i.e., $\mathrm{d}R_g/\mathrm{d}c > 0$ when c is close to c_A. Thus, there exists $c^* \in (c_A, c_0)$ such that $\mathrm{d}R_g/\mathrm{d}c > 0$ for all $c < c^*$. This completes the proof of Proposition 4.1.

D. Proof of Proposition 4.3

To prove the proposition, we first establish the following lemma:

LEMMA D.1. The following relations hold for all $p > 0$:

$$\gamma[1 - (1 - \gamma)p]\ln[1 - (1 - \gamma)p] - (1 - \gamma)(1 - \gamma p)\ln(1 - \gamma p) < 0, \tag{D.1}$$

$$\frac{\mathrm{d}}{\mathrm{d}p}\left(\frac{\ln[1 - (1 - \gamma)p]}{\ln(1 - \gamma p)}\right) < 0. \tag{D.2}$$

Proof. The left-hand side of (D.1) is a decreasing function of p and has a value zero when $p = 0$. Thus (D.1) is evident. Computing the derivative in (D.2) shows that it has the same sign as that of the left-hand side of (D.1) and so it is negative. ■

We now show that the equilibrium screening probability is a decreasing function of n. Differentiating the equation $EM_g(p, n) = c$, where $EM_g(p, n)$ is given by (3.9), yields

$$\frac{\mathrm{d}p}{\mathrm{d}n} = \frac{1 - (1 - \gamma)p}{(n - 1)(1 - \gamma)}\ln[1 - (1 - \gamma)p], \text{ if } c > c_A \text{ (i.e. } p < p_A), \tag{D.3}$$

$$\frac{\mathrm{d}p}{\mathrm{d}n} = \frac{1}{(n - 1)\Delta}\left\{\begin{array}{l}(y - 1)(1 - \gamma p)\ln(1 - \gamma p) \\ -\frac{(1-\alpha)(1-\gamma)}{\alpha\gamma}\cdot\frac{[1-(1-\gamma)p]^{n-1}}{(1-\gamma p)^{n-2}}\ln[1 - (1 - \gamma)p]\end{array}\right\}, \text{ if } c < c_A, \tag{D.4}$$

where

$$\Delta \equiv (y - 1)\gamma - \frac{(1 - \alpha)(1 - \gamma)^2}{\alpha\gamma}\left(\frac{1 - (1 - \gamma)p}{1 - \gamma p}\right)^{n-2}.$$

Clearly, $\mathrm{d}p/\mathrm{d}n < 0$ for $c > c_A$. When $0 < c < c_A$, $EM_g > 0$ implies

$$y > 1 + \frac{(1-\alpha)(1-\gamma)}{\alpha\gamma}\left(\frac{1-(1-\gamma)p}{1-\gamma p}\right)^{n-1}. \tag{D.5}$$

With (D.1) one can show that $-\mathrm{d}p/\mathrm{d}n$ is an increasing function of y and so (D.5) implies

$$-\frac{\mathrm{d}p}{\mathrm{d}n} > \frac{(1-\gamma p)[1-(1-\gamma)p]}{(n-1)(2\gamma-1)}\{\ln[1-(1-\gamma)p] - \ln(1-\gamma p)\} > 0. \tag{D.6}$$

Thus, $\mathrm{d}p/\mathrm{d}n < 0$ for $c \in (0, c_A)$ as well.

To show the dependence of T_g on n, examine first the case $c < c_A$. Differentiate $T_g = (1-\gamma p)^n$ and substitute (D.6) to obtain:

$$\begin{aligned}\frac{\mathrm{d}T_g}{\mathrm{d}n} &> \frac{n\gamma(1-\gamma p)^n}{(n-1)(2\gamma-1)}[-\ln(1-\gamma p)] \\ &\times\left\{\frac{n-1}{n}\left(\frac{1}{\gamma}-2\right) - [1-(1-\gamma)p]\left[\frac{\ln[1-(1-\gamma)p]}{\ln(1-\gamma p)} - 1\right]\right\}.\end{aligned}$$

Using (D.2) it can be verify that the expression in {.} is an increasing function of p and that its value at $p = 0$ is positive. Thus, $\mathrm{d}T_g/\mathrm{d}n > 0$ for $c < c_A$.

For $c > c_A$, let $\xi = 1/(n-1)$ and $\sigma = \alpha(y-1)/(1-\alpha) > 1$. Solving p as a function of c from $EM_g = c$ and substituting p_A yields

$$\ln T_g = \left(1+\frac{1}{\xi}\right)\left\{\ln\left[1-\left(\frac{c}{c_0}\right)^{\xi}\right] - \ln(\sigma^{\xi}-1) - \ln\left(\frac{1-\gamma}{2\gamma-1}\right)\right\}.$$

The following two properties can be verified: (i) $\mathrm{d}\ln T_g/\mathrm{d}\xi$ is an increasing function of c/c_0; (ii) $\mathrm{d}\ln T_g/\mathrm{d}\xi > 0$ when $c \to c_0$. Thus, there exists $c_1 < c_0$ such that T_g is an increasing function of ξ and hence a decreasing function of n if and only if $c > c_1$. The level c_1 may or may not be greater than c_A. Since the current case is restricted to $c > c_A$, let $c^{**} = \max\{c_1, c_A\}$. Then T_g is an increasing function of n if and only if $c < c^{**}$. This completes the proof of Proposition 4.3. ■

E. Proof of Lemma 5.1

To begin, note that the function $N(p_s)$ defined in (5.2) is a step function. With the function $P(i)$ defined in (5.5), we have

$$N(p_s) = \begin{cases} 0, & \text{if } p_s \in [0, P(0)) \\ i, & \text{if } p_s \in [P(i-1), P(i)) \text{ and } i \in \{1, ..., \bar{N}-1\}, \\ \bar{N}, & \text{if } p_s \in [P(\bar{N}-1), 1]. \end{cases} \tag{E.1}$$

Define

$$J(p) = \alpha(y-1)(1-p\gamma)^n - (1-\alpha)[1-p(1-\gamma)]^n.$$

Let $J^{(i)}(p)$ be the ith order derivative of $J(p)$. The condition (5.1) is equivalent to $(-1)^i J^{(i)}(p_s) \geq 0$. That is, for $i \in \{0, 1, ..., \bar{N}-1\}$,

$$i \geq N(p_s) \Longleftrightarrow p_s \leq P(i) \Longleftrightarrow (-1)^i J^{(i)}(p_s) \geq 0, \tag{E.2}$$

where the equality holds if and only if $p_s = P(i)$.

Next, we rewrite the expected social surplus function $\Delta(p)$ in (5.3). Divide the interval [0, 1] into $(\bar{N}+1)$ sub-intervals as in (E.1) and let $\Delta_i(p)$ be the segment of $\Delta(p)$ for $p \in [P(i-1), P(i))$. Because $N(p_s) = i$ for $p \in [P(i-1), P(i))$, we have

$$\begin{aligned}\Delta_i(p_s) \ &= \textstyle\sum_{n_g=i}^{n} \frac{n!}{n_g!(n-n_g)!}(p_s)^{n_g} \left\{\alpha(y-1)\gamma^{n_g}(1-p_s\gamma)^{n-n_g}\right. \\ &\qquad \left. -(1-\alpha)(1-\gamma)^{n_g}[1-p_s(1-\gamma)]^{n-n_g}\right\}.\end{aligned}$$

Computing the sum $\sum_{n_g=0}^{n}(.)$ and using the functions $J^{(n_g)}(p)$, we have

$$\Delta_i(p_s) = \alpha y - 1 - \sum_{n_g=0}^{i-1} \frac{(-1)^{n_g}}{n_g!}(p_s)^{n_g} J^{(n_g)}(p_s). \tag{E.3}$$

Now, we establish feature (i) in the lemma. Because $J^{(k)}(p)$ is continuous for any non-negative integer k, $\Delta_i(p_s)$ is continuous for $p_s \in (P(i-1), P(i))$. Noticing $J^{(i)}(P(i)) = 0$, we have

$$\begin{aligned}\lim_{p_s \downarrow P(i)} \Delta_{i+1}(p_s) \ &= \alpha y - 1 - \textstyle\sum_{n_g=0}^{i} \frac{(-1)^{n_g}}{n_g!}(P(i))^{n_g} J^{(n_g)}(P(i)) \\ &= \alpha y - 1 - \textstyle\sum_{n_g=0}^{i-1} \frac{(-1)^{n_g}}{n_g!}(P(i))^{n_g} J^{(n_g)}(P(i)) \\ &= \lim_{p_s \uparrow P(i)} \Delta_i(p_s).\end{aligned}$$

That is, the function $\Delta(p_s)$ is also continuous at the endpoints of each sub-interval $(P(i-1), P(i))$. Therefore, $\Delta(p_s)$ is continuous for all $p_s \in [0, 1]$.

For other features in the lemma, we compute the derivative of $\Delta(p_s)$ as

$$\Delta'(p_s) = \begin{cases} 0, & \text{if } p_s \in [0, P(0)) \\ \Delta_i'(p_s), & \text{if } p_s \in [P(i-1), P(i)) \text{ and } i \in \{1, \ldots, \bar{N}-1\}, \\ \Delta_{\bar{N}}'(p_s), & \text{if } p_s \in [P(\bar{N}-1), 1]. \end{cases}$$

The first line in the above expression verifies feature (iii) in the lemma. For $1 \leq i \leq \bar{N}$, we use (E.3) to compute

$$\Delta_i'(p_s) = \frac{(-1)^i}{(i-1)!}(p_s)^{i-1} J^{(i)}(p_s).$$

This is clearly a continuous function in the interior of the sub-interval $(P(i-1), P(i))$. Also, $(-1)^i J^{(i)}(p_s) > 0$ in the interior of the sub-interval (see (E.2)). Thus, $\Delta_i'(p_s) > 0$ for $p_s \in (P(i-1), P(i))$. Finally, because $J^{(i)}(P(i)) = 0$, $\lim_{p_s \uparrow P(i)} \Delta_i'(p_s) = 0 < \lim_{p_s \downarrow P(i)} \Delta_{i+1}'(p_s)$. ■

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