Chapter 8

Charge/Mass Ratio of Electron & Induction

Experiments in the 19th C. caused great changes in the concepts of Physics, changes that we are still exploring today. In today's lab session we shall examine two of those experiments, experiments which led to the concept of the FIELD (instead of action-at-a-distance) and to the ideas of fundamental particles and the "graininess" of matter. These same experiments along with a few others, led to a search (yet to be fully realized) for a mechanism which would underlie and unify electricity, light, magnetism, and gravity.

Objective

1. To determine the charge/mass ratio for an electron.
2. To measure the electromotive force induced in a coil.

Apparatus Complete unit to measure charge/mass ratio, bar magnet, compass needle, air coils, digital multimeter, signal generator, safety glasses, graph paper.

8.1 Charge/Mass ratio for electrons

8.1.1 Introduction

The relationship between magnetism and electricity was discovered in 1819 by the Danish physicist H. Oersted, who found that an electric current in a wire causes a deflection of a nearby compass needle. Later Ampere found that magnetic fields exert a force on a current-carrying wire placed in the magnetic field. The mutual interaction between an electric current and a magnetic field can be explained by the assumption that the electric current is the source of the magnetic field.

The magnetic field produced by the electric current can then interact with the magnetic field of a permanent magnet. It was discovered that the magnetism of a permanent magnet also results from tiny currents formed by electrons spinning and rotating around the nucleus of an atom (Andre Ampere).

The electric current is caused by moving electric charges. Thus we can expect that a magnetic force acts on moving charges. This force is

$$\vec{F} = q\vec{v} \times \vec{B}$$
$$q = \text{ electric charge, includes a + or - sign}$$
$$\vec{v} = \text{ velocity of the charge}$$
$$\vec{B} = \text{ magnetic field}$$

The direction of the force $\vec{F}$ is determined by the cross product $\vec{v} \times \vec{B}$ and whether the charge is negative or positive.

Fig. 8.1 shows how to apply a right-hand rule to find the direction of for a positively charged particle (turn the vector $\vec{v}$ into $\vec{B}$ using the four fingers of the right hand, with the palm facing vector $\vec{B}$;the thumb shows the direction of $\vec{F}$). For a negatively charged particle the direction is reversed.

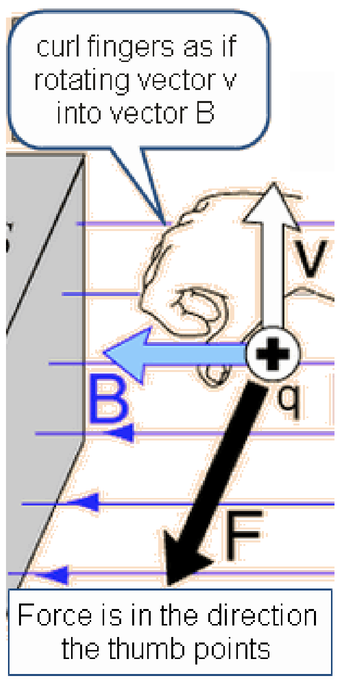

Figure 8.1: Right-hand rule for Faraday's Law

As seen in Fig. 8.1, the magnetic force $\vec{F}$ acting on a moving charged particle is always perpendicular to both the velocity $\vec{v}$ and magnetic field $\vec{B}$. Consider the case when an electron of charge e is injected into a magnetic field (Fig. 8.2) whose direction is into the page.

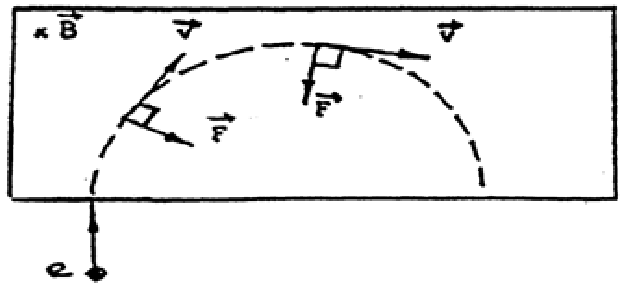

Figure 8.2: Charge moving in a magnetic field

The magnetic field in Fig. 8.2 is confined to the rectangular region (difficult to do in practice) and perpendicular to the page (into the page). When an electron enters the magnetic field the force F of size $= evB$ starts to act on the electron. The force will deflect the electrons slightly. As the direction of $\vec{v}$ changes, so does the direction of $\vec{F}$. As a result the electron moves in a circle perpendicular to the direction of $\vec{B}$. The magnetic force F of size evB is a centripetal force, so that

$$evB = \frac{mv^2}{r} \quad \text{where m is the mass of an electron}$$

$$r = \frac{mv}{eB} \tag{8.1}$$

The purpose of this experiment is to determine the value of the ratio e/m. from Eq. 8.1.

$$\frac{e}{m} = \frac{v}{rB} \tag{8.2}$$

8.1.2 Experimental

Electrons are produced by the electron gun in a partially evacuated glass tube. The tube contains helium vapour (at a very low pressure) to make the electron beam visible. After collisions with the electrons, helium molecules get excited and radiate visible light. Electrons are emitted by a heated cathode (thermal emission) and accelerated in an electric field between the cathode and the anode. The increase in the kinetic energy of the electrons $mv^2/2$ comes from the change of potential energy eV of the electrons in the electric field.

$$\frac{1}{2}mv^2 = eV + \frac{1}{2}mv_0^2 \quad (v_0 \approx 0m/s)$$

$$v = (\frac{2eV}{m})^{\frac{1}{2}} \tag{8.3}$$

The magnetic field is produced by a constant current flowing in two coils surrounding the tube. The coils, called Helmholtz coils, produce a relatively uniform magnetic field in the central volume of the tube. The magnitude of the magnetic field depends on the number of turns, the radius of the coils and the magnitude of the current.

The relationship for magnetic field strength to current for the particular Helmholtz coils you will be using is:

$$B = 10.27 \times 10^{-4} I \tag{8.4}$$

To use Formula 8.4, the current I must be in amperes [A] and the magnetic field will be in Tesla [T].

Our goal will be to determine the charge to mass ratio e/m of the electron. By combining equations 8.2, 8.3 and 8.4 we obtain an expression for the electron e/m in terms of experimentally measurable quantities:

$$\frac{e}{m} = \frac{V}{52.73 \times 10^{-8} I^2 r^2} \tag{8.5}$$

The apparatus is shown in Fig. 8.3. The e/m tube is mounted on a rotatable socket and is located between two Helmholtz coils. The accelerating voltage is controlled by the second knob from the left. The knob and the switch on the left-hand-side of the control panel play no role in the experiment and should not be used. The current control knob serves to change the current in the Helmholtz coils. The direction of the current can be changed between clockwise to counter-clockwise using a switch. The tube, Helmholtz coils, voltmeter, ammeter and a reversing switch are "integrated" into one compact unit.

The diameter of the electron beam path is measured using a scale and a moveable eyepiece, which is a hollow tube fitted with cross wires. The entire scale is movable up and down and should pass through the center of the circular path of electrons.

1. Electron Tube	6. Fixed Index	11. Deflection V polarity switch
2. Helmholtz Coil	7. CCW current Indicator	12. Accelerating V Control
3. Sliding Index	8. CW current Indicator	13. Current Direction Switch
4. Scale	9. Dark Box	14. Current Control
5. Angular Index	10. Deflection V Control	15. Power Switch

8.1.3 Prelab Exercise 1

1. In Fig. 8.2, the electron moves in a circle of radius $r = 0.051cm$. The magnetic field $B = 0.64T$. Determine the speed of the electron. The mass and charge of the electron can be found in the textbook. How many times is the speed of the electron smaller than speed of light $c = 3.0 \times 10^8 m/s$?

2. The magnetic field due to Earth is approximately equal to $0.5 \times 10^{-4} T$. What current is required in Helmholtz coils used in the lab to produce magnetic field at least ten times larger than the magnetic field due to Earth (use equation 8.4)?

Figure 8.3: Electron e/m apparatus

8.1.4 Measurement and Calculations

1. Set the apparatus controls as follows: Deflection Voltage to minimum, Deflection Polarity to OFF, Accelerating Voltage to minimum, magnetizing current to minimum, magnetizing current direction to OFF.

2. Turn the power switch to ON, and allow the tube to warm up (verify the heating

element attached to the cathode starts glowing).

3. Slowly increase the accelerating voltage to about 200 V. You should now see a straight blue/green beam of electrons.

4. Set the magnetizing current direction to CW and adjust the Current Control of the B-field coils to 1 Ampere. Now, the electron beam is immersed in a magnetic field which is parallel to the axis of the coils.

5. Being very careful not to crack the delicate glass electron tube, bring a bar magnet near the tube and observe its effect on the beam. Using the right-hand rule (Fig. 8.1) determine the sign of charged particles.

6. By gripping the black base of the electron tube, rotate the socket of the tube until the path is a closed circle. Observe the effect of changing the magnitude of the current. Change the direction of the current to the counter-clockwise. Record your observations.

7. Using a compass needle, find the direction of the magnetic field inside the Helmholtz coils for the clockwise and counter-clockwise direction of the current. Draw a clear diagram.

8. Set the current in the Helmholtz coils to about $1.0A$. Vary the electron acceleration voltage from 250 to $150V$ in steps of $20V$ and record the diameter of the circular beam for these six voltages. Record the outer diameter of the beam in each case, since the inner diameter is less sharp, and due to slowed-down electrons.

9. **Switch all control dials to minimum and shut off the main power switch.**

10. To plot the radius squared vs the acceleration voltage use the Excel worksheet on the desktop called *Charge to Mass.xlsx*. Enter your values in the preformatted table. A least squares fit to the values will be automatically generated. Use the values from the excel table and create a graph of the radius squared vs the acceleration voltage using the provided graph paper.

8.1.5 Questions

1. Why do electrons spread out (the beam gets broader) as they move along the circular path?

2. Sketch the trajectory of electrons when their velocity is not perpendicular to the magnetic field produced by the Helmholtz coils. Verify your prediction by rotating the e/m tube. Be extremely careful and use safety glasses. Sketch also the trajectories of electrons with decreasing speeds in the same magnetic field.

3. Does the magnetic field due to the Earth affect your result of the e/m ratio? Explain!

4. Compare trajectories of electrons and protons moving with the same speed in the magnetic field produced by the Helmholtz coils?

8.2 Electromagnetic Induction

8.2.1 Introduction

Experiments conducted by M. Faraday in 1831 showed that an electric current could be induced in a circuit by a changing magnetic field. Faraday showed that current could be set up in the circuit even though there were no batteries in the circuit! This result, combined with Oersted's experiment suggested an inter connection of electricity and magnetism.

Faraday's Law of Induction says that the induced potential difference in the circuit called electromotive force (emf) is equal to the negative of the time rate of change of the magnetic flux ϕ through the circuit.

$$\epsilon = -\frac{d\phi}{dt} \tag{8.6}$$

The magnetic flux through the planar loop of area A is equal to

$$\phi = BA\cos\theta = \vec{B}\cdot\vec{A} \tag{8.7}$$

B = magnetic field

θ = angle between $\vec{B}$ vector and the direction perpendicular to the loop,i.e. the direction of $\vec{A}$.

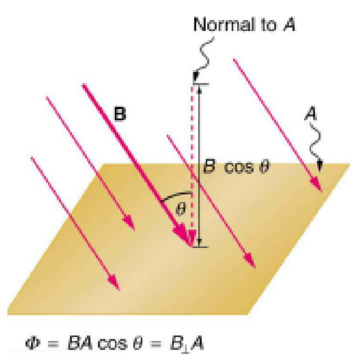

Figure 8.4: Magnetic flux through a surface

The electromotive force can be induced in the circuit in three different ways:

1. the magnitude of $\vec{B}$ can vary with time
2. the area of the circuit $\vec{A}$ can change with time
3. the angle θ can be time dependent

You will study case 1.

The source of the magnetic field is the current flowing in the solenoid. A solenoid is a long wire wound in the form of a helix.

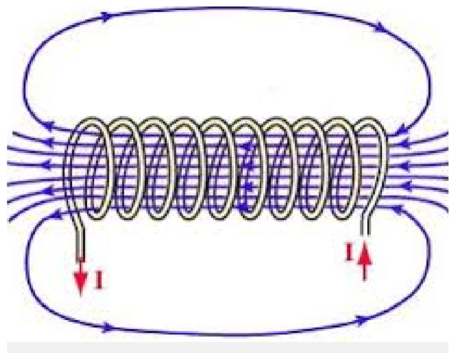

Figure 8.5: Solenoid with its generated magnetic field.

The size of the magnetic field inside the solenoid depends on the magnitude of the current I and the number of turns N, in the solenoid.

$$B \propto NI \tag{8.8}$$

If the turns of the solenoid are closely spaced, the magnetic field inside the solenoid is relatively uniform and strong. The magnetic field distribution is similar to that caused by a bar magnet.

If the current flowing in the solenoid is changing sinusoidally when being driven by a signal generator, then the magnetic field produced by this current changes sinusoidally as well. Imagine that another small coil (called a secondary coil) which is not connected to a voltage source is put inside the solenoid in such a way that the axes of the solenoid and small coil coincide. The variable magnetic field inside the solenoid will induce an emf in the secondary coil.

According to Faraday's Law, the emf induced in the secondary coil is

$$\begin{aligned} \epsilon = -N_2\frac{d\phi}{dt} &\quad N_2 \text{ the number of turns in the secondary coil} \\ \phi = BA, \text{ as } \cos\theta = 1 &\quad \text{because } B \text{ is perpendicular to the surface area of the secondary coil, } A \\ B \propto N_1 I &\quad N_1 \text{ the number of turns in the primary coil} \\ &\quad I \quad \text{the current in the solenoid} \end{aligned}$$

Taking into account all three of the above relations we obtain:

$$\epsilon \propto N_1 A N_2 \frac{dI}{dt} \tag{8.9}$$

which can be written as

$$\epsilon = -M\frac{dI}{dt}, \quad \text{where } M \propto N_1 N_2 A$$

M is called the mutual inductance and depends on the number of turns, the surface area, and the mutual orientation of the coils.

8.2.2 Prelab Exercise 2

1. Describe the magnetic field, in terms of magnitude and direction, produced inside a solenoid when it is driven by
   (a) a signal generator
   (b) a DC volt power supply or battery
2. In the experimental setup shown below, the solenoid and the secondary coil have many turns. Explain why?

8.2.3 Experimental

Connect the circuit shown below.

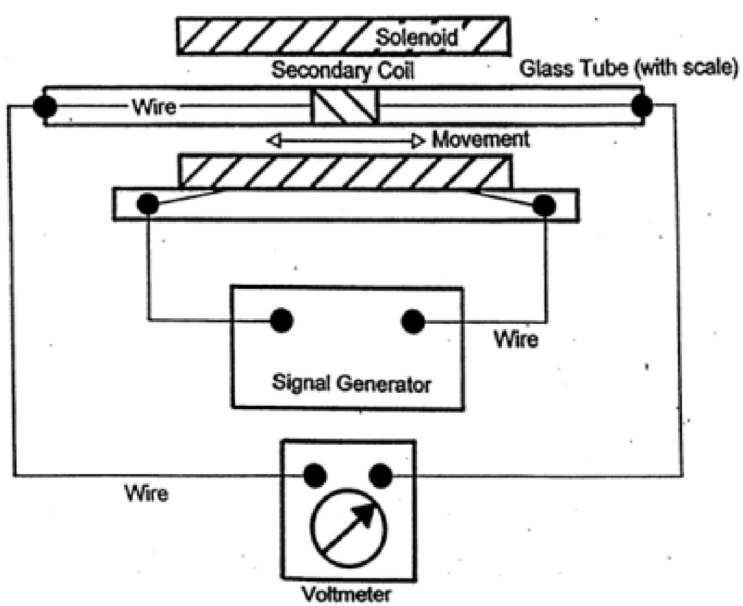

Figure 8.6: Setup to test electromagnetic induction

Note that there is a "loop" formed by the wires connected to the voltmeter. However, the area of this loop is perpendicular to $\vec{B}$ so that the flux $\phi = \vec{B} \cdot \vec{A}$ vanishes.

The current in the solenoid is driven by the signal generator, with frequency f

$$I = I_0 \sin 2\pi ft \quad \text{, amplitude of the current} \tag{8.10}$$

Using equation 8.9 and finding the derivative of I, we have

$$\epsilon \ \propto \ N_1 N_2 A I_0 2\pi f \cos(2\pi ft)$$

But $N_1 I_0$ is proportional to the amplitude of the magnetic field at a given point inside the solenoid, $B_0 \ \sim \ N_1 I_0$, so that

$$\epsilon = 2\pi N_2 A B_0 f cos(2\pi ft)$$

Notice that in the above equation an equality sign is used instead of proportionality sign. The proportionality sign was introduced in equation 8.8, defining the magnetic field inside the solenoid. Now, when magnetic field B_0 is incorporated back to the equation, the proportionality sign can be replaced by an equality symbol.

The above equation can be written as

$$\epsilon = \epsilon_{max} cos(2\pi ft) \quad \text{, where } \epsilon_{max} = 2\pi N_2 A B_0 f \tag{8.11}$$

This is the signal you are viewing on the oscilloscope- a sinusoidal voltage vs. time waveform. As seen from equation 8.11 ϵ is a cosine function while the current I given by equation 8.10 is a sine function $2\pi ft$. Thus the induced emf in the secondary coil oscillates 90° out of phase as compared to current in the solenoid.

You will use a digital voltmeter to measure the emf. Meters which measure alternating voltage or current are calibrated to read so called root-mean square (rms) or effective values. There is a simple relationship between the amplitude and rms values.

$$\epsilon_{rms} = \frac{\epsilon_{max}}{\sqrt{2}} \ \approx \ 0.707\epsilon_{max}$$

$$\text{Thus,} \quad \epsilon_{rms} = \sqrt{2}\pi N_2 A B_0 f$$

8.2.4 Measurement and Calculations

1. Put the secondary coil at the centre of the solenoid.
2. Increase the amplitude of the signal generator to about half of its maximum value.
3. Increase the frequency f in steps of 50 Hz, starting from zero and going up to 300 Hz and measure ϵ_{rms} using a digital voltmeter. Plot its dependence vs frequency f. Verify Faraday's Law which states that the induced emf is linearly dependent on the frequency of the magnetic field inside the solenoid. How would the slope of the line change if the number of turns in the secondary coil were four times smaller?

4. Instead of using a digital multimeter to measure the induced emf in the secondary coil, use an oscilloscope. Connect the output of the signal generator to CH I and the secondary coil to CH II of the oscilloscope. Adjust the sensitivity knobs (Volts/Div) for both channels and the Time/Div. knob so that you see two sinusoid signals on the screen of the oscilloscope. Answer the following questions:

   - How many times is the amplitude ϵ_0 of the induced emf in the secondary coil smaller than the amplitude of the voltage at the output of the signal generator?
   - Is the frequency of the induced emf in the secondary coil the same, smaller or larger than the frequency produced by the signal generator?
   - Measure the value of ϵ_0 to calculate root-mean square (rms)

     $$\epsilon_{rms} = \frac{\epsilon_0}{\sqrt{2}} \approx 0.707\epsilon_0$$

     Does this value agree with the value measured by the digital multimeter (obviously for the same frequency)?

8.2.5 Questions

1. What is the effect of the Earth's magnetic field on the emf induced in the secondary coil? Explain!
2. What are the advantages of using the oscilloscope compared to the digital meter to measure the induced emf in the secondary coil.

END OF LAB

Was this lab useful, instructive, and did it work well? If not, send an email to thatlabsucked@gmail.com and tell us your issues. In the subject line, be sure to reference the your course, the experiment, and session. example subject: *PHYS1010 Linear Motion monday 2:30*. We won't promise a response, but we will promise to read and consider all feedback.