Vulnerable Options, Risky Corporate Bond, and Credit Spread

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In the current literature, the focus of credit-risk analysis has been either on the valuation of risky corporate bond and credit spread or on the valuation of vulnerable options, but never both in the same context. There are two main concerns with existing studies. First, corporate bonds and credit spreads are generally analyzed in a context where corporate debt is the only liability of the firm and a firm's value follows a continuous stochastic process. This setup implies a zero short-term spread, which is strongly rejected by empirical observations. The failure of generating non-zero short-term credit spreads may be attributed to the simplified assumption on corporate liabilities. Because a corporation generally has more than one type of liability, modeling multiple liabilities may help to incorporate discontinuity in a firm's value and thereby lead to realistic credit term

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structures. Second, vulnerable options are generally valued under the assumption that a firm can fully pay off the option if the firm's value is above the default barrier at the option's maturity. Such an assumption is not realistic because a corporation can find itself in a solvent position at option's maturity but with assets insufficient to pay off the option. The main contribution of this study is to address these concerns. The proposed framework extends the existing equity-bond capital structure to an equity-bond-derivative setting and encompasses many existing models as special cases. The firm under study has two types of liabilities: a corporate bond and a short position in a call option. The risky corporate bond, credit spreads, and vulnerable options are analyzed and compared with their counterparts from previous models. Numerical results show that adding a derivative type of liability can lead to positive short-term credit spreads and various shapes of credit-spread term structures that were not possible in previous models. In addition, we found that vulnerable options need not always be worth less than their default-free counterparts. Jrl Fut Mark 21:301–327, 2001

INTRODUCTION

The focus of credit-risk analysis has been either on the valuation of risky corporate bonds and credit spreads or on the valuation of vulnerable options, but never on both in the same context. Risky corporate bonds and credit spreads have been modeled by Black and Scholes (1973) and Merton (1974) and extended by Black and Cox (1976), Longstaff and Schwartz (1995), Briys and de Varenne (1997), and others. Studies of vulnerable options were pioneered by Johnson and Stulz (1987) and subsequently advanced by Hull and White (1995) and Jarrow and Turnbull (1995). The existing studies can be improved in several aspects. First, corporate bonds and credit spreads are generally analyzed in a context where corporate debt is the only liability of the firm and a firm's value follows a continuous stochastic process. The obvious implication of this setup is a zero short-term credit spread, which is strongly rejected by empirical observations (see Fons, 1994; Jones, Mason, & Rosenfeld, 1984; Sarig & Warga, 1989). Because a corporation generally has more than one type of liability and one maturing liability may cause a sharp decrease in a firm's value, modeling multiple liabilities may help to incorporate discontinuity in a firm's value and thereby lead to non-zero short-term credit spreads.

Second, vulnerable options are priced either under the assumption that the option is the only liability of the firm (e.g., Johnson & Stulz, 1987) or under the assumption that the option is fully paid off if the firm's value is above the default barrier at the option's maturity (e.g., Hull & White, 1995). Both assumptions are questionable. On the one hand, most firms have debts in their capital structure, and contingent liabilities (in the form of derivatives) are only part of the total liability. On the other hand, it is not appropriate to evaluate an option's vulnerability by focusing only on technical solvency because a corporation can find itself in a solvent position at the option's maturity but with assets insufficient to pay off the option. For example, suppose the value of the firm's assets until the option's maturity has always been above the default barrier, which is \$40. Further suppose that the firm's value is \$50 at the option's maturity. If the option is \$20 in-the-money, the firm is threshold-solvent but unable to pay \$20 in full to the option holder. The downfall of Barings Bank serves as a convincing illustration: The default on debentures was purely due to the large loss on derivative positions.

Third, risky corporate bonds (or corporate credit spreads) are usually analyzed separately from other liabilities, especially short positions on derivatives. However, many firms take derivative positions and at the same time have outstanding corporate debts. It will be useful to examine how risky corporate bonds affect the valuation of vulnerable derivatives, and vice versa. As a matter of fact, some authors (e.g., Bodnar, Hayt, & Marston, 1998; Howton & Perfect, 1998; Levich, Hyat, & Ripston, 1998) have demonstrated the increasing usage of derivatives by financial and nonfinancial firms. In a wide survey of nonfinancial firms, Bodnar et al. (1998) found that 50 of the responding firms reported the use of derivatives. Of the derivative users, 42 indicated that usage had increased over the previous year (Bodnar et al., 1998, p. 71). Moreover, the implementation of FAS133 requires that firms report their derivative positions at fair value on their balance sheet, which effectively crystallizes the contribution of any short positions on derivatives to a firm's overall capital structure. In addition, as pointed out by a reviewer, firms also take indirect derivative positions. For example, an investment bank may hold an option position in a takeover target; a financing firm frequently issues letters of credit that represents contingent liabilities; a large conglomerate may act as a third-party loan guarantor, which again results in contingent liabilities; and so on.

The main objective of this study is to overcome the aforementioned shortcomings in the existing literature. We propose a framework that incorporates two types of liabilities: a corporate bond that represents the conventional debt of the firm and a short position in a call option that represents contingent liabilities. The default barrier is stochastic with respect to the interest rate and can be a function of either the initial option's value or the market value of the option. Under each default-barrier specification, we also examine two alternative settlement rules. When the default barrier is determined by the market value of the option, the option buyer essentially imposes a mark-to-market style of covenant. The framework is quite general and encompasses many existing models as special cases, including Merton (1974), Black and Cox (1976), Johnson and Stulz (1987), Longstaff and Schwartz (1995), and Briys and de Varenne (1997). Numerical analyses demonstrate that including an additional liability can generate positive short-term credit spreads. Under reasonable parameter values, credit spreads can exhibit upward-shaped, downward-shaped, and hump-shaped term structures. In contrast, within the existing models a downward credit-spread term structure is possible only when the firm is already bankrupt. It is shown that the bond maturity, the moneyness of the option (or significance of the other type of liability), the default-barrier requirement, the correlation between a firm's value, and the optioned stock price all play a role in determining the level and shape of credit-spread curves. Moreover, we show that a vulnerable European option may be worth more than its default-free counterpart if the option holder is paid off according to a particular claim rule on default.

The article is organized as follows. The next section outlines the model settings and discusses possible payoff rules under different scenarios for call holders, bond holders, and shareholders. The Numerical Analysis section presents simulation results for vulnerable options, risky bonds, and credit spreads under reasonable parameters and compares our results with those generated by some of the existing models. The last section concludes the article.

MODEL

Model Setup

Consider a firm that has as liabilities a zero-coupon bond with maturity T_b and face value F and a short position on a call option written on another firm's stock with strike price K and maturity T_c ($<T_b$).[1] We assume a continuous-time economy where financial markets are complete and frictionless so that we can apply Harrison and Kreps' (1979) equivalent martingale pricing principle to price the securities under consideration.

[1]We use a call option as an example. The analysis can also be performed for other types of derivatives.

The short-term riskless interest rate r_t at time t is assumed to evolve according to a Vasicek (1977) type of mean-reverting process under the equivalent probability measure:

$$dr_t = a(b - r_t)dt + \sigma_r dz_{1t} \tag{1}$$

where a, b, and σ_r are constant and z_{1t} is a standard Wiener process. Without a loss of generality, we assume a zero market price of risk. Under the short-rate process, the default-free zero-coupon bond $P(t,T)$ maturing at time T is governed by the following process:

$$\frac{dP(t,T)}{P(t,T)} = r_t dt - \sigma_r \frac{1 - e^{-a(T-t)}}{a} dz_{1t} \tag{2}$$

The firm's asset value under the equivalent probability measure is assumed to follow

$$\frac{dA_t}{A_t} = r_t dt + \sigma_A dz_{2t} \tag{3}$$

where σ_A is the standard deviation of the assets return and z_{2t} is a Wiener process. The correlation between the short-term risk-free rate and asset value is ρ_{Ar}. The firm value is assumed to be independent of the capital structure of the firm. Finally, the price of the optioned stock under the equivalent probability measure is described by the following stochastic process:

$$\frac{dS_t}{S_t} = r_t dt + \sigma_s dz_{3t} \tag{4}$$

where σ_s is the instantaneous volatility and z_{3t} is a standard Wiener process. The correlation between the short-term risk-free rate and the optioned stock price is ρ_{sr}, whereas that between the optioned stock price and the firm value is ρ_{sA}.

The default barrier or threshold level at time $t, \nu(t)$, is specified as

$$v(t) = \begin{cases} \alpha_1 FP(t,T_b) + \alpha_2 C & \forall\ t < T_c \\ \alpha_1 FP(t,T_b) & \forall\ T_c \le t < T_b \\ F & \text{for } t = T_b \end{cases}$$

As soon as the firm value A_t falls below $\nu(t)$, the safety covenant would trigger bankruptcy. Prior to the option's maturity, this threshold represents the sum of the minimum requirements imposed by bond holders and option holders. After the option's expiration, the threshold represents bond holders' minimum requirement. Depending on the specific features of the covenant, C could be the initial value, C_0 or the (varying) market value C_{mkt} of an otherwise default-free option. In the former case, the option holders impose a fixed amount of default protection, whereas in the latter case, the protection amount varies according to the market value of the option, which amounts to mark-to-market margin requirements. The parameters α_1 and α_2 represent the degree of protection for the two classes of liability holders and are assumed to be positive constants. The maximum value of α_1 is 1.0, corresponding to full protection to bond holders. However, α_2 can be bigger than 1.0, especially when the initial value of the option, C_0, is used in the covenant specification. The default barrier is stochastic, with the bond portion depending on the evolution of the short-term risk-free interest rate and the option portion depending on both the interest rate and the optioned stock's price when the covenant is based on C_{mkt}.

Next, we specify two distribution rules on default. The first approach is the proportional distribution rule based on the default-barrier requirements, whereas the second is based on the market values of the corresponding default-free instruments. Under the default-barrier distribution rule, the payoff proportion to the option holder is based on the initial value of the option, except when the default occurs at the option maturity, in which case we use the option's intrinsic value, $\max(S(t_2) - K, 0)$, to calculate the proportions. To allow for potential violations of the strict priority rule at default resolutions, we adjust the payoff proportions for the bond holders and option holders respectively by fractional numbers, γ_1 and γ_2. When $\gamma_1 = 1$ and $\gamma_2 = 1$, there is no leakage and the two classes of creditors receive the full amount to which they are entitled. If $T_{A,\nu}$ is the first passage time for the process A to go through the barrier ν, then $T_{A,\nu} = \inf\{t \geq 0, A_t = \nu(t)\}\ \forall\ 0 < t \leq T_b$ and $T_c < T_b$. The payoffs are summarized in Table I.

The payoffs under Scenarios 1, 3, and 4 are straightforward and easy to understand. However, under Scenario 2, when $A(T_c) \leq \nu(T_c) + [S(T_c) - K]^+$, although the asset value is above the threshold, on the exercise of the call the remaining value of the asset is below the threshold required by the bond holder. This is a case where the firm meets the covenant requirements before the option's maturity but is triggered into default by the exercise of the option. Barings' bankruptcy serves as the best example to illustrate the point. Barings was financially strong and solvent before the huge loss on derivatives trading was revealed. The unfortunate bankruptcy was triggered by the large obligations that resulted from the Nikkei

TABLE I

Payoff Descriptions for Different Scenarios

Scenario 1: Default Prior to Option's Maturity: $T_{A,v} < T_c$

	Default-Barrier Rule	*Market-Value Rule*
Call Holder	$\alpha_2 C \gamma_2$	$w_1 \gamma_2 A(T_{A,v})$
Bondholder	$\alpha_1 FP(T_{A,v}, T_b)\gamma_1$	$(1 - w_1)\gamma_1 A(T_{A,v})$
Shareholder	0	0

Scenario 2: No Default Prior to Option's Maturity

	$A(T_c) \le v(T_c)$ (*Default*)		$A(T_c) > v(T_c)$		
			$A(T_c) \le v(T_c) + [S(T_c) - K]^+$ (*Default*)		$A(T_c) > v(T_c) + [S(T_c) - K]^+$
	Default Barrier Rule	Market Value Rule	Default Barrier Rule	Market Value Rule	
Callholder	$w_2 \gamma_2 A(T_c)$	$w_3 \gamma_2 A(T_c)$	$w_2 \gamma_2 A(T_c)$	$w_3 \gamma_2 A(T_c)$	$[S(T_c) - K]^+$
Bondholder	$(1 - w_2)\gamma_1 A(T_c)$	$(1 - w_3)\gamma_1 A(T_c)$	$(1 - w_2)\gamma_1 A(T_c)$	$(1 - w_3)\gamma_1 A(T_c)$	
Shareholder	0	0	0	0	

Scenario 3: Default Prior to Bond Maturity: $T_c < T_{A,v} < T_b$

Bondholder	$A(T_{A,v})\gamma_1$
Shareholder	0

Scenario 4: No Default Prior to Bond Maturity

	$A(T_b) < F$	$A(T_b) > F$
Bondholder	$A(T_b)\gamma_1$	F
Shareholder	0	$A(T_b) - F$

Note: $\max[S(T_c) - K, 0] = [S(T_c) - K]^+$. $w_1 = C_{mkt}/(FP(T_{A,v}, T_b) + C_{mkt})$ $w_2 = \alpha_2 C/(\alpha_1 FP(T_c, T_b) + \alpha_2 C)$. $w_3 = [S(T_c) - K](FP(T_c, T_b) + [S(T_c) - K]^+)$. When the default barrier is based on the initial option value, $C = C_0$; when the default barrier is based on the market value of the option, $C = C_{mkt}$.

index futures. Our article represents the initial effort in addressing the consequences of such a possible outcome on the credit spreads and vulnerable option values. We show in the numerical analysis that incorporating this possible outcome helps us to generate positive short-term credit spreads and a downward sloping term structure of credit spreads, which the existing models can generate only if the firm is already insolvent.

Discussion

We have specified the option's maturity to be shorter than the bond's, which by and large reflects reality.[2] One could easily study a case where the opposite applies. The only conceivable practical scenario corresponding to this setup would be a case where a large amount of corporate debt is maturing. In such a case, debt default is a less important issue given the imminent maturity, but there could be implications for vulnerable options. Such a case is straightforward to study but is omitted for brevity.

An acute reader may realize that we do not explicitly specify a recovery rate as many other authors have done. In our model, prior to the option's maturity, the recovery rate is stochastic and endogenous and dependent on the interest rate and the severeness of the bankruptcy. This is an important improvement over the existing literature, which mostly employs a constant exogenous recovery rate. (As apparent in Table I, the recovery rate beyond the option's maturity will be $\alpha_1\gamma_1$ before the debt's maturity and between $\alpha_1\gamma_1$ and 1.0 at the debt's maturity.)

Several additional properties of our setup need to be pointed out. First, when the covenant is based on the market value of the option, C_{mkt}, the two distribution rules become identical if $\alpha_1 = \alpha_2$. In the general case where $\alpha_1 < \alpha_2$, it can be seen that the market-value-based distribution rule would be favorable to bond holders and detrimental to option holders in comparison with the barrier-based distribution rule. Second, as long as $\alpha_1 \neq 1$, the bond will always be risky, regardless of the level of other parameters. This mimics reality well. Third, under the default-barrier-based distribution rule and the market-value (of option) -based covenant, the option would be completely default-free if $\gamma_2 = 1$ and $\alpha_2 = 1$. This is as if the option is marked to market continuously and margin deposits are adjusted accordingly. With over-the-counter (OTC) derivatives, this is highly unlikely, although counterparties have lately moved toward fuller protection in the context of credit-risk management. Our setup is capable of accommodating this reality. For example, under the market-value-based distribution rule, even if $\gamma_2 = 1$ and $\alpha_2 = 1$, the call option is still vulnerable. This situation is equivalent to the following: Daily revaluation of the option is indeed performed and the covenant requirement is readjusted, but margin deposits are either not required or not adjusted as frequently as the revaluation. This is a realistic scenario in light of the widespread implementation of Value at Risk (which means

[2]Survey results from Bodnar et al. (1998) indicate that most firms use derivatives with short maturities. For example, 77% of the foreign currency derivatives being used have an original maturity of 91 to 180 days.

frequent reevaluation and reassessment of counterparty risk) and the nonexistence of an established margin mechanism for OTC products.

Because of its general and realistic features, our model can be reduced to many previous models as special cases. For vulnerable options, setting the default barrier to zero (by setting $F = 0$ and $\alpha_2 = 0$) and the priority parameter to one ($\gamma_2 = 1$) leads to the model by Johnson and Stulz (1987). For risky debts, our model encompasses several existing models. First, our setup reduces to the simple framework of Merton (1974) by setting the option position to zero ($S = 0$) and removing the bond covenant protection (by setting $\alpha_1 = 0$).[3] Second, the model of Black and Cox (1976) assumes a constant risk-free interest rate and no violation of the strict priority rule, which can be achieved in our model by setting the option position to zero ($S = 0$) and $a = 0$, $\sigma_r = 0$, and $\gamma_1 = 1$. Third, Longstaff and Schwartz (1995), using the same stochastic processes as in Equations 1 and 3, derived a closed-form formula for the risky bond with a constant default barrier. In our framework, this is equivalent to setting the option position to zero ($S = 0$) and the default barrier to a constant, $\nu(t) = K_0$ ($\forall\, t \leq T_b$). Finally, Briys and de Varenne (1997) extended the model by Longstaff and Schwartz by allowing the default barrier to vary according to the market value of an otherwise risk-free bond. In spirit, our model is close to Briys and de Varenne's, although they studied risky bonds only. We can obtain their model by setting $S = 0$. None of the aforementioned models can generate non-zero short-term spreads and a downward sloping credit term structure for a solvent firm, although such features have been observed empirically (Fons, 1994; Sarig & Warga, 1989) (To generate a positive short term credit spread, Zhou (1998) uses a mixed jump-diffusion process to model firm's value while maintaining debts as the sole liability of the firm).

Once we specify the stochastic processes and the payoff rules, the value of the vulnerable option and the defaultable bond can be expressed by way of risk-neutral discounting. Unfortunately, the valuation expressions do not admit closed-form solutions because of the complex recovery-rate specifications or the distribution rules. As a result, we must resort to Monte Carlo simulations, which are delineated in the following section.

[3]Throughout the discussions here, setting the option position to zero means $S = 0$, which is different from $\alpha_2 = 0$. In the latter case, although the option's position does not affect the default barrier, paying off the option at maturity will alter the asset value process and, therefore, the bond-default dynamics.

NUMERICAL ANALYSIS

The stochastic processes for the firm value, risk-free interest rate, and stock price are discretized into daily intervals under the assumption of 250 days in a year. Correlated paths of the three variables and that of the default barrier are then generated. At the beginning of each daily interval, the firm value is compared against the default barrier. If default occurs, the two classes of liability holders are paid off according to the prespecified distribution rule, and a new run is started, and so on. Each numerical estimate is based on 10,000 runs. To reduce the simulation variance, we employ both the antithetic method and the control variate technique.

To apply the antithetic method, for each regular path of the variable in question, we generate a companion path that uses the same random innovations as used by the regular path, but we reverse the signs. For each run, the simulated value in question is simply the average of the two simultaneously generated values.[4] To apply the control variate technique, we need to choose a control variate for the risky bond and the vulnerable option. For the risky bond, the control variate is the defaultable bond studied by Briys and de Varenne (1997) that admits a closed-form formula; for the vulnerable option, the control variate is the default-free counterpart whose formula was derived by Merton (1973). The difference between the theoretical value and the simulated value of the control variate will then be used to adjust the simulated value of the instrument in question. For details on the two variance reduction techniques, please consult Hull (2000).

The parameter values for the simulation are chosen such that they are consistent with empirical observations. Without a loss of generality, we scale the current value of the firm to $A_0 = 100$. The instantaneous volatility of the asset value or firm value is set at $\sigma_A = 0.2$, consistent with empirical findings of Jones et al. (1984) and the discussions in Leland and Toft (1996). For the interest-rate process, the mean-reversion speed and the instantaneous volatility are set according to estimation results of Chan, Karolyi, Longstaff, and Sanders (1992) and are consistent with the values used by Briys and de Varenne (1997). Specifically, $a = 0.2$ and $\sigma_r = 0.02$. The spot interest rate and the long-run reversion target are both set at 10% ($r_0 = b = 10\%$) to produce roughly a flat term structure of the risk-free rate, which will simplify interpretations of the risky term structure. The instantaneous volatility of the stock return is σ_S

[4]Unlike a typical simulation where the antithetic path always mirrors the regular path, in our setting the two paths may terminate at different times because we are essentially simulating the first passage time. Nonetheless, the mirroring effect or variance reduction is achieved at least for the time period when both paths are in the solvent region.

= 0.25, the level for a typical stock. In accordance with Footnote 3, the option's maturity T_c and the bond maturity T_b are set at 0.5 and 5 years, respectively, for the base case, and other values are examined in a comparative analysis. The current stock price S_0 is set at 100 so that when the option is at-the-money, it is about 10% of the firm's asset value. Realizing that equity and asset values tend to be negatively correlated with interest rates, we set the three correlations at $\rho_{sA} = 0.3$, $\rho_{sr} = -0.4$, and $\rho_{Ar} = -0.25$, consistent with Briys and de Varenne. The base case values for the covenant protection parameters are $\alpha_1 = 0.85$ and $\alpha_2 = 1.00$, and the priority rule parameters are $\gamma_1 = 0.90$ and $\gamma_2 = 1.00$. Finally, the face value of the bond is set such that the debt ratio is 50% initially: $F = 0.5A_0/P(0,T_b)$.

To enhance our insight, we compare our results with those of the existing models. For risky bonds and credit spreads, because our model is a direct extension of Briys and de Varenne (1997), who in turn extended the previous models, we include the credit spreads from their model for comparison. The credit spread generated in our model will always be larger than that in Briys and de Varenne because we introduce another liability, the option. For vulnerable options, because our framework is a direct extension of Johnson and Stulz (1987), we use their model for comparison. In the framework of Johnson and Stulz, the vulnerable option only depends on the firm value dynamic and is assumed to be unaffected by the capital structure of the firm. Therefore, our model extends that of Johnson and Stulz in three ways. First, we allow default prior to the option's maturity; second, we allow bonds to compete with the option for payoffs at the option's maturity; and third, we allow a stochastic interest rate. Because the last extension is a straightforward generalization of the constant interest-rate assumption, we assume that it is already a feature of Johnson and Stulz. In other words, when we refer to Johnson and Stulz, we mean their model implemented with a stochastic interest rate.

We examine various combinations of modeling parameters to fully understand the behavior of vulnerable options and credit spreads. Schematically, we first start with two main dimensions: whether the option component of the default barrier ($\alpha_2 C$) is based on the initial option value C_0 or the market value C_{mkt}. With each barrier specification, two distribution rules at default are possible, either default-barrier-based or market-value-based, as outlined in Table I. For each combination, we examine how vulnerable options and credit spreads are affected by such variables as bond maturity, capital structure, extent of covenant protection, and various correlations. For similar results, we report only representative data for brevity. Considering that daily mark-to-market and margin requirements are not yet an established practice for OTC derivatives, we start first with the barrier specification based on the initial value of the option. Later, in the section of Default Barrier Based on the Market Value of the Option, we study the market-value-based default barrier and compare the two.

Default Barrier Based on the Initial Option Value

Here the two distribution rules, the default-barrier rule and the market-value rule, generate very similar results. For brevity, we only present the results under the default-barrier rule.

Vulnerable Options and Term Structure of Credit Spreads

We first study how the credit spreads vary across maturities and simultaneously examine the value of the vulnerable option. To this end, in Exhibit 1 we vary the bond maturity from 1 to 10 years and accordingly adjust the face value of the bond so that the debt ratio remains at 0.5. Given our assumption of a firm's value invariance to the capital structure, the sum of the defaultable bond value, the vulnerable option value, and the equity value is always $A_0 = 100$ if there is no leakage. Whenever there is a leakage (i.e., $\gamma_1 \neq 1$), we credit the amount of the bond payoff leakage to equity value so that the conservation of value is obtained. This is done mainly as a way of double-checking the numerical accuracy.

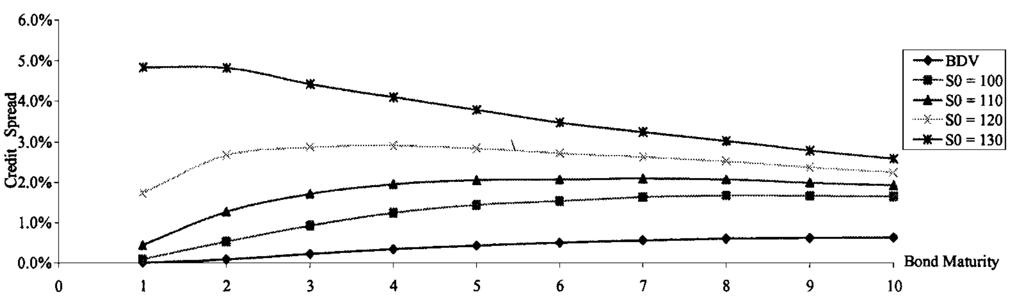
The effects of debt maturity are shown in Panels A and B, where the option's maturity is fixed at 0.5 years. We made several interesting observations. First, the default-free call and the vulnerable call based on Johnson and Stulz (1987) are both independent of the bond maturities, as they should be. Second, in both panels the value of the default-free call is higher than that of the vulnerable call based on Johnson and Stulz, which in turn is higher than the vulnerable call's value generated from our model. The option is the most vulnerable in our model because default can occur not only at option's maturity but also at any time before the option's maturity. Although the results make intuitive sense, as we see later, the vulnerable option in our model can be worth more than its counterpart in Johnson and Stulz, or even its default-free counterpart. Nonetheless, both panels show that the vulnerable option's value is not very sensitive to the bond maturity. Third, the credit spread from our model is higher than its counterpart from Briys and de Varenne (1997),

EXHIBIT 1

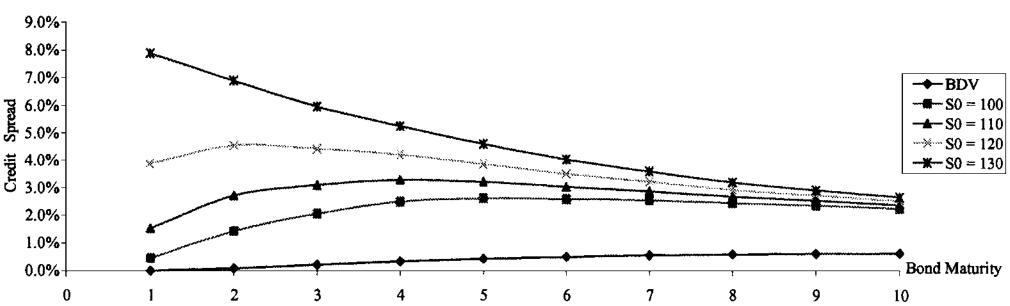
Vulnerable Options and Term Structure of Credit Spreads

Bond Maturity (years)	*Bond Value*	*Call Value*	*Equity Value*	*Risk-free Yield*	*Credit Spread_CW*	*Credit Spread_BDV*
Panel A: $S_0 = 100, K = 100, C_{df} = 9.5323, C_{js} = 9.5176$						
1	49.7759	9.5002	40.7300	9.99%	0.45%	0.00%
2	49.4872	9.4992	41.0200	9.98%	0.52%	0.08%
3	49.0633	9.4983	41.4400	9.96%	0.63%	0.21%
4	48.5807	9.4975	41.9300	9.94%	0.72%	0.33%
5	48.0878	9.4969	42.4200	9.92%	0.78%	0.42%
10	45.9594	9.4948	44.5500	9.81%	0.85%	0.63%
Panel B: $S_0 = 120, K = 100, C_{df} = 25.6847, C_{js} = 25.6441$						
1	48.0989	25.1400	26.7700	9.99%	3.88%	0.00%
2	47.4139	25.1327	27.4600	9.98%	2.66%	0.08%
3	46.7585	25.1264	28.1200	9.96%	2.23%	0.21%
4	46.1598	25.1211	28.7300	9.94%	2.00%	0.33%
5	45.6224	25.1165	29.2700	9.92%	1.83%	0.42%
10	43.6634	25.1024	31.2400	9.81%	1.36%	0.63%

Credit Spread: Option's Maturity is 1/4 of the Bond's Maturity



Credit Spread: Option's Maturity is 1/2 of the Bond's Maturity



Note: Unless otherwise specified, all results were computed with $S_0 = 100$, $K = 100$, $A_0 = 100$, $r_0 = 10\%$, $a = 0.2$, $b = 0.1$, $\sigma_r = 0.02$, $\sigma_A = 0.2$, $\sigma_S = 0.25$, $\rho_{sr} = -0.4$, $\rho_{sA} = 0.3$, $\rho_{Ar} = -0.25$, $\alpha_1 = 0.85$, $\alpha_2 = 1.00$, $\gamma_1 = 0.90$, $\gamma_2 = 1.00$, $T_c = 0.5$, and a debt ratio of 0.5. Spread_CW is the spread generated from our model, Spread_BDV is the spread generated from the model by Briys and de Varenne (1997), C_{df} is the default-free call and C_{js} is the vulnerable call based on Johson and Stulz (1987).

as expected. When the option is in-the-money, the credit spread is much larger, which makes intuitive sense. In this case, the model by Briys and de Varenne will significantly underestimate the credit spreads. The importance of incorporating nondebt liabilities becomes obvious. Fourth, the shape of the credit-spread term structure depends on the moneyness of the option. It is upward sloping when the option is at-the-money and downward sloping when the option is deep in-the-money.[5] As discussed earlier, in previous models, including that of Briys and de Varenne, a downward sloping term structure of credit spreads is possible only when the firm is already insolvent. In our model, a short position on an option can easily lead to the empirically observed result.

These finding begs a natural question: What if the option's maturity increases with the bond's? To answer this question, we rerun the simulations for two cases: to maintain the option's maturity at one quarter of the bond's in one case and at one half of the bond's in the other. As the two figures in Exhibit 1 show, sizable short-term credit spreads are still present when the option is in-the-money. For example, when the option's maturity is maintained at one quarter of the bond's, the 1-year credit spread is around 5 when the stock price is 130. In addition, the option's moneyness also determines the varieties of the term structure's shape. Downward sloping, upward sloping, and humped structures are all observed. In contrast, the term structure generated from Briys and de Varenne (1997) is always upward sloping. Again, given the empirical observations of flat or downward sloping credit-spread term structures by Saig and Warga (1989) and Fons (1994), our model has proved its versatility and potential.

Effects of the Capital Structure

In our model, capital structure or debt ratio is defined as the ratio of the market value of an otherwise risk-free bond over the current firm value: $F \times P(0,T_b)/A_0$. To study its impact, we repeat the calculations for Panel A in Exhibit 1 for different debt ratios and report the results in Exhibit 2. The credit spread is very small when the debt ratio is 0.3. When the debt ratio is 0.7, credit spreads are sizeable in both models. Here, our model generates a downward sloping credit-spread term structure, which

[5]When we perform simulations for in-the-money cases, we set the initial barrier using the value of an at-the-money option. This also holds for all subsequent simulations involving different moneyness situations. Essentially, we are simulating situations where the barrier was set some time ago and the option's moneyness has since changed. In addition, we omit the results for out-of-the-money cases because our model generates virtually the same results as those of Briys and de Varenne (1997) because of the minimal effect of the option's position.

EXHIBIT 2

Effects of the Capital Structure

Bond Maturity (years)	*Bond Value*	*Call Value*	*Equity Value*	*Risk-free Yield*	*Credit Spread_CW*	*Credit Spread_BDV*
Panel A: Debt Ratio = 0.3, S_0 = K = 100, C_{df} = 9.5323, C_{js} = 9.5176						
1	29.9863	9.5266	60.4900	9.99%	0.05%	0.00%
2	29.9777	9.5264	60.5000	9.98%	0.04%	0.00%
3	29.9596	9.5262	60.5200	9.96%	0.04%	0.00%
4	29.9249	9.5260	60.5500	9.94%	0.06%	0.01%
5	29.8694	9.5259	60.6100	9.92%	0.09%	0.02%
10	29.3388	9.5255	61.1400	9.81%	0.22%	0.14%
Panel B: Debt Ratio = 0.7, S_0 = K = 100, C_{df} = 9.5323, C_{js} = 9.5176						
1	67.5925	9.4383	23.0600	9.99%	3.50%	0.75%
2	65.8745	9.4297	24.7900	9.98%	3.04%	1.36%
3	64.4841	9.4227	26.1900	9.96%	2.74%	1.56%
4	63.3781	9.4171	27.3100	9.94%	2.48%	1.60%
5	62.4759	9.4127	28.2100	9.92%	2.27%	1.58%
10	59.6752	9.4010	31.0400	9.81%	1.60%	1.30%

Panel C: Changing Debt Ratio With the Bond Maturity Fixed at 5 Years S_0 = K = 100, C_{df} = 9.5323, C_{js} = 9.5176

Debt Ratio	*Bond Value*	*Call Value*	*Equity Value*	*Risk-Free Yield*	*Credit Spread_CW*	*Credit Spread_BDV*
0.1	9.9992	9.5317	80.4700	9.92%	0.00%	0.00%
0.2	19.9869	9.5300	70.4900	9.92%	0.01%	0.00%
0.3	29.8694	9.5259	60.6100	9.92%	0.09%	0.02%
0.4	39.3612	9.5167	51.1300	9.92%	0.32%	0.14%
0.5	48.0878	9.4969	42.4200	9.92%	0.78%	0.42%
0.6	55.8054	9.4567	34.7500	9.92%	1.45%	0.91%
0.7	62.4759	9.4127	28.2100	9.92%	2.27%	1.58%
0.8	68.1251	9.5047	22.4200	9.92%	3.21%	2.36%
0.9	73.0600	9.8121	17.1700	9.92%	4.17%	3.20%

Note: Unless otherwise specified, all results were computed with $S_0 = 100$, $K = 100$, $A_0 = 100$, $r_0 = 10\%$, $a = 0.2$, $b = 0.1$, $\sigma_r = 0.02$, $\sigma_A = 0.2$, $\sigma_S = 0.25$, $\rho_{sr} = -0.4$, $\rho_{sA} = 0.3$, $\rho_{Ar} = -0.25$, $\alpha_1 = 0.85$, $\alpha_2 = 1.00$, $\gamma_1 = 0.90$, $\gamma_2 = 1.00$, $T_c = 0.5$, and $T_b = 5$. Spread_CW is the spread generated from our model, Spread_BDV is the spread generated from the model by Briys and de Varenne (1997), C_{df} is the default-free call and C_{js} is the vulnerable call based on Johson and Stulz (1987).

is in contrast to the upward sloping curve apparent in Panel A of Exhibit 1 with a debt ratio of 0.5. This implies that capital structure determines not only the level but also the shape of the credit term structure. Moreover, with a debt ratio of 0.3, the credit term structure is U-shaped in our model but upward sloping in Briys and de Varenne (1997); with a debt ratio of 0.7, the term structure is humped in Briys and de Varenne. In Panel C of Exhibit 2, we vary the debt ratio between 0.1 and 0.9. It is seen that the value of the vulnerable option generally decreases and the credit spread increases, as expected. However, we notice a peculiar phenomenon at a debt ratio of 0.9, whereby the vulnerable option's value is higher than its default-free counterpart and its vulnerable counterpart in Johnson and Stulz (1987). The price inversion with respect to the vulnerable counterpart in Johnson and Stulz also exists when the debt ratio is 0.1, 0.2, and 0.3. We later show in Exhibit 6 that it is entirely possible for a vulnerable option in our model to be worth more than its default-free counterpart.

Degree of Covenant Protection

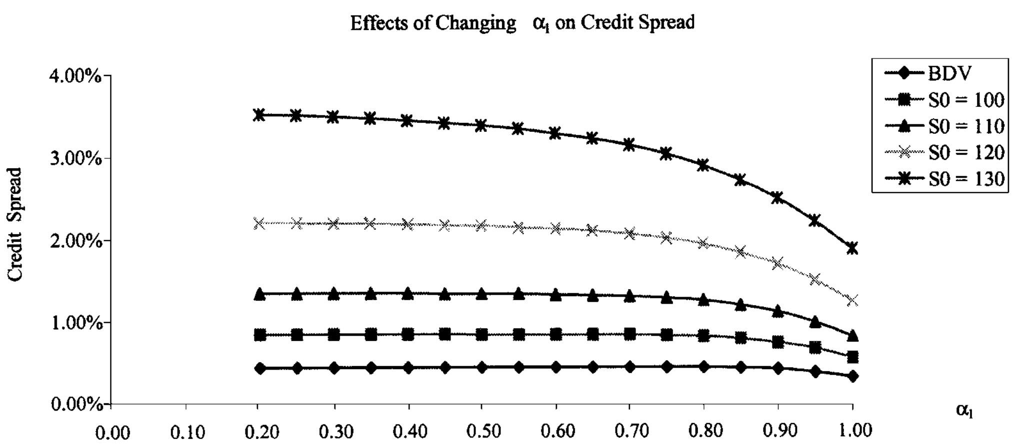
Recall that the strictness of the covenant is captured by the two parameters, α_1 and α_2. Because derivatives' position (as a percentage of the firm's asset value) is usually smaller than debt's, we still set $\alpha_2 = 1$ and only examine the effect of varying α_1. Exhibit 3 reports the results. In both models, the credit spread decreases as the degree of protection increases. When full protection is in place, the credit spread is not zero in either model because of the violation of the strict priority rule (i.e., $\gamma_1 = 0.9$). In our model, even if $\alpha_2 = 1$ and $\gamma_1 = 1.0$, the bond is still risky because default can be triggered by the option's position, in which case bond holders' proportional claim to the asset value may be less than the market value of the risk-free bond if the option is deep in-the-money. Similar observations can be made for other moneyness situations, for which we have plotted at the bottom of the exhibit the credit-spread term structures. The credit spread is almost constant for most of the moneyness situations when α_1 is not very high. This is especially true in the framework of Briys and de Varenne (1997). Intuitively, when the default barrier is lower than a certain threshold, the possibility of default before the option's maturity is almost zero (and, hence, α_1 ceases to play a role), and the spread is almost entirely due to defaults at the option's maturity. For Briys and de Varenne, the constant spread is almost entirely due to the violation of the strict priority rule.

In the framework of Johnson and Stulz (1987), the vulnerable option is not affected by the bond covenant protection. However, in our model it is seen that the vulnerable option's value goes down as the bond covenant protection increases. This is a result of a subtle trade-off. When the bond covenant protection increases, the overall default barrier is raised, and as a result, both the bond holders and option holders enjoy better protection against dramatic losses. However, because the option holders' covenant protection remains unchanged, an increase in α_1 means a higher claim proportion for bond holders (and a lower proportion for

EXHIBIT 3

Degree of Covenant Protection (α_1)

α_1	*Blood Value*	*Call Value*	*Equity Value*	*Risk-Free Yield*	*Credit Spread_CW*	*Credit Spread_BDV*
S_0 = 100, K = 100, C_{df} = 9.5323, C_{js} = 9.5176						
0.20	47.9515	9.5315	42.5200	9.92%	0.84%	0.43%
0.25	47.9519	9.5311	42.5200	9.92%	0.84%	0.43%
0.30	47.9525	9.5305	42.5200	9.92%	0.84%	0.43%
0.35	47.9531	9.5298	42.5200	9.92%	0.84%	0.43%
0.40	47.9540	9.5288	42.5200	9.92%	0.84%	0.43%
0.45	47.9550	9.5277	42.5200	9.92%	0.84%	0.43%
0.50	47.9564	9.5263	42.5200	9.92%	0.83%	0.43%
0.55	47.9583	9.5243	42.5200	9.92%	0.83%	0.43%
0.60	47.9612	9.5218	42.5200	9.92%	0.83%	0.43%
0.65	47.9659	9.5189	42.5200	9.92%	0.83%	0.43%
0.70	47.9746	9.5151	42.5200	9.92%	0.83%	0.43%
0.75	47.9916	9.5103	42.5000	9.92%	0.82%	0.43%
0.80	48.0250	9.5044	42.4800	9.92%	0.81%	0.43%
0.85	48.0878	9.4969	42.4200	9.92%	0.78%	0.42%
0.90	48.1988	9.4878	42.3200	9.92%	0.73%	0.41%
0.95	48.3846	9.4762	42.1500	9.92%	0.66%	0.37%
1.00	48.6705	9.4617	41.8800	9.92%	0.54%	0.31%



Note: Unless otherwise specified, all results were computed with S_0 = 100, K = 100, A_0 = 100, r_0 = 10%, a = 0.2, b = 0.1, σ_r = 0.02, σ_A = 0.2, σ_S = 0.25, ρ_{sr} = −0.4, ρ_{sA} = 0.3, ρ_{Ar} = −0.25, α_2 = 1.00, γ_1 = 0.90, γ_2 = 1.00, T_c = 0.5, T_b = 5, and a debt ratio of 0.5. Spread_CW is the spread generated from our model, Spread_BDV is the spread generated from the model by Briys and de Varenne (1997), C_{df} is the default-free call, and C_{js} is the vulnerable call based on Johson and Stulz (1987).

option holders) at default. It is apparent from the exhibit that the latter effect dominates the former. An immediate implication is that option holders should not only ensure a higher overall default barrier but also ensure that their claim in case of default is commensurate with the default barrier.

Effects of the Correlation between the Firm Value and the Optioned Stock Price (ρ_{sA})

Both option holders and bond holders should be concerned about how the firm value and the optioned stock price are correlated. For option holders, a knowledge of the impact of the correlation on the option value can help them structure the covenant initially; it is also true for bond holders in that they have equal claim priority as the option holders. To gain some insight, we repeat the previous simulations for two additional values of ρ_{sA}, 0.0 and -0.3, and plot the results in Exhibit 4, together with the case for $\rho_{sA} = 0.3$.

First, we notice the usual patterns whereby the credit term structure generally slopes upward when the option is at-the-money or near-the-money and downward when the option is deep in-the-money. Second, as the correlation moves toward zero and becomes more negative, the credit spread becomes bigger for a particular maturity. Although not reported, the vulnerable option's value goes down as the correlation becomes negative. The results make intuitive sense. When the firm value and stock price are negatively correlated, a higher stock price is likely to be accompanied by a lower firm value, in which case both groups of liability holders stand to lose more than otherwise in case of default. However, a positive correlation will on average ensure that any gain on option values is supported by an increase in the firm value, which will make default less likely, and as a result, both types of liability holders benefit. This implies that a relatively lax covenant is warranted if the firm value is positively correlated with the optioned stock price, and vice versa.

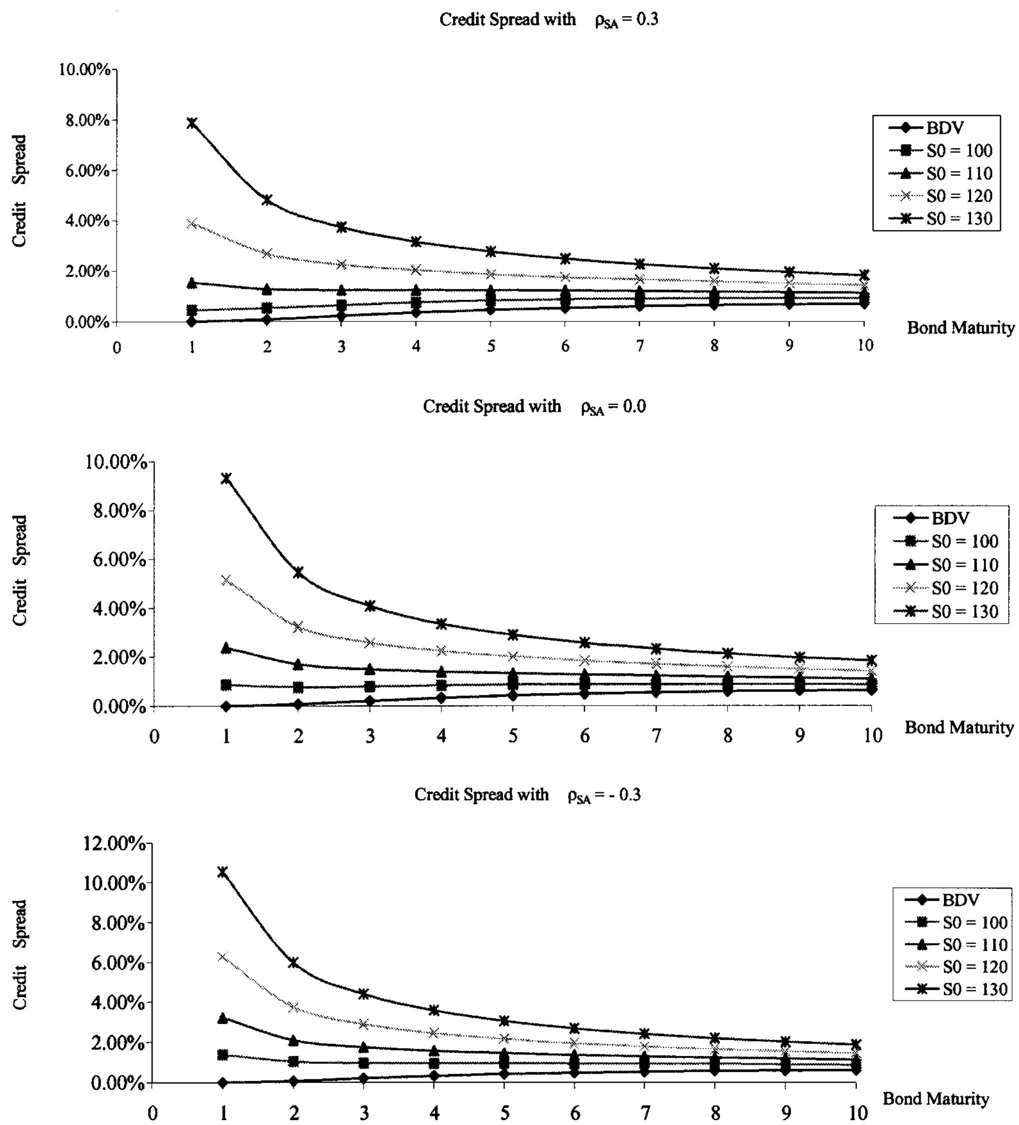
It is also apparent from the figures that the impact of the correlation diminishes as the bond maturity increases. This makes intuitive sense because the option's maturity is fixed at 0.5 years. As the bond maturity increases, the effect from the option's position is being spread out. Notice that the credit spread in Briys and de Varenne (1997) is independent of ρ_{sA}.

Effects of the Correlation between the Firm Value and the Interest Rate (ρ_{Ar})

Insofar as credit spread is jointly affected by the behavior of the firm value and the interest rate, it would be useful to see how the comovement of

EXHIBIT 4

Effects of the Correlation between the Firm Value and the Optioned Stock Price (ρ_{sA})



Note: Unless otherwise specified, all results were computed with $S_0 = 100$, $K = 100$, $A_0 = 100$, $r_0 = 10\%$, $a = 0.2$, $b = 0.1$, $\sigma_r = 0.02$, $\sigma_A = 0.2$, $\sigma_S = 0.25$, $\rho_{sr} = -0.4$, $\rho_{Ar} = -0.25$, $\alpha_1 = 0.85$, $\alpha_2 = 1.00$, $\gamma_1 = 0.90$, $\gamma_2 = 1.00$, $T_c = 0.5$, and a debt ratio of 0.5. BDV is the credit spread based on the model of Briys and de Varenne (1997).

the two variables affects the credit spreads and vulnerable options. To this end, we repeat previous simulations for various levels of ρ_{Ar} and report the result in Exhibit 5. The correlation ρ_{Ar} does not affect the shape of the credit term structure in any significant way, although it does slightly affect the level of credit spreads, as it affects the vulnerable option's value. Specifically, as the correlation moves from negative to positive, the vulnerable option's value in our model decreases, whereas the credit spread increases or, equivalently, the bond price decreases. Let us first examine the bond. With a positive correlation, when the interest rate is high, the firm value tends to be high, but the bond value is low. In this case, the higher firm value does not benefit bond holders very much because their claim is lower anyway. However, when the interest rate is low, the bond value is high and the firm value tends to be low, and default is more likely. If default does occur, bond holders stand to lose more because of the lower firm value. Therefore, a positive correlation between firm value and interest rate leads to situations where helps are bountiful when not required and scarce when needed.

As for the vulnerable options in our model, it is first to be recognized that a higher interest rate leads to a higher call option value, other things being equal. However, we have specified a negative correlation between the stock price and the interest rate (i.e., $\rho_{sr} = -0.4$), which means a higher interest rate tends to be associated with a lower stock price and, hence, a lower option value. If the second effect dominates the first, we tend to see a lower option value associated with a higher interest rate. Now, a positive correlation between the firm value and interest rate would imply a higher firm value with a higher interest rate, and vice versa. Combining these, we obtain an explanation for the pattern of the vulnerable option's value similar to that for the bond's: Helps are not needed when available and are absent when desired. This is why we see a downward pattern in the option price when the correlation increases. Finally, within the model of Johnson and Stulz (1987), the vulnerable option's value goes up slightly as the correlation increases, and this is because the first effect dominates in this case because there is no interim default.

Closer Examination of Vulnerable Options

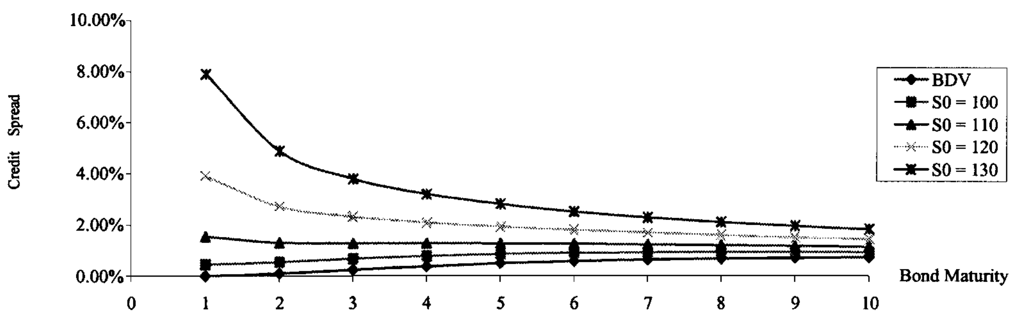
Up to this point, we have been studying vulnerable options and defaultable bonds when the option is either at-the-money or in-the-money and the option's maturity is relatively short. In those cases, the option component of the default barrier is either close to or lower than the market value of the option (most of the time). When default does occur, option

EXHIBIT 5

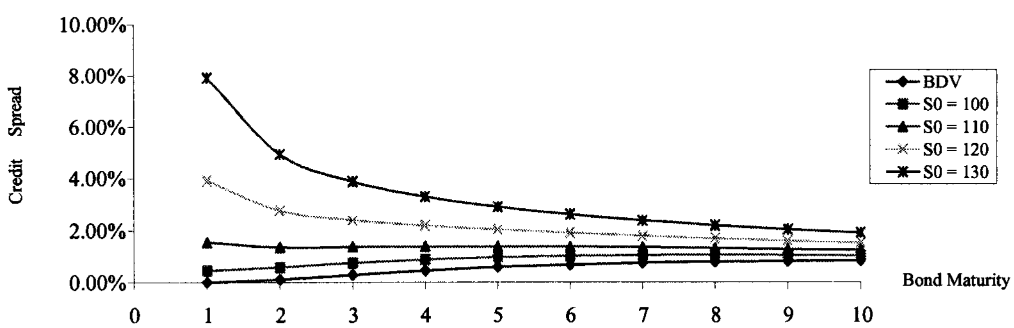
Effects of the Correlation between the Firm Value and the Interest Rate (ρ_{Ar})

ρ_{Ar}	Bond Value	Call Value	Call Value, C_{js}	Equity Value	Risk-Free Yield	Credit Spread_CW	Credit Spread_BDV
S_0 = 100, K = 100, C_{df} = 9.5323							
−0.55	48.3635	9.4983	9.5175	42.1400	9.92%	0.67%	0.33%
−0.45	48.2713	9.4978	9.5176	42.2400	9.92%	0.70%	0.36%
−0.35	48.1793	9.4974	9.5176	42.3300	9.92%	0.74%	0.39%
−0.25	48.0878	9.4969	9.5176	42.4200	9.92%	0.78%	0.42%
−0.15	47.9967	9.4964	9.5176	42.5100	9.92%	0.82%	0.46%
−0.05	47.9063	9.4960	9.5176	42.6000	9.92%	0.86%	0.49%
0.05	47.8166	9.4955	9.5176	42.6900	9.92%	0.89%	0.52%
0.15	47.7277	9.4950	9.5176	42.7800	9.92%	0.93%	0.56%
0.25	47.6396	9.4946	9.5176	42.8700	9.92%	0.97%	0.59%
0.35	47.5551	9.4943	9.5176	42.9700	9.92%	1.00%	0.62%
0.45	47.4689	9.4938	9.5177	43.0500	9.92%	1.04%	0.66%
0.55	47.3836	9.4933	9.5177	43.1400	9.92%	1.07%	0.69%

Credit Spread with ρ_{Ar} = 0.0



Credit Spread with ρ_{Ar} = 0.25



Note: Unless otherwise specified, all results were computed with S_0 = 100, K = 100, A_0 = 100, r_0 = 10%, a = 0.2, b = 0.1, σ_r = 0.02, σ_A = 0.2, σ_S = 0.25, ρ_{sr} = −0.4, ρ_{sA} = 0.3, α_1 = 0.85, α_2 = 1.00, γ_1 = 0.90, γ_2 = 1.00, T_c = 0.5, T_b = 5, and a debt ratio of 0.5. Spread_CW is the spread generated from our model, Spread_BDV is the spread generated from the model by Briys and de Varenne (1997), C_{df} is the default-free call, and C_{js} is the vulnerable call based on Johson and Stulz (1987).

EXHIBIT 6

A Closer Examination of Vulnerable Options

Option's Maturity (years)	*Bond Value*	*Call Value*	*Call Value C_{js}*	*Default-Free Call*	*Equity Value*	*Risk-Free Yield*	*Credit Spread_CW*	*Credit Spread_BDV*
Panel A: S_0 = 90, K = 100								
0.50	*48.6322*	*4.2216*	*4.2189*	*4.2272*	*47.1600*	*9.92%*	*0.55%*	*0.42%*
1.00	*48.0221*	*8.5912*	*8.5615*	*8.6084*	*43.4100*	*9.92%*	*0.81%*	*0.42%*
1.50	*47.1844*	*12.5047*	*12.4035*	*12.5706*	*40.3100*	*9.92%*	*1.16%*	*0.42%*
2.00	*46.0515*	*16.3657*	*15.8079*	*16.2384*	*37.4000*	*9.92%*	*1.65%*	*0.42%*
2.50	*45.0037*	*20.3369*	*18.8210*	*19.6751*	*34.5800*	*9.92%*	*2.11%*	*0.42%*
3.00	*44.0805*	*24.0737*	*21.4892*	*22.9171*	*32.1400*	*9.92%*	*2.52%*	*0.42%*
3.50	*43.1697*	*27.4735*	*23.8620*	*25.9878*	*29.4200*	*9.92%*	*2.94%*	*0.42%*
4.00	*42.4003*	*30.8444*	*25.9737*	*28.9038*	*27.0300*	*9.92%*	*3.30%*	*0.42%*
4.50	*41.6725*	*33.8648*	*27.8623*	*31.6775*	*24.4800*	*9.92%*	*3.64%*	*0.42%*
5.00	*41.0227*	*37.0710*	*29.5564*	*34.3191*	*21.9200*	*9.92%*	*3.96%*	*0.42%*
Panel B: S_0 = 100, K = 100								
0.50	48.0878	9.4969	9.5176	9.5323	42.4200	9.92%	0.78%	0.42%
1.00	47.1043	14.4461	14.7580	14.8497	38.3900	9.92%	1.19%	0.42%
1.50	46.0658	18.4858	19.0430	19.3946	35.4800	9.92%	1.64%	0.42%
2.00	44.8802	21.9583	22.6396	23.4925	32.9800	9.92%	2.16%	0.42%
2.50	43.8957	25.4712	25.6926	27.2698	30.5500	9.92%	2.60%	0.42%
3.00	43.1025	28.7068	28.3155	30.7919	28.5000	9.92%	2.97%	0.42%
3.50	42.3222	31.5660	30.5848	34.0983	26.1600	9.92%	3.33%	0.42%
4.00	41.6836	34.4460	32.5678	37.2159	24.1800	9.92%	3.64%	0.42%
4.50	41.0782	36.8955	34.3096	40.1640	22.0000	9.92%	3.93%	0.42%
5.00	40.5070	39.6592	35.8524	42.9576	19.8600	9.92%	4.21%	0.42%

Note: Unless otherwise specified, all results were computed with $K = 100$, $A_0 = 100$, $r_0 = 10\%$, $a = 0.2$, $b = 0.1$, $\sigma_r = 0.02$, $\sigma_A = 0.2$, $\sigma_S = 0.25$, $\rho_{sr} = -0.4$, $\rho_{sA} = 0.3$, $\rho_{Ar} = -0.25$, $\sigma_1 = 0.85$, $\alpha_2 = 1.00$, $\gamma_1 = 0.90$, $\gamma_2 = 1.00$, $T_c = 0.5$, and a debt ratio of 0.5. Spread_CW is the spread generated from our model, Spread_BDV is the spread generated from the model by Briys and de Varenne (1997), and C_{js} is the vulnerable call based on Johson and Stulz (1987).

holders tend to receive a settlement less than the market value of the option (under the default-barrier distribution rule that we are using). However, we sometimes observe in previous exhibits that the vulnerable option is worth more than its default-free counterpart. As apparent in Table I, option holders can potentially receive a settlement worth more than the market value of the option. This occurs when a default is triggered, yet the market value of the option is much lower than the initial value, C_0. A plausible corresponding scenario would be one where the option is struck at-the-money and the default barrier is set accordingly, but the option becomes out-of-the-money subsequently. To confirm this, we calculate option values and credit spreads for an out-of-the-money option with different maturities and report them in Exhibit 6. For comparison, we also calculate the same for an at-the-money option. (When we vary the option's maturity from 0.5 to 5 years, the bond's maturity is kept at 5 years.) When the option is out-of-the-money, it is worth more than its counterpart in Johnson and Stulz (1987) for all maturities and more than the default-free counterpart when the maturity is beyond 2.0 years. When the option is at-the-money, it is worth less than its default-free counterpart for all maturities but still more than its counterpart in Johnson and Stulz when the maturity is beyond 3 years.[6]

Bond holders do not enjoy this luck in the setup because their contribution to the barrier specification is fully market-value-based. That is why we did not observe a single negative credit spread. Unless covenants are fully market-value-based for OTC derivatives and margins are posted accordingly, which is unlikely to happen in practice, vulnerable options can always be potentially less vulnerable.

Finally, the credit spread becomes bigger as the option's maturity becomes longer. This reflects the bigger liability other than the debt incurred by the firm. This is in contrast with the constant spread of 0.42 produced by the model of Briys and de Varenne (1997).

Default Barrier Based on the Market Value of the Option

So far, the analyses are based on default barriers that depend on the initial value of the option. If the barrier is allowed to depend on the market value of the option, there will be altogether four possible combinations to consider, two default-barrier specifications and two distribution rules.

Some general discussion is in order. First, the bond is risky under all combinations except one, when the default barrier is based on the initial value of the option and the distribution rule is market-value-based. In this case, it is possible that a default is triggered when the option is deep out-of-the-money, and the settlement payoff to the bond holders is higher than the market value of the risk-free counterpart. This scenario, of course, does not make much sense. To avoid this, we arbitrarily stipulate that when the default barrier is breached, the firm is dissolved only if the firm value is lower than the default barrier based on the market value of the option at that point. Second, as far as option holders are concerned, it can be determined from Table I that, when $\alpha_2 = 1$, the default-barrier distribution rule is always preferred to the market-value distribution rule, no matter how the default barrier is specified. However, with the same

[6]The intuitive reason option maturity plays a role can be explained as follows. The default-free option's value is higher the longer the option maturity is. A higher initial (at-the-money) option value means a higher claim proportion in case of default. The higher the claim proportion is, the bigger the unfair claim portion is when the option is out-of-the-money at default and, hence, the pattern.

distribution rule, it is not entirely clear which default-barrier specification is preferable. Third, the opposite is true for bond holders. In other words, the bond will have a bigger credit spread under the default-barrier distribution rule. Again, under the same distribution rule, it cannot be determined ex ante which default-barrier specification is preferred.

The simulation results, which are omitted for brevity, confirm these general predictions. In addition, we find that the option is the least vulnerable when both the default barrier and the distribution rule are based on the market value of the option. It is the most vulnerable when the barrier is based on the initial option value, yet the distribution rule is based on the market value. Intuitively, this is because the option portion of the default barrier is static and provides only partial protection, and yet the distribution implies that the option holder will never receive more than the market value of the option on default.

As for credit spreads, although the general predictions are confirmed, when the option is at-the-money, or when the debt ratio is not very high, the differences in spreads among the alternative covenant and payoff specifications are generally not discernible. The difference in spreads becomes large only when, for example, the liability from the option's position is large (with in-the-money options).

Overall, the simulation results show that under general conditions, the two default-barrier specifications do not lead to very different valuations for vulnerable options and defaultable bonds. For vulnerable options, the best specification is one where both the default barrier and the distribution rule are based on the market value of the option. For defaultable bonds, the lowest spread is associated with a default barrier based on the market value of the option but a market-value-based distribution rule. Among the four barrier–distribution combinations, some are more plausible than others. The insights from our analysis help to determine how a covenant should be set up properly.

CONCLUSION

Most of the existing studies on credit-risk treat valuations of defaultable bonds and vulnerable options separately. There are two main drawbacks. First, the possibility of one type of liability going into default triggered by the other is totally ruled out, yet examples of such occurrences in practice are plentiful. Barings Bank is a case in point. Second, it is far too unrealistic to assume, for example, that bond holders have total claim against the firm in case of default. As derivatives become ever more prevalent in corporate treasuries, bond holders have found more and more companies as liability holders of the firm. The existing literature on vulnerable options has a drawback of its own. Most studies on this topic define default by comparing the asset value of the option-issuing firm with an exogenously specified default barrier, which usually takes the form of corporate debt. Option holders are fully paid off as long as the firm value is above the debt level. This flies in the face of both common sense and reality. Given that most derivative securities have theoretically infinite payoffs (e.g., a call), merely requiring the firm to be technically solvent with respect to regular debt does not guarantee full payoff to option holders. The end result of this erroneous assumption is the overestimation of the vulnerable option's value.

This article overcomes the aforementioned drawbacks by combining the two strands of literature. We introduce a second group of liability holders in the form of call option holders who are assumed to have equal claim priority as bond holders. The call option's maturity is assumed to be shorter than the bond's. In this framework, the default boundary is a sum of the minimum requirements imposed by bond holders and derivative holders. If default occurs before the option's maturity, the two groups of liability holders will claim against the firm's assets according to a distribution rule either based on prespecified requirements or based on the market value of the two instruments. In our model, both the default barrier and the firm value experience jump downward at the option's maturity. The former drops by the amount of the option holder's covenant requirement, whereas the latter drops by the amount of the intrinsic value of the option, which could be zero. Our framework contains many existing models as special cases, including Merton (1974), Black and Cox (1976), Johnson and Stulz (1987), Longstaff and Schwartz (1995), and Briys and de Varenne (1997).

To assess the full impact of the additional liability in the form of a short call, we examined two alternative default-barrier specifications, one based on the option value at initiation and the other based on the market value of an otherwise default-free option. The debt portion of the default barrier is always stochastic because of a stochastic interest rate. Under each default-barrier specification, we in turn examined two alternative settlement rules, one where the payoff proportions were based on the barrier specification and the other where the proportions were simply based on market value of the bond and the option.

Extensive simulations show that our model is capable of generating a variety of credit-spread term structure shapes, including upward sloping, downward sloping, and humped. Importantly, it can generate sizable short-term credit spreads that are impossible within the conventional diffusion setting. Moreover, unless the debt ratio is very high, or the option's position is large, the two different barrier specifications do not seem to produce very different valuations for the vulnerable option and the defaultable debt. With our parameters, it appears that the option is the least vulnerable when the default barrier is based on the market value of the option and the claim proportions (on default) are based on the barrier specifications (as opposed to simply the market value of the two instruments); the debt is the least risky under the default barrier but with a claim rule whereby the proportions are strictly based on the market values of the two liabilities.

For vulnerable options, we have an interesting and unique finding. Under a particular covenant and payoff specification (i.e., when both the default barrier and the claim rule are based on the initial value of the option), a vulnerable option can be worth more than its default-free counterpart or its counterpart that is not subject to early default: A vulnerable option need not be always vulnerable after all. This seemingly counter-intuitive result is due to the way we specify the default barrier and the claim rule. The default barrier is specified at the time the two types of liabilities are initiated and vary subsequently only because of interest-rate fluctuations. Although this setup suits the bond holder well, it does not take into account the market value change of the option due to fluctuations in the optioned stock price. As a result, sometimes the option holder can receive more than the fair value of the option at default. Unless the default barrier and the claim proportion are both market-value-based, this possibility always exists. This has profound implications in terms of the fair valuation of defaultable options.

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