Incentive Stocks and Options with Trading Restrictions
— Not as Restricted as We Thought —

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Abstract

Stock ownership and incentive options are important tools for companies to retain and motivate employees and managers. Such incentive schemes usually impose trading restrictions which make grantees’ total wealth highly undiversified. As a result, as shown by Meulbroek (2001), Ingersoll (2002), and Kahl, Liu and Longstaff (2003), grantees tend to value these incentive securities below market. The difference between the market value and the grantee’s private value represents a deadweight loss. In this paper, we show how this deadweight loss can be reduced while preserving the retention and long-term incentive effects. Specifically, using the continuous-time, consumption-portfolio framework as a backdrop, we demonstrate how a hedging index can alleviate the negative impact of vesting requirements (i.e., the deadweight loss) and help align the employee’s private valuation of the restricted assets with that of the market’s. The hedging index is especially effective in alleviating the deadweight loss and improving the incentive effects of restricted options.

Keywords: Restricted Stock and Stock Options, Optimal Portfolio-Consumption Selection, Deadweight loss.

JEL classifications: J33; G13

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Abstract

Stock ownership and incentive options are important tools for companies to retain and motivate employees and managers. Such incentive schemes usually impose trading restrictions which make grantees’ total wealth highly undiversified. As a result, as shown by Meulbroek (2001), Ingersoll (2002), and Kahl, Liu and Longstaff (2003), grantees tend to value these incentive securities below market. The difference between the market value and the grantee’s private value represents a deadweight loss. In this paper, we show how this deadweight loss can be reduced while preserving the retention and long-term incentive effects. Specifically, using the continuous-time, consumption-portfolio framework as a backdrop, we demonstrate how a hedging index can alleviate the negative impact of vesting requirements (i.e., the deadweight loss) and help align the employee’s private valuation of the restricted assets with that of the market’s. The hedging index is especially effective in alleviating the deadweight loss and improving the incentive effects of restricted options.

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1. Introduction

According to the National Center for Employee Ownership (NCEO), about 20 million employees and executives in the U.S. received stock and stock options. The employee ownership was pioneered by high-tech start-up companies as a practical way of retaining and motivating talented employees. Usually, these companies require their employees to continue working for a minimum number of years before the stocks and options are vested. During the vesting period, employees are not allowed to sell their stock or option holdings. Retention is achieved through vesting in that the employee must forgo the holdings if he leaves the company before the stocks and options are vested; long-term motivation is achieved by linking the employee’s personal wealth (i.e., the restricted holdings) to the company’s stock value. Granting stocks without trading restrictions is equivalent to granting cash bonuses, since the employee can sell the stocks immediately after receiving them. In this case, although the employee values his stock grant exactly as the market does, the long-term incentive is absent and the employee can leave the company without incurring any loss. Granting stocks with an infinite vesting period does not make any sense either since they are worthless to the employee. Clearly, trading restrictions imposed for a modest period (usually five to ten years) are necessary for both retention and long-term motivation purposes. But such restrictions will make part of the employee’s wealth illiquid and increase the unnecessary exposure to the company-specific or non-systematic risk. Consequently, a risk-averse employee will value such a restricted asset below its market value. In other words, there is a deadweight loss associated with vesting. Thus, bearing the company’s non-systematic risk is an undesirable side effect, not the intended purpose.

Meulbroek (2001) shows how costly bearing non-systematic risk can be from a diversification perspective. Ingersoll (2002), and Kahl, Liu and Longstaff (2003) (KLL hereafter) also demonstrate how much the employee discounts the restricted holdings in a portfolio-selection framework. Bearing the firm-specific risk does not create incentives. The ultimate incentive effects of restricted stocks and options depend on the holder’s private valuation. The higher the private valuation, the higher the incentive. In this sense, bearing the firm-specific risk hampers the incentive effects.

The objective of this paper is to propose a hedging strategy to improve the employee’s private
valuation of the incentive assets with trading restrictions and hence improve their incentive effects. To do so, we link the literatures on executive compensation and asset valuation with portfolio constraints, and go one step further by introducing a hedging index in the portfolio choice set.\footnote{For examples of the incentive literature, see Lambert, Larcker, Verrecchia (1991), Rubinstein (1995), Aboody (1996), Carpenter (1998, 2000), Hall and Murphy (2000a, 2000b), Meulbroek (2001), and Lambert and Larcker (2003). For examples of the asset valuation with portfolio constraints, see DeTemple and Sundaresan (1999) and references therein. Recent studies on constrained portfolio selections and asset valuation include Henderson and Hobson (2002), Ingersoll (2002), Browne, Milevsky and Salisbury (2003), and KLL (2003).} This hedging index can be considered as an industry index with a high correlation to the restricted stock. The portfolio strategy is optimized in a consumption-portfolio selection framework with respect to the employee’s intertemporal utility. Overall, we find that the hedging index can increase the employee’s private valuation of the restricted assets and thus reduce the deadweight loss associated with vesting. In other words, the hedging index can alleviate the side-effect of vesting while preserving the retention and long-term incentive effects. The specific findings are as follows.

With respect to the restricted stock, the hedging index can help improve the employee’s optimal consumption-investment policy and better align the employee’s private valuation of the restricted stock with the market’s. The difference between the stock’s market value and the employee’s private value represents the illiquidity discount or deadweight loss. As expected, the deadweight loss increases with the employee’s degree of risk aversion, the length of the vesting period and the volatility of the restricted stock. However, the deadweight loss is reduced when the residual correlation between the restricted stock and the hedging index is high. For example, an employee with a risk aversion parameter $\gamma = 4$ and a 50% holding in the firm’s stock would discount the stock by as much as 58% when the hedging index is not used. However, with a hedging index whose partial correlation with the stock is 0.6, the discount is reduced to 42%, a 16% improvement of the private valuation. When the residual correlation is perfect, the discount disappears and the side effect of the trading restriction is completely removed. In this case, the employee’s valuation of the restricted stock coincides with the market’s. Therefore, with an optimal position on the index, the negative effect of trading restrictions can be alleviated while the retention and long-term motivation effects are preserved. The trading restrictions are not as restrictive as we thought.

With respect to restricted options, the hedging index can also help align the employee’s private
valuation with the market’s or the Black-Scholes value, thus reducing the deadweight loss. Compared with the case of restricted stocks, the hedging index is much more effective in reducing the deadweight loss. Moreover, hedging also improves the incentive effects measured by the option’s delta. Again, with the hedging index, the trading restrictions on options are not as restrictive as they appear.

It is important to note that reducing the side effect of the trading restriction is different from granting unrestricted stocks because the employee must hold the company’s stock regardless of what he does with the rest of his investment; likewise, reducing the impact of the non-systematic risk will not undermine incentives because an employee’s personal wealth is still tied to the stock. In fact, with enhanced private valuation of the restricted assets, the incentive mechanism is actually improved since the ultimate source of motivation is a higher personal wealth and utility.

Although the continuous-time consumption-portfolio framework used here is similar to that of Ingersoll (2002) and KLL (2003), the current paper differs from these two studies in four aspects. First and foremost, our emphasis is on how to improve the private valuation of the restricted assets while their focus is on quantifying the discount or the deadweight loss. Our emphasis helps shed light on the incentive issues surrounding the restricted stocks and options. Second, we augment their portfolio choice set by including a hedging index. Specifically, our portfolio choice set includes the market portfolio, a hedging index, the restricted stock and the riskfree asset. Our analysis shows that the inclusion of the hedging index can help improve the employee’s intertemporal utility, increase the private valuation of the incentive stocks and options and enhance the incentive effects of these restricted assets.\(^2\) Third, we study the private valuation through the marginal rate of substitution, which is equivalent to the Euler equation approach in the equilibrium framework. In contrast, KLL (2003) defines the employee’s private valuation of the restricted stock as the cash amount which makes the employee indifferent between holding the restricted stock and cash.

\(^2\)The simplistic portfolio choice set in Ingersoll (2002) and KLL (2003) may be motivated by the well-known result that, as long as the financial market is perfect and the CAPM holds, the optimal portfolio strategy for an investor is to hold a combination of the risk-free bond and the market portfolio. However, with trading restrictions, including other assets in the feasible set will help to reduce the non-systematic risk, and as a result, improve the employee’s intertemporal utility. The benefit derived from the additional assets cannot be achieved by simply re-scaling the market portfolio’s weights (Section 2.3 and Table 1 delineate this point).
Fourth, we analyze both the incentive stocks and options while Ingersoll focuses on options and KLL on stocks.

The rest of the paper is organized as follows. Section 2 formulates and characterizes the employee’s lifetime consumption-investment decision with an expanded portfolio choice set. We illustrate numerically the hedging index’s contribution in enhancing the employee’s utility. Section 3 analyzes and demonstrates how the hedging index can alleviate the deadweight loss of the trading restrictions on the stock. Section 4 examines the hedging index’s role in enhancing the private valuation and incentive effects of various types of restricted stock options. Section 5 provides concluding remarks. Proofs and tables are relegated to the appendix.

2. Optimal Consumption-Portfolio Strategy with a Hedging Index

2.1. The Setup

Consider a risk-averse employee with a finite lifetime horizon $T$ whose preference is described by a smooth, time-additive expected utility function: $U(c) = E \left[ \int_0^T U(c_t,t) dt \right]$. Following the literature, we adopt the constant-relative-risk-aversion (CRRA) utility function for the employee:

$$U(c_t,t) = e^{-\phi t} c_t^{1-\gamma} \frac{1}{1-\gamma}, \quad (2.1)$$

where $\phi > 0$ is the rate of time preference and $\gamma \geq 1$ is the coefficient of relative risk aversion. The employee works for a company and, in return, receives a constant wage $w$ per unit of time. To retain and motivate employees, the company issues restricted stocks and options over time. Similar to Ingersoll (2002), we approximate the ongoing nature of the incentive scheme by assuming that the employee is required to hold a fixed fraction of his total wealth (defined later) in the company’s stock during the vesting period which ends at time $T < T$. By the end of the vesting period, the employee is free to sell these shares. The assumption of fixed fraction is equivalent to assuming proportional growths in the stock’s value and the total wealth.

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3Since the employee no longer faces trading constraints after the vesting period, the role of bequest is unimportant. We therefore assume a zero bequest function for simplicity.

4KLL (2003) assume that the employee receives only one grant of the company’s stock. Therefore, the number of the restricted shares is fixed during the vesting period. Such a restriction is realistic for IPO lock-up, but less so for an annual incentive scheme used by most of the companies.
It is well known that, in the absence of trading restrictions and assuming the continuous-time CAPM, the employee’s optimal portfolio strategy is to hold the market portfolio and the risk-free asset only. With trading restrictions on the stock, his optimal portfolio strategy is no longer clear-cut. The imposed stock holding will force the employee to bear some non-systematic risk, which can not be fully offset by the market portfolio. Introducing an additional asset will help reduce this non-systematic risk and improve the employee’s overall utility. The deadweight loss of vesting can therefore be alleviated while the retention and incentive effects are preserved.

To this end, the employee’s portfolio choice set is assumed to include a bond \( B \) earning the risk-free rate \( r \), the market portfolio \( M \), the restricted asset or the company’s stock \( S \), and the index \( I \). The price dynamics are:

\[
\frac{dM}{M} = (\mu_m - q_m) dt + \sigma_m dz_m, \quad \frac{dS}{S} = (\mu_s - q_s) dt + \sigma_s dz_s, \quad \frac{dI}{I} = (\mu_I - q_I) dt + \sigma_I dz_I,
\]

where the correlation coefficient matrix of \( z = (z_m \ z_s \ z_I) \)' is

\[
\Sigma = \begin{pmatrix}
1 & \rho_{ms} & \rho_{mI} \\
\rho_{ms} & 1 & \rho_{Is} \\
\rho_{mI} & \rho_{Is} & 1
\end{pmatrix},
\]

and the cum-dividend expected returns, dividend yields and the volatilities are

\[
\mu = \begin{pmatrix}
\mu_m \\
\mu_s = r + \beta_s (\mu_m - r) \\
\mu_I = r + \beta_I (\mu_m - r)
\end{pmatrix}, \quad q = \begin{pmatrix}
q_m \\
q_s \\
q_I
\end{pmatrix}, \quad \text{and} \quad \sigma = \begin{pmatrix}
\sigma_m \\
\sigma_s \\
\sigma_I
\end{pmatrix},
\]

with \( \beta_s = \rho_{ms} \sigma_m / \sigma_s \) and \( \beta_I = \rho_{mI} \sigma_m / \sigma_I \) and the non-systematic variances for the stock and the index as \( \nu_s^2 = (1 - \rho_{ms}^2) \sigma_s^2 \) and \( \nu_I^2 = (1 - \rho_{mI}^2) \sigma_I^2 \).

With the above elements, the employee chooses an optimal consumption and portfolio policy to maximize his expected lifetime utility subject to the usual budget constraint. The first order conditions yield the following stochastic Euler equation:

\[
X_t = E_t \left[ \int_t^T \frac{U_{c_t}}{U_{c_t}} dD_t \right], \quad (2.2)
\]

where \( X_t \) is the current price of an asset with a dividend yield of \( D \), and \( U_{c_t} \) stands for the partial derivative of the utility function with respect to the consumption \( c_t \). Intuitively, given his optimal consumption policy, the employee’s private valuation of any security is the expected present value of
the future yields or payoffs, discounted at his own marginal rate of substitution. It is important to emphasize that the employee takes as given the market prices of all securities (which are equilibrium outcomes under no trading restrictions). When he is free to trade any security, his private valuation equals the market’s. When he is restricted in trading some of the securities, however, his private valuation may deviate from the market’s. This does not imply any arbitrage since the employee is a price taker in the market place. In other words, we assume that the number of investors with trading restrictions is so small compared with the total population of investors that they can not influence the competitive financial market.

2.2. Optimal Consumption-Portfolio Policies and The Impacts of Trading Restrictions

To determine the employee’s valuation of the restricted stock and options, we first need to solve the consumption-investment problem to obtain the marginal rate of substitution. To this end, we define the employee’s total wealth as the portfolio wealth plus his human wealth. The former is \( \theta_B B_t + \theta_M M_t + \theta_S S_t + \theta_I I_t \) (where \( \theta_B, \theta_M, \theta_S, \) and \( \theta_I \) are portfolio holdings) and the latter is the present value of his labor income \( w \frac{1-e^{-r(T-t)}}{r} \). Thus, the total wealth is \( W_t = \theta_B B_t + \theta_M M_t + \theta_S S_t + \theta_I I_t + w \left(1 - e^{-r(T-t)}\right)/r \). Denote the percentages of his total wealth invested in the risky assets as \( x_t = (x_{mt}, x_{st}, x_{It})' \), then we have

\[
\begin{align*}
x_{mt} &= \frac{\theta_{mt}}{W_t}, & x_{st} &= \frac{\theta_{st}}{W_t}, & x_{It} &= \frac{\theta_{It}}{W_t}.
\end{align*}
\]

Similar to Merton (1971), the budget constraint is

\[
\frac{dW_t}{W_t} = \left(r + x_t' (\mu - r) - \frac{c_t}{W_t}\right) dt + (x_t \cdot \sigma)' dZ_t.
\]

Given the trading restrictions, the employee’s optimization must be solved for two distinct periods as shown below.\(^5\)

\[
\begin{array}{c|c|c}
\hline
\text{vesting period} & \text{post-vesting period} \\
\hline
\hline
T & \hline
\end{array}
\]

Since there is no trading restriction during the post-vesting period, the optimal solution is similar

\(^5\)For simplicity, we assume that the retirement time coincides with the finite horizon of the utility maximization.
to the standard Merton solution (1969). That is:

\[ x^*_{mt} = \frac{s_m}{\gamma \sigma_m}, \quad x^*_{st} = 0, \quad x^*_{It} = 0, \quad \forall \ T < t < T, \]

\[ c^*_t = f(a, t, T)W_t \quad \text{and} \quad J(W, t, f) = e^{-\phi t} f(a, t, T)^{-\gamma \frac{W^t_{1-\gamma}}{1-\gamma}}. \]

with \( s_m = \frac{\mu_m - r}{\sigma_m}, \quad a = \frac{\phi}{\gamma} - \frac{1-\gamma}{\gamma} (r + \frac{\sigma_m^2}{2\gamma}), \quad f(a, t, T) = \frac{a}{1-e^{-a(T-t)}}. \)

The optimal policy indicates that the holding of the market portfolio is positively related to the market’s Sharpe ratio \( s_m \), negatively related to the employee’s risk-aversion parameter \( \gamma \) and the volatility of the market portfolio \( \sigma_m \). Stated differently, the employee will invest heavily in the market portfolio if he is less risk-averse or when the market’s Sharpe ratio is high.

Now we turn to the vesting period. Let \( x_s \) be the fixed percentage of the employee’s total wealth in the restricted stock. For any \( x_s > 0 \), the optimality condition implies that this constraint is binding. The employee’s optimal portfolio should hold exactly \( x_s \) during the vesting period \( (0 < t < T) \). Unlike Ingersoll (2002) and KLL (2003), we augment the portfolio choice set by including the index to help the employee reduce the negative impact of the trading restrictions. Specifically, the employee takes \( x_s \) as given and optimizes the portfolio strategy on the market and the index. Appendix A provides the solution to this optimization problem. To facilitate the presentation of the optimal solution, we denote the residual or partial correlation between the restricted stock and the index, after controlling for the market impact, as \( \rho_{Is.m} = (\rho_{Is} - \rho_{ms}\rho_{mI}) / \sqrt{(1-\rho_{ms}^2)(1-\rho_{mI}^2)}. \)

**Proposition 2.1.** When there is no trading limit on the index, the employee’s optimal consumption-investment strategy during the vesting period consists of

\[ x^*_{mt} = \frac{s_m}{\gamma \sigma_m} - x_s \beta_s \rho_{ms} - \rho_{mI} \rho_{Is} \rho_{ms} \rho_{mI} (1-\rho_{mI}^2), \quad x^*_{st} = x_s, \quad x^*_{It} = -x_s \cdot \rho_{Is.m} \cdot \frac{v_s}{v_I} \equiv x^*_I, \]

\[ c^*_t = F(A, t, T)W_t, \quad \text{and} \quad J(W, t, F) = e^{-\phi t} F(A, t, T)^{-\gamma \frac{W^t_{1-\gamma}}{1-\gamma}}, \quad \forall \ 0 < t < T, \]

where \( A = a - \frac{1}{2}(\gamma - 1) \Omega^*, \quad \Omega^* = x^2_s v^2_s - x^2_I v^2_I \equiv x^2_s v^2_s (1-\rho_{Is.m}^2), \quad F(A, t, T) = \frac{A}{1+\left(\frac{A}{f(a,T,t)-1}\right)e^{-A(T-t)}}. \)

Proposition 2.1 indicates that the optimal position on the index depends on, among other things, the residual correlation between the stock and the index. If the residual correlation is positive, then the employee will short the index; if it is negative, a long position on the index will
be taken. The optimal position could be very large if the residual correlation is high and / or the ratio of the non-systematic risk is high. In reality, although the employee could take a long position on the index without any limit, he may face certain shorting restrictions imposed by his broker. To complete the framework, we now consider the optimal consumption-investment policy when the index position is limited. To this end, suppose the employee can not short more than $|x_I| (x_I < 0)$ of his total wealth. When this constraint is not binding, i.e., when $x_I < x^*_I$, the employee’s optimal consumption-portfolio strategy is still the one presented in Proposition 2.1. Otherwise, the following proposition characterizes the optimal profile (we omit the proof for brevity).

**Proposition 2.2.** When $\rho_{I,s,m} > 0$ and the shorting restriction on the index is $-x_s \cdot \rho_{I,s,m} \cdot \frac{v_s}{v_I} < x_I < 0$, the employee’s optimal consumption-investment strategy during the vesting period becomes

$$x^*_{mI} = \frac{s_m}{\gamma \sigma_m} - x_s \beta_s - x_I \beta_I, \quad x^*_{st} = x_s, \quad x^*_{It} = x_I,$$

$$c^*_t = \mathcal{F}(\overline{\Omega}, t, T) W_t, \quad \text{and} \quad J(W, t, \mathcal{F}) = e^{-\phi t} \mathcal{F}(\overline{\Omega}, t, T)^{-\gamma} W^{1-\gamma} \frac{1}{1-\gamma}, \quad 0 < t < T,$$

where $\overline{\Omega} = a - \frac{1}{2} (\gamma - 1) \Omega, \quad \overline{\Omega} = x^2_s v_s^2 + x^2_I v_I^2 + 2 \rho_{I,s,m} x_s x_I v_s v_I$,

$$F(\overline{\Omega}, t, T) = \frac{x}{1 + \left(\frac{x}{T-a} - 1\right) e^{-\gamma (T-t)}}.$$

In order to show how the consumption and investment decisions are affected by the trading restrictions, we examine the optimal consumption dynamics. During the post-vesting period, the consumption evolves according to

$$\frac{dc}{c} = \frac{1}{\gamma} \left[ r - \phi + \frac{1}{2} (1 + \frac{1}{\gamma}) s^2_m \right] dt + \frac{s_m}{\gamma} dz_m \equiv \mu_c dt + \sigma_c dz.$$

During the vesting period, the consumption process follows

$$\frac{dc}{c} = \left[ \mu_c + \frac{1}{2} (\gamma - 1) \overline{\Omega} \right] dt + \left( \frac{s_m}{\gamma} - \beta_s x_s s_m - \beta_I x^*_I \sigma_m \right) dz_m + x_s \sigma_s dz_s + x^*_I \sigma_I dz_I,$$

where $\begin{cases} \Omega = \Omega^* & \text{and} \quad x^*_I = x^*_I \quad \text{with a non-binding index shorting limit} ; \\ \Omega = \overline{\Omega} & \text{and} \quad x^*_I = x_I \quad \text{with a binding index shorting limit} . \end{cases}$

During the post-vesting period, the employee’s consumption uncertainty is completely induced by the market portfolio. During the vesting period, regardless of the index shorting limit, his consumption growth rate and volatility are affected by the uncertainty in the market portfolio, as well as by those in the stock and the index. With a well defined residual correlation between the
index and the stock (i.e., $|\rho_{ts,m}| < 1$), we have $\Omega > 0$. Given that $\gamma \geq 1$, the consumption growth rate during the vesting period is higher than that during the post vesting period. The reason is that the vesting requirement makes part of the employee’s personal wealth illiquid and the employee tends to consume at a lower level. As time approaches the end of the vesting, the employee tends to increase his consumption, resulting in a higher consumption growth rate. The consumption variance during the vesting period can be easily shown as $\frac{\sigma^2}{\gamma} + \Omega$ and is higher than that of the post vesting period. The excess consumption variance $\Omega$ reaches to its maximum when the index is excluded and reduces to its minimum $\Omega^*$ when the employee can optimally invest in the index. Given the negative relation between the employee’s intertemporal utility and the consumption volatility, it is clear that the employee obtains the highest utility when the index shorting restriction is absent or non-binding, and the lowest utility when the index is not included in the portfolio choice set. As shown later, this result translates to how much the employee discounts the market value of the restricted securities.

It should be stressed that the inclusion of the hedging index unambiguously increases the employee’s intertemporal utility, and hence improves the effectiveness of the compensation package. Contrary to casual perceptions, adding the index into the optimization does not simply boil down to a re-scaling of the market portfolio’s weight. If the index’s role is trivial, then the optimized weight on the index would have been either zero or non-zero but inconsequential to the utility level. In the follow subsection, we will use a simple, three-risky-asset setup to show that the introduction of an additional asset does improve the overall consumption-portfolio decision.

2.3. Non-Trivial Role of the Index

Let us consider a simple economy where there are only three risky assets and one risk-free bond in the financial market. Suppose the risk-free rate is $r = 6\%$ and the return characteristics of the three risky assets are given in Table 1. The market portfolio is the tangent portfolio formed with the three risky assets. The weights are: 36.2% in Asset 1, 34.5% in Asset 2, and 29.3% in Asset 3. As shown in Panel A of Table 1, under no trading restrictions, an investor with risk aversion $\gamma = 4$ will invest 34.8% in the risk-free bond and 65.2% in the market portfolio. In other words,
the investor ultimately holds 34.8% of his wealth in the risk-free bond, 23.6% in Asset 1, 22.5% in Asset 2, and 19.1% in Asset 3. The intertemporal expected utility is calculated via the value function in Proposition 2.1 and re-scaled to 100.

Without loss of generality, let us now impose a trading restriction on Asset 1. To see how the introduction of additional assets (other than the market portfolio) can affect the consumption-portfolio decisions, we examine three indices. Index_{12} is the tangent portfolio formed with Assets 1 and 2. The index’s weights and residual correlation with Asset 1 (controlling for the market) are given in the top portion of Table 1. Index_{23} is defined in a similar fashion. Index_{23,m} consists of Assets 2 and 3 whose weights are proportional to those in the market portfolio. Naturally, the residual correlation between Asset 1 and Index_{23,m} is -1.0.

Panel B of Table 1 presents results for the situation where the imposed holding on Asset 1 is 10%, which is lower than the optimal weight of 23.6% in the market portfolio. Without the hedging index (Case 1), the ultimate weight on Asset 1 is 31.9%, higher than 23.6% (the optimal weight of Asset 1 in the market portfolio), and the utility level is now lower. The holding restriction makes the investor worse-off. The overall weight on Asset 1 is higher than 10%, since the investor is also holding the market portfolio which contains Asset 1. Note that this case corresponds to the simplified portfolio choice set as in Ingersoll (2002) and KLL (2003) where the employee can only optimize his strategy over the risk-free bond and the market portfolio.

When we include Index_{12} into the choice set (Case 2), a short position is taken on this index, and the ultimate weights on all assets are different from those of Case 1. More importantly, the utility level improves. The inclusion of Index_{23} (Case 3) has similar effects, except that the utility improvement is larger since the residual correlation (0.542) is bigger than that in Case 2 (0.392). Moving on to Case 4 where we add Index_{23,m} into the portfolio choice set, the ultimate portfolio weights are all restored to their optimal level, and so is the utility level.

The contribution of the additional asset is bigger when the restricted position is larger. To

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*There are many other ways in which we can introduce an additional asset into the choice set. For instance, we could introduce a tangent portfolio consisting of Assets 1 and 3. This index will be similar to Index_{12} in nature. Alternatively, we could include Assets 2 and 3 as individual assets, which will be equivalent to including Index_{23,m} alone.*
appreciate this point, we repeat the calculations in Panel A by setting the restriction on Asset 1 to 50%, which is higher than the optimal weight of 23.6% in the market portfolio. Panel C shows that the restriction reduces the utility by about 15% when the investor can only trade the market portfolio and the risk-free bond. But with the right index (Index$_{23,m}$), the restriction can be totally undone, and the loss of utility can be completely recovered.

This simple exercise clearly demonstrates that, when there are trading restrictions on certain assets, allowing only the market portfolio and the risk-free bond in the portfolio choice set will lead to sub-optimal decisions. Introducing other assets into the choice set will improve the consumption-portfolio decisions and enhance the overall utility level. In the case of restricted stock and options, the deadweight loss will be reduced when the hedging index is available.

3. Alleviating the Restricted Stock’s Deadweight Loss Using the Hedging Index

3.1. The State Price Deflator

In order to show how the hedging index can alleviate the deadweight loss due to vesting, we need to determine how the employee values the restricted security with and without the hedging index. The employee’s private valuation of an incentive security hinges upon his state price deflator $\frac{U_{ct}}{U_{ct}}$, i.e., the marginal rate of substitution. Given the CRRA utility function, we have $U_{ct} = e^{-\phi t}c_t^{-\gamma}$.

Applying Ito’s lemma, we obtain the following processes for the marginal utility:

$$\frac{dU_{ct}}{U_{ct}} = -(r - \gamma \Omega)dt - (s_m - \gamma \beta_s \sigma_m - \gamma \beta_I x_I^* \sigma_I) dz_m$$

$$-\gamma x_s \sigma_s dz_s - \gamma x_I^* \sigma_I I dz_I,$$

$\forall \quad t < T$ \hspace{1cm} (3.1)

$$\frac{dU_c}{U_c} = -rdt - s_m dz_m,$$

$\forall \quad t > T$, \hspace{1cm} (3.2)

where $\Omega$ and $x_I^*$ are defined in equation (2.3).

The processes in (3.1) and (3.2) apply to the vesting and post-vesting periods, respectively. Beyond the vesting period, the employee’s state price deflator is determined by the risk-free rate and the Sharpe ratio of the market portfolio and is independent of the risk preference parameters $\phi$ and $\gamma$. In this case, the employee’s valuation of any incentive security is equal to the market
valuation, and the deadweight loss is zero. Such results don’t hold within the vesting period, since the discount rate is \( r - \gamma \Omega \) and the variance of the marginal utility is \( s^2_m + \gamma^2 \Omega \). In the following section, we will determine precisely how much the employee discounts the restricted stock.

### 3.2. Deadweight Loss of the Restricted Stock

As mentioned above, when the employee faces trading restrictions, his private valuation of an incentive security no longer equals the market’s. Instead, it is determined via the Euler equation (2.2) under the state price deflator (3.1). Taking the continuous dividend yield into consideration, the employee’s subjective valuation of the restricted stock is computed as

\[
\hat{S}_t = E_t \left( \frac{V_{c_T}}{V_{c_t}} e^{\psi(T-t)} S_T \right).
\]

Straightforward, albeit tedious algebra leads to:

\[
\hat{S}_t = S_t e^{-l_s(T-t)},
\]

where the illiquidity discount \( l_s \) is defined as

\[
l_s = \begin{cases} 
  l^*_s \equiv \gamma(x_s - x^2_s)v_s^2(1 - \rho_{I,s,m}^2) & \text{if the index shorting restriction is not binding,} \\
  l^*_s + \gamma v_I^2(x_I - x^*_I)(x^*_I - x_I - x^2_s) & \text{if the index shorting restriction is binding.}
\end{cases}
\]

When the trading restriction on the stock is removed, i.e., when \( x_s = 0 \), there is no need to take a position on the index. The illiquidity discount \( l^*_s \) dissipates and the employee’s private valuation is equal to the market’s. Also, the private valuation reverts to the market value at the end of the vesting period where \( t = T \). However, when \( x_s \neq 0 \) or \( t < T \), the private value is always lower than the market value, and granting the restricted stock will always have a deadweight loss. The extent to which the employee discounts the restricted stock depends on the length of the vesting period \( T - t \) and the illiquidity discount rate, \( l^*_s \). The illiquidity discount increases with the degree of risk aversion and the volatility of the company’s stock. However, the discount decreases with the correlation between the company’s stock and the index. The discount reduces to its minimum when the index position is optimized since freely optimizing the position on the index can help the employee better hedge the non-systematic risk and hence reduce the illiquidity discount while leaving the incentive effects intact. With an optimal index weight, a perfect residual correlation between the stock and the index (i.e., \( |\rho_{I,s,m}| = 1 \)) can make the illiquidity discount
completely vanish. In this case, the inherent non-systematic risk can be completely nullified, and the employee’s private valuation of the stock coincides with the market’s, reducing the deadweight loss to zero. Moreover, in contrast to the results in Ingersoll (2002) and KLL (2003) which predict a negative relationship between the illiquidity discount and the correlation between the stock and the market, the relationship in our framework is not always one way. It depends on the residual correlation between the stock and the market, after controlling for the index. When $\rho_{ms,I} > 0$, the illiquidity discount is negatively related to $\rho_{ms}$; When $\rho_{ms,I} < 0$, the relationship is positive.

To gain further insights, we report in Table 2 the private stock value as a fraction of its market value for various parameter combinations. One minus the fraction measures the size of the deadweight loss in percentage. Under each stock restriction, we examine three scenarios: no hedging (corresponding to Ingersoll, 2002 and KLL, 2003), hedging with a binding restriction on the index, and optimal hedging.\footnote{To maintain a consistent comparison across all scenarios, in this and subsequent tables, we set the index restriction as 50% of the optimal level. We set the restriction as a fraction of the optimal level so that it is always binding to the same extent across all scenarios.} The difference in the fractions between the no-hedging ($x_I = 0$) column and optimal-hedging ($x_I = x_I^*$) column measures the reduction of deadweight loss due to hedging. Other than the obvious (e.g., a higher risk aversion and a longer vesting period lead to a deeper discount or a bigger deadweight loss), the table bears out the previous discussions: allowing the employee to optimally choose the position on the index will always reduce the illiquidity discount or the deadweight loss. For example, for an employee whose risk-aversion is 4 and who holds 50% of his wealth in the restricted stock, he discounts the holding by about 58% when $\rho_{ms} = 0.2$, $\rho_{Is} = 0.61$, and $T - t = 10$ years. With a hedging index ($\rho_{Is,m} = 0.6$), the discount is about 42%, representing an improvement of 16%.

Regardless of whether the stock is more correlated with the index or with the market, as long as the residual correlation $\rho_{Is,m}$ is not zero, hedging with the index will always enhance the utility level and the private valuation of the stock. The higher the residual correlation, the better the index can help align the private valuation with the market’s. For instance, when $\gamma = 2$, $\rho_{ms} = 0.2$, $\rho_{mI} = 0.5$, $\rho_{Is} = 0.95$, and $x_s = 0.5$, a ten-year vesting period will lead to a deadweight loss of 35% when hedging is not in place; but with optimal hedging, this entire loss can be avoided. It
should be noted that many correlation combinations can lead to a perfect or a very high residual correlation $\rho_{I_8.m}$. To see this, we can set $\rho_{m_8} = 0.2$, $\rho_{m_8} = 0.7$, and $\rho_{I_8} = 0.84$ to have $\rho_{I_8.m} = 1$.

4. Alleviating the Deadweight Loss and Improving the Incentive Effects of Options on the Restricted Stock

4.1. Conventional European Options

Similar to the analyses for restricted stocks, we first obtain the marginal valuation of options and then determine how much the employee discounts the options due to trading restrictions. The valuation depends on whether the stock is still being restricted at the time of the option’s exercise. When the option’s maturity $T_o$ is before the end of the vesting period $T$, the stock will have a private value at the option’s expiry date, and an adjustment for the remaining vesting period must be made, which amounts to a discount to the stock price. When $T_o > T$, the stock attains its market value at the option’s maturity and no adjustment is necessary. In this case we must use both state price deflators. The valuation under the two cases can be summarized as follows:

$$\hat{C}_t = E_t \left[ \frac{U_{ct}}{U_{ct}} \max(S_{T_0} e^{-l_s(T-T_o)} - K, 0) \right], \quad \forall \ T_o < T,$$

$$\hat{C}_t = E_t \left[ \frac{U_{ct}}{U_{ct}} E_T \left( \frac{U_{ct}}{U_{ct}} \max(S_{T_0} - K, 0) \right) \right], \quad \forall \ T_o > T,$$

where $K$ is the exercise price of the option. To facilitate presentation, let $C_{BS}$ be the Black-Scholes call value expressed as:

$$C_{BS}(S_t, T - t, K, r, q, \sigma) = S_t e^{-q(T-t)} N [d_1(r, q)] - K e^{-r(T-t)} N [d_2(r, q)],$$

where

$$d_1(r, q) = \frac{\ln(S_t/K) + (r - q + \sigma^2/2)(T-t)}{\sigma \sqrt{T-t}}$$

and

$$d_2(r, q) = d_1(r, q) - \sigma \sqrt{T-t}.$$

Then the employee’s valuation of the European option on the restricted stock is (see Appendix B for a proof)

$$\hat{C}_t = C_{BS}(S_t e^{-l_s(T-t)}, T_o - t, K, r - \gamma \Omega, q_8, \sigma_s), \quad \forall \ T_o < T,$$

$$\hat{C}_t = C_{BS}(S_t e^{-l_s(T-t)}, T_o - t, K, r - \gamma \Omega \frac{T_o - t}{T_o - t}, q_8, \sigma_s), \quad \forall \ T_o > T.$$ (4.1)

Since a European call option’s value is increasing in the stock price and the interest rate and since the effective stock price ($S_t e^{-l_s(T-t)}$) and the interest rate ($r - \gamma \Omega$ or $r - \gamma \Omega \frac{T_o - t}{T_o - t}$) are
both reduced from their normal levels, restricted European calls are always worth less than their Black-Scholes counterparts. In other words, there is a deadweight loss when granting European options. We showed earlier that the illiquidity discount $l_s$ and the excess variance $\Omega$ can both be reduced when the index is in place. Thus the hedging index will narrow the gap between the private valuation and the market’s. In other words, it will alleviate the deadweight loss. When the residual correlation between the stock and the index is perfect: $|\rho_{I,s,m}| = 1$, the employee’s private valuation of the option is equal to the Black-Scholes value and the deadweight loss is zero.

As for incentive effects, we follow the literature and examine the option’s delta (i.e., the option’s sensitivity to the stock price). In our case, delta is $S_t e^{-l_s(T-t)} e^{-q(T_0-t)} N \left( d_1 \left( r - \gamma \Omega, q_s \right) \right)$ or $S_t e^{-l_s(T-t)} e^{-q(T_0-t)} N \left( d_1 \left( r - \gamma \Omega \frac{T-t}{T_0-t}, q_s \right) \right)$, clearly lower than its Black-Scholes counterpart. The insights regarding values also apply to delta. That is, the hedging index can increase delta and thus improve incentive effects of restricted options; when the residual correlation is perfect, i.e., when $|\rho_{I,s,m}| = 1$, the incentive effects are restored to their Black-Scholes level as if there were no trading restrictions.

To quantify the above observations, we report the option values as a fraction of their Black-Scholes counterparts in Panel A and the option delta’s in Panel B of Table 3. For all moneyness and maturity combinations, we fix the residual correlation between the index and the stock at $\rho_{I,s,m} = 0.756$. For simplicity, we assume that the option’s maturity coincides with the end of the stock’s vesting period. Again, for each given weight on the restricted stock, $x_s$, we examine three scenarios: $x_I = 0$, $x_I = 0.5x_I^*$ and $x_I = x_I^*$.

Panel A of Table 3 reveals that the deadweight loss associated with options is generally much bigger than that associated with the restricted stock (by comparing with Table 2). The bigger loss is primarily due to the non-linear nature of the option’s payoff. Table 3 also reveals that the deadweight loss is bigger when 1) the weight on the restricted stock is higher, 2) the option is further out-of-the-money, and 3) the option’s time to maturity is longer. More importantly, the introduction of the hedging index can alleviate the deadweight loss substantially. For instance, when $\gamma = 2.0$, $x_s = 0.5$, and $T - t = 10$ years, the deadweight loss for the at-the-money option is about $1 - 36\% = 64\%$ (of its fair market value) when the index is not available; but this loss is
reduced to 33% when the index is optimally held, an improvement of 31%. Moreover, even when the index hedging is constrained at 50% of its optimal level, in most cases, the reduction in deadweight loss is still substantial.

Similar observations can be made regarding delta’s in Panel B of Table 3. In most cases, the improvements due to the introduction of the hedging index are substantial. The results in Table 3 indicate that the hedging index can play an important role in alleviating the deadweight loss and in improving the incentive effects for options.

4.2. Indexed European Options

Johnson and Tian (2000) argue that indexed options are more effective in aligning performance with compensation. Here we consider an index option whose strike price is linked to the index, not the market portfolio. Specifically, the strike price is defined as $K_t = \frac{S_0}{I_0} I_t$. The valuation of such indexed options can be easily carried out using the procedure in Appendix B. For instance, when the option’s maturity coincides with the vesting period, the employee’s private valuation of the indexed option is

$$\tilde{C}_t = C_{BS}(S_t e^{-l_s(T-t)}, T - t, \frac{S_0}{I_0} I_t e^{l_I(T-t)}, q_I, q_s, \Sigma_{ls}) \quad \text{with} \quad \Sigma_{ls}^2 = \sigma_s^2 - 2\rho_{ls}\sigma_s\sigma_I + \sigma_I^2.$$ 

The indexed option on the restricted stock is always worth less than its Black-Scholes counterpart (i.e., the indexed option written on the non-restricted stock) because the stock price is discounted at $l_s$ and the indexed exercise price is compounded at $l_I$. Moreover, the delta of the indexed option on the restricted stock is also lower than its Black-Scholes counterpart. Since the hedging index can reduce both $l_s$ and $l_I$, we know that the deadweight loss of the indexed option will be reduced and its incentive effect improved when the index is included in the employee’s portfolio choice set.

To gain further insights, we report the value of the indexed option on the restricted stock as a fraction of its Black-Scholes counterpart in Panel A and report the delta’s in Panel B of Table 4. Two observations are in order. First, comparing Table 4 with Table 3, we see that the discount is deeper with indexed options. The deadweight loss of granting indexed options is bigger than granting non-indexed options. Intuitively, when the exercise price is indexed, hedging need (by trading the stock)
is higher. Trading restrictions on the stock would therefore lead to a bigger discount for indexed options. Second, being able to take an optimal position on the index can reduce the deadweight loss and improve the incentive effect substantially. Even constrained hedging is always better than no hedging. Other things being equal, the reduction in deadweight loss and the improvement in incentive effects are more pronounced when the correlation between the stock and the index is high. The said correlation plays a critical role. Without hedging, the correlation only enters the valuation through the total variance $\Sigma_{I_4}$, and its impact is clearly negative as apparent in the above formula and in the results in Table 4 (when $x_I = 0$). With hedging though, the correlation affects not only the total variance but also the illiquidity discount $I_4$.

4.3. Options with Early Exercise and Vesting Features

In reality, vesting requirements are usually imposed not only on the stock, but also on options. The vesting period for stock options is usually up to five years within which no exercise is allowed. Beyond the vesting period, options can typically be exercised any time before maturity. Obviously, the vesting requirement for options does not have any impact on European options, but it may reduce the value of American options.

To quantify the above, we calculate option values for the simple case where the option’s maturity coincides with the stock’s vesting period. For this purpose, we construct a 5000-step binomial tree for the dynamics of the private value with a starting price of $S_t e^{-I_s(T-t)}$ during the vesting period. The effective discount rate is $r - \gamma \Omega$.

Table 5 reports the American option value with vesting features as a fraction of its Black-Scholes counterpart. The table shows several vesting scenarios under various parameter combinations. For comparisons, we also report European values in the first panel, and the theoretical market value of European and American options under no trading restrictions. There are several interesting observations. First, American options are always worth more than their European counterparts no matter how long the vesting period is. The difference is larger when the risk aversion and the weight on the restricted stock are high. In addition, the impact of the restricted stock’s weight and the hedging schemes is similar to that for European options shown in Table 3, namely, partial hedging.
or optimal hedging can reduce the option discount substantially. In other words, the deadweight loss can be reduced substantially when the employee is allowed to trade the index.

Second, with only a few exceptions, the ratio of the restricted option’s value over its Black-Scholes counterpart remains almost unchanged (to the second decimal place) across different vesting periods. We can infer that the length of the option’s vesting period does not affect the extent of deadweight loss reduction. What is more important is the vesting period imposed on the stock itself, which directly affects the starting price, $S_t e^{-J_s(T-t)}$ and the effective discount rate, $r - \gamma \Omega$.

Third, the incremental impact of vesting is slightly more pronounced when the vesting period is longer. For instance, for the parameter combination of $x_s = 0.5$, $\gamma = 4.0$, and $x_I = x_I^*$, the non-restricted option’s value remains unchanged (to the second decimal place) when the vesting period increases from 1 year to 3 years; however, it goes from $41.04$ to $40.68$ when the vesting period increases from 7 years to 9 years. Naturally, when the vesting period equals the option’s maturity, the American option becomes an European option.

Incidentally, the difference in value between American and European options is generally small. In fact, the difference is caused purely by the dividend yield. If we allow the stock to be vested right after the exercise, then the American option will be worth more. But the valuation in this case becomes very challenging because the amount of discount on the restricted stock is random and depends on the optimal exercise time. Nonetheless, the value of an American option with vesting on a freely traded stock (i.e., the “theoretical market value” presented in each panel’s heading) serves as the upper bound for the option on the restricted stock.

To examine the incentive effects, we calculate delta for each entry in Table 5 and report the results in Table 6. Unlike values, delta’s of European and American options on the restricted stock tend to be quite different. Delta’s of American options are much higher than their European counterparts. A longer vesting period makes delta lower. Like values, the incremental effects of vesting is larger when the vesting period is longer. Of course, when the vesting period is equal to the option’s maturity, the American option becomes a European option. More important is the hedging effect of the index. It is seen that introducing the index can increase delta, especially when
the stock’s weight is large and when the employee is more risk-averse. The upshot is that, like other types of options, options with early exercise and vesting features will also have an enhanced incentive effect when the employee can trade the index.

Before concluding the study, we would like to offer a few remarks about the incentive implications of our results. Intuitively, the reason that the incentive stocks and options can create incentive effects is because they enhance the employee’s personal wealth. The higher the employee values these assets, the higher the incentive effects these assets can create. However, the vesting requirement forces the employee to bear non-systematic or firm-specific risk, which makes the employee’s personal wealth illiquid. The employee thus values the incentive assets below the market prices. Introducing the hedging index can improve the employee’s consumption-portfolio decisions and align his private valuation with the market’s, thus alleviating the deadweight loss. In contrast to the common belief that forcing employees to bear non-systematic risk is a way to maintain incentives, we argue and demonstrate that employees are actually better off when they can hedge away the exposure to non-systematic risk.

Undoubtedly, the employee’s wealth must be tied to the firm’s fortune in order for incentives to exist. Granting stocks without trading restrictions will not create long-term incentives because the employee can convert the stock into cash and leave the company at his wish. Vesting requirements will achieve the purpose of retaining and motivating employees. The retention effect is obvious and the motivation effect is through the higher value of personal wealth in the form of higher value of the stock. Therefore, bearing the firm’s non-systematic risk is an inherent side-effect, not an intended incentive mechanism. Being stuck with the firm’s non-systematic risk does not mean that the employee will work harder. His ultimate motivation is to increase his personal wealth, which in turn depends on how he manages his personal wealth and how much he values the granted stocks and options. Nothing can create a stronger incentive than a higher private valuation of the incentive securities, and the hedging index is the perfect tool to enhance private valuations.

Retaining talents and enhancing incentives are two aspects of a complete incentive package. Cash bonus and non-restricted stock grants are retrospective incentive vehicles. They are rewards
for realized good performance but do not ensure the long-run stay of the employee. Retaining employees through, say, legal contracting does not necessarily guarantee a higher incentive. An incentive scheme is effective only when both aspects are properly considered. Installing a vesting feature in the compensation scheme would ensure the retention and proper motivation of the employee. But it brings about a negative side-effect which is the deadweight loss. Introducing a hedging index can effectively alleviate this side-effect. The combination of vesting and hedging can help achieve the overall goal. With a hedging index, the trading restrictions will not be as restrictive as they appear.

5. Conclusion

This paper proposes a hedging strategy to reduce the deadweight loss caused by the vesting requirement associated with incentive stocks and options. The analysis is carried out in a continuous-time consumption-portfolio framework. We show that a hedging index can align the employee’s private valuation of the restricted assets with the market’s, thus reducing the deadweight loss while preserving and improving the incentive effects. The main results can be summarized as follows.

First, the hedging index can reduce the deadweight loss of restricted stocks. The deadweight loss is reduced to the lowest level when the index position is optimized. The optimal amount of the index positively depends on the stock’s volatility, the residual correlation between the stock and the index, and negatively depends on the index’s volatility, the correlation between the stock and the market. The higher the residual correlation between the stock and the index, the more effective is the index in reducing the deadweight loss. When the residual correlation is perfect, the deadweight loss can be completely removed. Therefore, the trading restrictions on the incentive stocks may not be as restrictive as we thought. Second, the hedging index can also reduce the deadweight loss of stock options, and does so more effectively than for restricted stocks. Moreover, the hedging index also improves the incentive effects of stock options. The value enhancement and the incentive improvement are observed for conventional European options, indexed European options and options with early exercise and vesting features.
In a nutshell, the vesting requirement lowers the private valuation of the restricted stocks and options, leading to a deadweight loss, in spite of its usefulness in retaining employees and in ensuring long-term incentives. Such a negative impact undermines the overall effectiveness of the incentive mechanism since a high private valuation of the restricted asset is the ultimate source of motivation. Our analysis shows that the inclusion of a hedging index can enhance the employee’s private valuation of restricted stocks and options and therefore improve the effectiveness of the incentive scheme.
References


Appendix

A. Proof of Proposition 2.1

In order to determine the optimal consumption, \(c_t^*\) and portfolio holdings, \(x_{mt}^*\) and \(x_{Bt}^*\), we apply the optimal control rule:

\[
J(W, M, S, I, t) = \max_{c,t} E_t \left[ \int_t^T U(c(\tau), \tau) d\tau \right],
\]

\[
\Psi(x, c; W, M, S, I, t) = U(c(t), t) + L[J],
\]

where \(L[J]\) is the differential generator of \(J\) associated with its control function:

\[
L[J] = \frac{\partial J}{\partial t} + \frac{\partial J}{\partial W} W \mu_W + \frac{\partial^2 J}{\partial W \partial \mu} W \mu^2 + \frac{\partial J}{\partial M} M \mu_M + \frac{\partial^2 J}{\partial W \partial M} WM \text{cov}(W, M) + \frac{1}{2} \frac{\partial^2 J}{\partial M^2} M^2 \sigma^2_m
\]

\[
+ \frac{\partial J}{\partial S} S \mu_S + \frac{\partial^2 J}{\partial W \partial S} WS \text{cov}(W, S) + \frac{\partial^2 J}{\partial M \partial S} \rho_{ms} S \sigma_m M \sigma_M + \frac{1}{2} \frac{\partial^2 J}{\partial S^2} S^2 \sigma^2_s
\]

\[
+ \frac{\partial J}{\partial I} I \mu_I + \frac{\partial^2 J}{\partial W \partial I} WI \text{cov}(W, I) + \frac{\partial^2 J}{\partial M \partial I} \rho_{mi} M \sigma_m I \sigma_I + \frac{\partial^2 J}{\partial I \partial S} \rho_{is} I \sigma_i S \sigma_s + \frac{1}{2} \frac{\partial^2 J}{\partial I^2} I^2 \sigma^2_i.
\]

Now, the bequest function for the employee at time \(T\) is \(J(W, M, S, I, T) = J(W, T)\). The optimal consumption, \(c_t^*\) and the optimal weights, \(x_{mt}^*, x_{Bt}^*\) are solved by maximizing \(\Psi(x, c; W, M, S, I, t)\).

The first order conditions are:

\[
\frac{\partial \Psi}{\partial c} = \frac{\partial U}{\partial c} - \frac{\partial J}{\partial W} = 0, \quad \text{(A.1)}
\]

\[
\frac{\partial \Psi}{\partial x_m} = \frac{\partial J}{\partial W} W \mu_W \frac{\partial x_m}{\partial x_m} + \frac{\partial^2 J}{\partial W^2} W^2 \sigma_W \frac{\partial x_m}{\partial x_m} + \frac{\partial^2 J}{\partial W \partial M} WM \frac{\partial \text{cov}(W, M)}{\partial x_m}
\]

\[
+ \frac{\partial^2 J}{\partial W \partial S} WS \frac{\partial \text{cov}(W, S)}{\partial x_m} + \frac{\partial^2 J}{\partial W \partial I} WI \frac{\partial \text{cov}(W, I)}{\partial x_m} = 0, \quad \text{(A.2)}
\]

\[
\frac{\partial \Psi}{\partial x_I} = \frac{\partial J}{\partial W} W \mu_W \frac{\partial x_I}{\partial x_I} + \frac{\partial^2 J}{\partial W^2} W^2 \sigma_W \frac{\partial x_I}{\partial x_I} + \frac{\partial^2 J}{\partial W \partial M} WM \frac{\partial \text{cov}(W, M)}{\partial x_I}
\]

\[
+ \frac{\partial^2 J}{\partial W \partial S} WS \frac{\partial \text{cov}(W, S)}{\partial x_I} + \frac{\partial^2 J}{\partial W \partial I} WI \frac{\partial \text{cov}(W, I)}{\partial x_I} = 0, \quad \text{(A.3)}
\]

together with

\[
\Psi(x^*, c^*; W, M, S, I, t) = U(c^*(t) + w, t) + L[J] = 0, \quad \text{(A.4)}
\]

\[
J(W, M, S, I, T) = J(W, T). \quad \text{(A.5)}
\]

Tedious algebra confirms the results in Proposition 2.1.
Let $T_0$ be the maturity of the option. We first prove the result for the case $T_0 < T$. In this case, at the time of exercise, the stock is still being restricted. The employee’s value of the non-tradable stock option is therefore given by:

$$\hat{C}_t = E_t \left[ \frac{U_{cT_0}}{U_{ct}} \max(S_{T_0}e^{-\delta(T-T_0)} - K, 0) \right].$$

To compute, We need the joint conditional distribution for the state price deflator and the company’s stock. We have

$$\frac{dU_c}{U_c} = -(r - \gamma \Omega)dt - (s_m - \gamma \beta_s x_s \sigma_m - \gamma \beta_I x^{*}_m \sigma_m)dz_m - \gamma x_s \sigma_s dz_s - \gamma x^{*}_I \sigma_I dz_I$$

and

$$\frac{dS}{S} = (\mu_s - q_s)dt + \sigma_s dz_s.$$

Let $\tau = T_0 - t$, and define

$$x_{T_0} = \ln \frac{U_{cT_0}}{U_{ct}} - \Psi_u$$

and

$$y_{T_0} = \ln \frac{S_{T_0}}{S_t} - \Psi_s$$

with

$$\Psi_u = -(r - \gamma \Omega + \frac{s^2_m + \gamma^2 \Omega}{2}) \tau, \quad \sigma_u = \sqrt{s^2_m + \gamma^2 \Omega},$$

$$\Psi_s = (\mu_s - q_s - \frac{s^2_m}{2}) \tau, \quad \rho = -\frac{1}{\sigma_s \sigma_u} (\mu_s - r + \frac{\gamma \Omega}{\sigma_s}).$$

Then the joint conditional density for $x_{T_0}$ and $y_{T_0}$ is:

$$f(x_{T_0}, y_{T_0}) = \frac{1}{2\pi \sqrt{1 - \rho^2}} \exp \left( -\frac{x_{T_0}^2 + 2\rho x_{T_0} y_{T_0} + y_{T_0}^2}{2(1 - \rho^2)} \right).$$

It is easy to show that

$$\frac{U_{cT_0}}{U_{ct}} S_{T_0} f(x_{T_0}, y_{T_0}) = S_t e^{-\delta(T_0-t)} e^{\Psi_u + \Psi_s + \frac{1}{2} (s^2_u + 2\rho \sigma_u \sigma_s + \sigma^2_s) \times \frac{1}{\sqrt{2\pi}} \exp \left( -\frac{1}{2} \left( y_{T_0} - \rho \sigma_u \sqrt{1 - \sigma_s \sqrt{1}} \right)^2 \right)}$$

$$\times \frac{1}{\sqrt{2\pi(1 - \rho^2)}} \exp \left( -\frac{(x_{T_0} - \rho \sigma_u \sqrt{1 - \sigma_s \sqrt{1}})^2}{2(1 - \rho^2)} \right).$$

Therefore,

$$\int_{-\infty}^{\infty} \int_{\ln K - \Psi_u}^{\Psi_s} \frac{U_{cT_0}}{U_{ct}} S_{T_0} e^{-\delta(T-T_0)} \cdot f(x_{T_0}, y_{T_0}) dx_{T_0} dy_{T_0}$$

$$= S_t e^{-\delta(T-T_0)} \exp \left[ \Psi_s + \Psi_u + \frac{1}{2} \left( s^2_u + 2\rho \gamma \sigma_u \sigma_s + \sigma^2_s \right) \right] \int_{-\infty}^{\infty} \frac{e^{-\frac{1}{2} \omega^2}}{\sqrt{2\pi}} d\omega$$

$$= S_t e^{-\delta(T-t)} e^{-q_0(T_0 - t)} N \left[ d_1 (r - \gamma \Omega, q_s) \right].$$
where \( d_1(\cdot, \cdot) \) is defined in the text and stock price takes the value \( S_t e^{-r(T-t)} \). Similarly, we have

\[
\frac{U_{cT}}{U_{cT}} \cdot f(x_{T_0}, y_{T_0}) = e^{\Psi_u + \frac{1}{2} \sigma_u^2 \tau} \times \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2} (y_{T_0} - \rho \sigma_u \sqrt{\tau})^2\right) \times 1 - \frac{1}{\sqrt{2\pi(1-\rho^2)}} \exp\left(-\frac{(x_{T_0} - \rho y_{T_0} - (1-\rho^2) \sigma_u \sqrt{\tau})^2}{2(1-\rho^2)}\right).
\]

Therefore,

\[
\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{U_{cT}}{U_{cT}} K \cdot f(x_{T_0}, y_{T_0}) dx_{T_0} dy_{T_0} = K e^{\Psi_u + \frac{1}{2} \sigma_u^2 \tau} \int_{-d_2(r-\gamma \Omega, q_s)}^{d_2(r-\gamma \Omega, q_s)} e^{-\frac{\omega^2}{2\pi}} d\omega = K e^{- (r - \gamma \Omega)(T_0 - t)} N \left[ d_2(r - \gamma \Omega, q_s) \right].
\]

Now, for the case \( T_o > T \), the stock obtains its market value at the time of exercise, but the discounting has to be done in two stages. Specifically, the employee’s value of the non-tradable stock option is given by:

\[
\tilde{C}_t = E_t \left[ \frac{U_{cT}}{U_{cT}} ET \left( \frac{U_{cT_o}}{U_{cT}} \max(S_{T_o} - K, 0) \right) \right].
\]

Following similar procedures as above and using the fact that

\[
\int_{-\infty}^{\infty} N(A + Bz) \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz = N\left(\frac{A}{\sqrt{1+B^2}}\right)
\]

we obtain the formulas as shown in the text.
Table 1: Optimal Portfolio Holdings and Utility Level with or without Index

<table>
<thead>
<tr>
<th>Portfolio Feasible Set</th>
<th>Optimal Portfolio Holdings</th>
<th>Ultimate Portfolio Holdings</th>
<th>Utility Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>B, M and Assets 1, 2, 3</td>
<td>0.348 0.652</td>
<td>0.348 0.236 0.225 0.191</td>
<td>100.00</td>
</tr>
</tbody>
</table>

**Panel A: Without Trading Restriction**

**Case 1:** B, M and Asset 1
- \( x_k = 0.295 \) 0.605 0.100
- \( x_m = 0.295 0.319 0.209 0.178 \)
- \( w \_b = 99.36 \)

**Case 2:** B, M, Asset 1 and Index 12
- \( x_k = 0.311 0.652 0.100 -0.062 \)
- \( x_m = 0.311 0.304 0.194 0.191 \)
- \( w \_b = 99.46 \)

**Case 3:** B, M, Asset 1 and Index 23
- \( x_k = 0.265 0.731 0.100 -0.095 \)
- \( x_m = 0.265 0.364 0.202 0.169 \)
- \( w \_b = 99.55 \)

**Case 4:** B, M, Asset 1 and Index 23.m
- \( x_k = 0.348 0.375 0.100 0.176 \)
- \( x_m = 0.348 0.236 0.225 0.191 \)
- \( w \_b = 100.00 \)

**Panel B: Trading Restrictions:** \( x = 0.1 \) 0.236)

**Case 1:** B, M and Asset 1
- \( x_k = 0.079 0.421 0.500 0.079 0.652 0.145 0.123 \)
- \( w \_b = 85.16 \)

**Case 2:** B, M, Asset 1 and Index 12
- \( x_k = 0.159 0.652 0.500 -0.311 0.159 0.576 0.073 0.191 \)
- \( w \_b = 87.30 \)

**Case 3:** B, M, Asset 1 and Index 23
- \( x_k = -0.071 1.047 0.500 -0.476 -0.071 0.879 0.112 0.080 \)
- \( w \_b = 89.29 \)

**Case 4:** B, M, Asset 1 and Index 23.m
- \( x_k = 0.348 -0.731 0.500 0.882 0.348 0.236 0.225 0.191 \)
- \( w \_b = 100.00 \)

**Panel C: Trading Restrictions:** \( x = 0.5 \) 0.236)

**Note:**
1. Parameter values: \( \phi = 0.03, \gamma = 4.0, r = 0.06, \mu_1 = 0.10, \mu_2 = 0.15, \mu_3 = 0.20, \sigma_1 = 0.15, \sigma_2 = 0.25, \sigma_3 = 0.35, \rho_{12} = 0.2, \rho_{13} = 0.3, \rho_{23} = 0.4. \)

2. To calculate the utility level, the agent’s life horizon is set to be \( T = 40 \) and the vesting period is set at \( T = 10 \). The utility level under no trading restriction is normalized to 100.
Table 2: Private Value of the Restricted Stock as a Fraction of its Market Value

<table>
<thead>
<tr>
<th></th>
<th>γ</th>
<th>ρ_{ms}</th>
<th>ρ_{l}</th>
<th>T-t</th>
<th>ρ_{I, l}</th>
<th>x_{I} = 0.1</th>
<th>x_{I} = 0.3</th>
<th>x_{I} = 0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2</td>
<td>0.2</td>
<td>0.6</td>
<td>0.6</td>
<td>0.6</td>
<td>0.985</td>
<td>0.990</td>
<td>0.990</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.2</td>
<td>1</td>
<td>0.6</td>
<td>0.6</td>
<td>0.990</td>
<td>0.990</td>
<td>0.990</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.6</td>
<td>0.6</td>
<td>0.6</td>
<td>0.6</td>
<td>0.990</td>
<td>0.990</td>
<td>0.990</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.6</td>
<td>0.6</td>
<td>0.6</td>
<td>0.6</td>
<td>0.990</td>
<td>0.990</td>
<td>0.990</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.2</td>
<td>0.6</td>
<td>0.6</td>
<td>0.6</td>
<td>0.990</td>
<td>0.990</td>
<td>0.990</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.2</td>
<td>1</td>
<td>0.6</td>
<td>0.6</td>
<td>0.990</td>
<td>0.990</td>
<td>0.990</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.6</td>
<td>0.6</td>
<td>0.6</td>
<td>0.6</td>
<td>0.990</td>
<td>0.990</td>
<td>0.990</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.6</td>
<td>0.6</td>
<td>0.6</td>
<td>0.6</td>
<td>0.990</td>
<td>0.990</td>
<td>0.990</td>
</tr>
</tbody>
</table>

Note:
1. This table reports the implied value of the restricted stock as a fraction of the market value for different combinations of the risk aversion, γ, the correlation between the stock and the market, ρ_{ms}, the correlation between the stock and the hedging index, ρ_{I, l}, and the vesting period, T-t. For each parameter combination, we examine two levels of the stock's weight relative to the total wealth. Under each stock weight, we examine three index scenarios: no hedging (x_{I} = 0), hedging with a binding constraint on the index (50% of the optimal weight), and hedging with the optimal weight (x_{I} = x_{I}^{*}).
2. Other parameter inputs: σ_{s} = 0.30, σ_{l} = 0.25, ρ_{ml} = 0.5.
Table 3: Value Discount and Incentive Effect (Delta) of European Stock Options

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>$\gamma$</th>
<th>$\gamma$</th>
<th>$\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>$\delta$</td>
<td>$\delta$</td>
<td>$\delta$</td>
</tr>
</tbody>
</table>

Panel A: Option Value (as a fraction of Black-Scholes)

<table>
<thead>
<tr>
<th>$x_I$</th>
<th>$x_I$</th>
<th>$x_I$</th>
<th>$x_I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_I$</td>
<td>$x_I$</td>
<td>$x_I$</td>
<td>$x_I$</td>
</tr>
<tr>
<td>$K = $115, \ T - t = 10\text{ years}, \ Black-Scholes = $36.73$</td>
<td>$K = $115, \ T - t = 10\text{ years}, \ Black-Scholes = $36.73$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$0.1$</td>
<td>$0.0$</td>
<td>$0.3$</td>
<td>$0.5$</td>
</tr>
<tr>
<td>$0.3$</td>
<td>$0.0$</td>
<td>$0.3$</td>
<td>$0.5$</td>
</tr>
<tr>
<td>$0.5$</td>
<td>$0.0$</td>
<td>$0.3$</td>
<td>$0.5$</td>
</tr>
</tbody>
</table>

Panel B: Option Delta

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>$\gamma$</th>
<th>$\gamma$</th>
<th>$\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>$\delta$</td>
<td>$\delta$</td>
<td>$\delta$</td>
</tr>
</tbody>
</table>

Note:
1. This table reports the ratio of the European stock option’s value over its Black-Scholes counterpart (Panel A) and delta of the European stock option (Panel B). For each moneyness and maturity combination, we examine the ratio or value discount and delta for two levels of risk aversion ($\gamma = 2, 4$), three levels of the stock’s weight relative to the total wealth ($x_I = 0, 0.3, 0.5$), and three index scenarios: no hedging ($x_I = 0$), hedging with a binding constraint equal to 50% of the optimal weight ($x_I = 0.5x_1^*$), and hedging with the optimal weight ($x_I = x_1^*$).
2. Other parameter inputs: $\sigma_s = 0.3, \sigma_I = 0.25, \rho_{ms} = 0.4, \rho_{mI} = 0.5, \rho_{ts} = 0.8, \rho_{ts.m} = 0.756, r = 0.06, q_s = 0.02$.
3. For simplicity, we assume that the option’s maturity coincides with the stock’s vesting period.
### Table 4: Value Discount and Incentive Effect (Delta) of Indexed European Stock Options

<table>
<thead>
<tr>
<th>$x_1$</th>
<th>T-t</th>
<th>Panel A: Option Value (as a fraction of Black-Scholes)</th>
<th>Panel B: Option Delta</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\gamma = 2.0$</td>
<td>$\gamma = 4.0$</td>
</tr>
<tr>
<td></td>
<td>$x_1 = 0$</td>
<td>$x_1 = 0.5x_1$</td>
<td>$x_1 = x_1^*$</td>
</tr>
<tr>
<td>10 years</td>
<td>$\rho_h = 0.4$ ($\rho_{BS} = 0.252$), BS(non-indexed) = $40.43$, BS(indexed) = $32.84$</td>
<td>$\rho_h = 0.4$ ($\rho_{BS} = 0.252$), BS(non-indexed) = $0.667$, BS(indexed) = $0.590$</td>
<td></td>
</tr>
<tr>
<td>$x_1 = 0$</td>
<td>0.750</td>
<td>0.765</td>
<td>0.782</td>
</tr>
<tr>
<td>$x_1 = 0.5x_1$</td>
<td>0.441</td>
<td>0.468</td>
<td>0.502</td>
</tr>
<tr>
<td>$x_1 = x_1^*$</td>
<td>0.275</td>
<td>0.301</td>
<td>0.343</td>
</tr>
<tr>
<td>5 years</td>
<td>$\rho_h = 0.6$ ($\rho_{BS} = 0.504$), BS(non-indexed) = $40.43$, BS(indexed) = $28.04$</td>
<td>$\rho_h = 0.6$ ($\rho_{BS} = 0.504$), BS(non-indexed) = $0.667$, BS(indexed) = $0.572$</td>
<td></td>
</tr>
<tr>
<td>$x_1 = 0$</td>
<td>0.694</td>
<td>0.743</td>
<td>0.799</td>
</tr>
<tr>
<td>$x_1 = 0.5x_1$</td>
<td>0.350</td>
<td>0.418</td>
<td>0.528</td>
</tr>
<tr>
<td>$x_1 = x_1^*$</td>
<td>0.196</td>
<td>0.242</td>
<td>0.363</td>
</tr>
</tbody>
</table>

Note:
1. This table reports the ratio of the indexed European stock option’s value over its Black-Scholes counterpart (Panel A) and delta of the indexed European stock option (Panel B). For each correlation level $\rho_{S I}$, we examine the ratio or value discount and delta for two levels of risk aversion ($\gamma = 2, 4$), three levels of the stock’s weight relative to the total wealth ($x_S = 0.1, 0.3, 0.5$), and three index scenarios: no hedging ($x_I = 0$), hedging with a binding constraint equal to 50% of the optimal weight ($x_I = 0.5x_I^*$), and hedging with the optimal weight ($x_I = x_I^*$).
2. Other parameter inputs: $\sigma_S = 0.3$, $\sigma_I = 0.25$, $\rho_{MS} = 0.4$, $\rho_{MI} = 0.5$, $T - t = 10$ years, $r = 0.06$, $q_S = 0.02$, $q_I = 0.03$, $S_0/I_0 = 1$, $S_I = $100, and $I_I = $100.
3. For simplicity, we assume that the option’s maturity coincides with the stock’s vesting period.
Table 5: Private vs. Market Values of Stock Options with Vesting and Early Exercise Features

<table>
<thead>
<tr>
<th>$x_s$</th>
<th>$x_I = 0$</th>
<th>$x_I = 0.5x_s^*$</th>
<th>$x_I = x_s^*$</th>
<th>$x_I = 0$</th>
<th>$x_I = 0.5x_s^*$</th>
<th>$x_I = x_s^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.786</td>
<td>0.841</td>
<td>0.903</td>
<td>0.611</td>
<td>0.704</td>
<td>0.814</td>
</tr>
<tr>
<td>0.3</td>
<td>0.513</td>
<td>0.616</td>
<td>0.762</td>
<td>0.232</td>
<td>0.357</td>
<td>0.568</td>
</tr>
<tr>
<td>0.5</td>
<td>0.357</td>
<td>0.470</td>
<td>0.670</td>
<td>0.087</td>
<td>0.184</td>
<td>0.422</td>
</tr>
</tbody>
</table>

European Option (Black-Scholes = $40.43$)

<table>
<thead>
<tr>
<th>$x_s$</th>
<th>$x_I = 0$</th>
<th>$x_I = 0.5x_s^*$</th>
<th>$x_I = x_s^*$</th>
<th>$x_I = 0$</th>
<th>$x_I = 0.5x_s^*$</th>
<th>$x_I = x_s^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.784</td>
<td>0.840</td>
<td>0.902</td>
<td>0.608</td>
<td>0.701</td>
<td>0.813</td>
</tr>
<tr>
<td>0.3</td>
<td>0.513</td>
<td>0.615</td>
<td>0.761</td>
<td>0.233</td>
<td>0.356</td>
<td>0.568</td>
</tr>
<tr>
<td>0.5</td>
<td>0.365</td>
<td>0.473</td>
<td>0.674</td>
<td>0.095</td>
<td>0.188</td>
<td>0.430</td>
</tr>
</tbody>
</table>

American Option, vesting = 0 year (Black-Scholes = $41.26$)

<table>
<thead>
<tr>
<th>$x_s$</th>
<th>$x_I = 0$</th>
<th>$x_I = 0.5x_s^*$</th>
<th>$x_I = x_s^*$</th>
<th>$x_I = 0$</th>
<th>$x_I = 0.5x_s^*$</th>
<th>$x_I = x_s^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.784</td>
<td>0.840</td>
<td>0.902</td>
<td>0.608</td>
<td>0.701</td>
<td>0.813</td>
</tr>
<tr>
<td>0.3</td>
<td>0.513</td>
<td>0.615</td>
<td>0.761</td>
<td>0.233</td>
<td>0.356</td>
<td>0.568</td>
</tr>
<tr>
<td>0.5</td>
<td>0.365</td>
<td>0.473</td>
<td>0.674</td>
<td>0.095</td>
<td>0.188</td>
<td>0.430</td>
</tr>
</tbody>
</table>

American Option, vesting = 1 year (Black-Scholes = $41.26$)

<table>
<thead>
<tr>
<th>$x_s$</th>
<th>$x_I = 0$</th>
<th>$x_I = 0.5x_s^*$</th>
<th>$x_I = x_s^*$</th>
<th>$x_I = 0$</th>
<th>$x_I = 0.5x_s^*$</th>
<th>$x_I = x_s^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.785</td>
<td>0.840</td>
<td>0.903</td>
<td>0.609</td>
<td>0.702</td>
<td>0.813</td>
</tr>
<tr>
<td>0.3</td>
<td>0.513</td>
<td>0.615</td>
<td>0.761</td>
<td>0.233</td>
<td>0.356</td>
<td>0.568</td>
</tr>
<tr>
<td>0.5</td>
<td>0.365</td>
<td>0.473</td>
<td>0.674</td>
<td>0.094</td>
<td>0.188</td>
<td>0.430</td>
</tr>
</tbody>
</table>

American Option, vesting = 3 years (Black-Scholes = $41.26$)

<table>
<thead>
<tr>
<th>$x_s$</th>
<th>$x_I = 0$</th>
<th>$x_I = 0.5x_s^*$</th>
<th>$x_I = x_s^*$</th>
<th>$x_I = 0$</th>
<th>$x_I = 0.5x_s^*$</th>
<th>$x_I = x_s^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.785</td>
<td>0.840</td>
<td>0.903</td>
<td>0.609</td>
<td>0.702</td>
<td>0.813</td>
</tr>
<tr>
<td>0.3</td>
<td>0.513</td>
<td>0.615</td>
<td>0.761</td>
<td>0.233</td>
<td>0.356</td>
<td>0.568</td>
</tr>
<tr>
<td>0.5</td>
<td>0.364</td>
<td>0.472</td>
<td>0.673</td>
<td>0.093</td>
<td>0.188</td>
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</table>

American Option, vesting = 5 years (Black-Scholes = $41.21$)

<table>
<thead>
<tr>
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<th>$x_I = 0.5x_s^*$</th>
<th>$x_I = x_s^*$</th>
<th>$x_I = 0$</th>
<th>$x_I = 0.5x_s^*$</th>
<th>$x_I = x_s^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.786</td>
<td>0.841</td>
<td>0.903</td>
<td>0.610</td>
<td>0.703</td>
<td>0.814</td>
</tr>
<tr>
<td>0.3</td>
<td>0.513</td>
<td>0.616</td>
<td>0.762</td>
<td>0.233</td>
<td>0.357</td>
<td>0.568</td>
</tr>
<tr>
<td>0.5</td>
<td>0.360</td>
<td>0.471</td>
<td>0.671</td>
<td>0.090</td>
<td>0.186</td>
<td>0.425</td>
</tr>
</tbody>
</table>

American Option, vesting = 7 years (Black-Scholes = $40.68$)

<table>
<thead>
<tr>
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<th>$x_I = 0.5x_s^*$</th>
<th>$x_I = x_s^*$</th>
<th>$x_I = 0$</th>
<th>$x_I = 0.5x_s^*$</th>
<th>$x_I = x_s^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.786</td>
<td>0.841</td>
<td>0.903</td>
<td>0.610</td>
<td>0.703</td>
<td>0.814</td>
</tr>
<tr>
<td>0.3</td>
<td>0.513</td>
<td>0.616</td>
<td>0.762</td>
<td>0.233</td>
<td>0.357</td>
<td>0.568</td>
</tr>
<tr>
<td>0.5</td>
<td>0.360</td>
<td>0.471</td>
<td>0.671</td>
<td>0.090</td>
<td>0.186</td>
<td>0.425</td>
</tr>
</tbody>
</table>

American Option, vesting = 9 years (Black-Scholes = $40.68$)

Note:
1. This table reports the value of stock options with vesting and early exercise features as a fraction of their Black-Scholes counterpart. For comparison, we also report European values in the first panel. For each vesting period, we examine the fractions for two levels of risk aversions ($\gamma = 2, 4$), three levels of the stock’s weight relative to the total wealth ($x_s = 0.1, 0.3, 0.5$), and three index scenarios: no hedging ($x_I = 0$), hedging with a binding constraint equal to 50% of the optimal weight ($x_I = 0.5x_s^*$), and hedging with the optimal weight ($x_I = x_s^*$).
2. Other parameter inputs: $\sigma_s = 0.3$, $\sigma_I = 0.25$, $\rho_{ms} = 0.4$, $\rho_{mI} = 0.5$, $\rho_{Is} = 0.8$, $r = 0.06$, $q_s = 0.02$, $S_t = $100, $K = $100, $T - t = 10$ years.
3. For simplicity, we assume that the option’s maturity coincides with the end of the stock’s vesting period.
4. American option values are calculated using a binomial tree with 5000 steps.
Table 6: Incentive Effects (Delta) of Stock Options with Vesting and Early Exercise Features

<table>
<thead>
<tr>
<th>$x_s$</th>
<th>$\gamma = 2.0$</th>
<th>$\gamma = 4.0$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$x_t = 0.0$</td>
<td>$x_t = 0.5 x_I$</td>
</tr>
<tr>
<td></td>
<td>$x_I$ = 0.0</td>
<td>$x_I$ = 0.5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>European Option (Black-Scholes = 0.667)</th>
<th>American Option, vesting = 0 year (Black-Scholes = 0.695)</th>
<th>American Option, vesting = 1 year (Black-Scholes = 0.695)</th>
<th>American Option, vesting = 3 years (Black-Scholes = 0.695)</th>
<th>American Option, vesting = 5 years (Black-Scholes = 0.692)</th>
<th>American Option, vesting = 7 years (Black-Scholes = 0.686)</th>
<th>American Option, vesting = 9 years (Black-Scholes = 0.676)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.550</td>
<td>0.680</td>
<td>0.680</td>
<td>0.680</td>
<td>0.680</td>
<td>0.680</td>
<td>0.680</td>
</tr>
<tr>
<td>0.3</td>
<td>0.395</td>
<td>0.656</td>
<td>0.656</td>
<td>0.656</td>
<td>0.656</td>
<td>0.656</td>
<td>0.656</td>
</tr>
<tr>
<td>0.5</td>
<td>0.303</td>
<td>0.615</td>
<td>0.615</td>
<td>0.615</td>
<td>0.615</td>
<td>0.615</td>
<td>0.615</td>
</tr>
</tbody>
</table>

Note:
1. This table reports delta of stock options with vesting and early exercise features. For comparison, we also report delta’s for European options in the first panel. For each vesting period, we examine delta’s for two levels of risk aversions ($\gamma = 2, 4$), three levels of the stock’s weight relative to the total wealth ($x_s = 0.1, 0.3, 0.5$), and three index scenarios: no hedging ($x_I = 0$), hedging with a binding constraint equal to 50% of the optimal weight ($x_I = 0.5 x_I^*$), and hedging with the optimal weight ($x_I = x_I^*$).
2. Other parameter inputs: $\sigma_s = 0.3$, $\sigma_I = 0.25$, $\rho_{ms} = 0.4$, $\rho_{mI} = 0.5$, $\rho_{Is} = 0.8$, $r = 0.06$, $q_s = 0.02$, $S_t = $100, $K = $100, $T - t = 10$ years.
3. For simplicity, we assume that the option’s maturity coincides with the end of the stock’s vesting period.
4. Delta’s for American options are calculated using a binomial tree with 5000 steps.