# Chapter 0

# Introductory Information

## 0.1 General Information

### 0.1.1 Why do Laboratory Work?

Firstly, all science is based on a foundation of experimental data and your experiments will exemplify and illuminate many of the principles studied in the lectures. We try to timetable experiments as near as possible to the related material in the lecture schedule. However, details in the operation of the laboratories prevent us from achieving a perfect match and we ask you to be tolerant in this regard.

Secondly, the laboratory will be a medium for teaching some new material which will not be covered in the lecture course.

Thirdly, it gives you an opportunity to train your brain, eyes and hands in good experimental techniques, while familiarizing yourself with some of the instruments used in experimental science.

Obtaining good results is important, particularly if you intend to go on to more difficult labs. But do not get so involved in the mechanics of ”doing” that you lose sight of the goal of the experiment, the theory behind it, and its wider applications.

**You will need:**

- this manual (print your name on it)
- the usual writing materials (graph paper is provided)
- a calculator

### 0.1.2 Lab Schedules and Attendance

The laboratories are located in 102C and 102D Bethune College.

You will have been assigned to a particular 3-hour laboratory time at registration. In the first few days of the course, this can be changed through the online registration system.

Otherwise, this may only be changed by arrangement with Lab Coordinator. The schedule is included at the front of the manual. Students are required to attend all laboratory sessions to which they are assigned. **Please be certain to sign the demonstrator's mark list as proof of attendance.** Absence due to illness or other legitimate cause should be reported to Lab Coordinator as soon as possible so that credit may be obtained or an alternate lab assigned.

### 0.1.3 Prelab Preparation and Reports

You will know from the posted schedule which experiment you will be doing. **Before coming to do the experiment, you are expected to read the appropriate section of this manual.** Be sure you understand the theory involved, consult your textbook, and plan your practical work. Most of the lab outlines contain prelab exercises which must be completed on a separate sheet of paper before you come to the lab. **This preparation is most important.** It is unlikely that you will be able to finish the experiment satisfactorily or learn from them if you do not prepare beforehand. There may be short, unannounced quizzes on the experiment during some labs.

A sample lab report is included in the manual (Appendix C).

We do not require you to write an elaborate report for each experiment. The report should include name, name of partner, title and date. The experimental data, whenever possible, should be summarized in the form of a table, with title, column headings, units and experimental uncertainties. Graphs should have titles, axes labeled and units included. Uncertainties of all measured quantities should be indicated on graphs in the form of error bars. Calculations should be shown and organized in a logical way, with short comments and explanations. Just formulas with substituted data are not acceptable. Calculations of uncertainties is an important part of the lab report (next section in the manual provides more information regarding uncertainty calculations and rounding of final result and its uncertainty).

You are encouraged to record in your report for future reference any comments regarding the theory or method or apparatus which enhance your understanding. Your report should resemble a research scientist's day-to-day experimental log rather than a polished scientific paper.

It is preferred that you write laboratory reports in notebooks, which encourage better organization and neatness. Do not tear pages out of the books, if a mistake is made, simply cross out the mistake neatly. Two books will be required to be used alternately throughout the year. Light weight coil notebooks are suitable. Put your name and lab time clearly on the outside.

The three-hour session should be sufficient for the taking of measurements and for calculations and conclusions, etc. Be punctual - latecomers will find it difficult to complete the assignment. **All lab reports, finished or unfinished, must be handed in to your demonstrator by the end of the three-hour lab session.**

Your report will be marked by the demonstrator whose name appears on the top of the attendance list which you sign. It will be your responsibility to collect your report from this demonstrator during your next laboratory session. At this time you should discuss with your demonstrator any matters concerning the report(s).

### 0.1.4 Lab Marks

The final lab mark will contribute approximately 10-20% (depending on the course) to the final grade. It will take into consideration prelab questions and quizzes, and the weekly lab reports. Students should keep their lab reports for reference and as a record of marks. All your lab marks will be posted on Moodle for you to check.

### 0.1.5 Lab Partners

Some students claim that they learn more while working with a lab partner; others prefer to work alone. For certain experiments where basic techniques, etc. are explored, you will be required to work individually - this will be stated in the lab outline for those particular experiments. For the other experiments we will try to provide sufficient apparatus so that you may work with another student who has been assigned the same experiment or alone, as you prefer. For a few of the experiments the mechanical work is so difficult that one person cannot perform the experiment satisfactorily. If two students work together, each should take a turn at reading all the instruments and although both will have the same data, **each student must submit an independent report, with independent calculations. No more than two students working together as lab partners is allowed.**

Lab partners are randomly assigned. This facilitates meeting many friends, promotes social skills as well as reduces the probability of dishonesty when doing lab work. The details of how lab partners are assigned will be explained in the first lab.

### 0.1.6 Cleanliness and Care of Equipment

We do not charge you for accidental breakages, but please report them to the demonstrator or lab technologist immediately, so that equipment can be replaced or repaired.

Students must leave their place of work in the lab neat with all the apparatus complete. Each experimental set-up will be used by approximately forty students before it is retired for the year, so leave it for the next student in the state in which you would like to find it.

**When a student hands in a report, the demonstrator will check their place of work to see that it is left in satisfactory condition. When satisfied, the demonstrator will accept the report.**

### 0.1.7 Lab Safety

Scientists very commonly live to a grand old age in spite of their daily encounters with many hazards. The main reason for this is that a scientist doing an experiment is paying very close attention to everything that happens, is expecting the unknown and can react quickly to it. Your best protection against accidents in the lab is a constant thoughtful alertness which never permits your actions to become *mechanical* and *reflex*.

Specific hazards which exist in particular experiments will be stressed in the respective lab outline. Please pay very careful attention to these warnings and act accordingly.

Notify the demonstrator or lab technician of any accident or injury no matter how insignificant it may seem.

In the case of a fire, at the sound of the fire alarm in the building, the university stipulates that everyone must leave the building. In the case of a fire in the lab students and demonstrators must leave the building immediately. The demonstrator is responsible for taking the appropriate action to scale the lab marks.

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**A 24-hour Emergency Services Telephone Centre operates on York Campus and can be alerted by calling 33333 on all campus telephones or 736-2100 Ext. 33333 on public or off-campus telephones.**

**Health services are located in York Lanes.**

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### 0.1.8 Academic Honesty

Students will certainly discuss and talk about their studies with their friends and this can be very useful; but any work that you hand in must have been done by yourself. This is the only way to test your own competence and to prepare yourself for positions of responsibility after graduation. If scientists are dishonest, they are useless.

**THE UNIVERSITY CONSIDERS ALL FORMS OF COPYING AND CHEATING TO BE SERIOUS OFFENCES.**

## 0.2 Measurement and Uncertainties

### 0.2.1 Measurements

There are several requirements that must be met if a measurement is to be useful in a scientific experiment:

**The Number of Determinations** It is a fundamental law of laboratory work that a single measurement is of little value because of the liability not only to gross mistakes but also to smaller random errors. Accordingly, it is customary to repeat all measurements as many times as possible. The laws of statistics lead to the conclusion that the value having the highest possibility of being correct is the arithmetic mean or average, obtained by dividing the sum of the individual readings by the total number of observations. Because of time limitations, we often suggest you do a minimal number of repetitive measurements but remember this reduces the reliability and respectability of your results.

**Zero Reading** Every measurement is really a difference between two readings, although for convenience, most instruments are calibrated so that one of these readings will be zero. In many instruments, this zero is not exact for all time but may shift slightly due to wear or usage. Thus it is essential that the zero be checked before every measurement where it is one of the two readings. In some cases the zero can be reset manually, while in others it is necessary to record the exact zero reading and correct all subsequent readings accordingly. e.g. When measuring the length AB shown in Fig. 1, a ruler could be placed (1) with 1.2 cm at A, then length AB = (4.0 - 1.2) cm = 2.8 cm. The more usual ruler position (2) allows the length AB to be read as 2.8 cm directly, but remember this is still the difference between two readings: 2.8 cm and 0.0 cm.

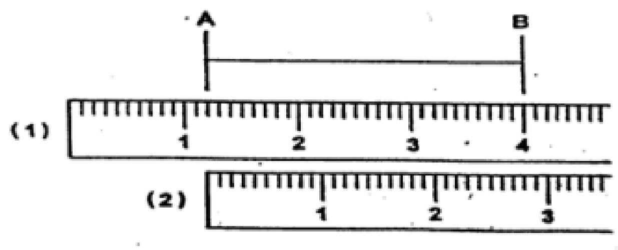


Figure 1: Example of measuring length

**Accuracy** Quantitative work requires that each measurement be made as accurately as possible. The main units of a scale are usually divided, and the eye can easily subdivide a small a distance of 1 mm into five parts reasonably accurately. Thus, if a linear scale is divided into millimeters, e.g. on a high quality ruler, a reading could be expressed to 0.2 of a millimeter; e.g. 4.6 mm, 27.42 cm, where 3/5 and 1/5 of a mm are estimated by eye. In cases where the reading falls exactly on a scale division, the estimated figure would be 0; e.g. 48.50 cm, indicating that you know the reading more accurately than 48.5 cm. But it would not be possible to take a reading with greater accuracy then 0.2 mm with this equipment. If the scale is not finely engraved, the lab meter sticks for example, it could probably only be read as 0.5 mm.

The accuracy desired from a measurement dictates the choice of instrument. For example, a distance of 4 m should not be measured by a car's odometer, nor a distance of 2 km with a micrometer. The student learns to decide which instrument is most appropriate for a certain measurement. Ideally, all measurements for any one experiment should have about the same percentage accuracy.

**Significant Figures** A significant figure is a digit which is reasonably trustworthy. One and only one estimated or doubtful figure can be retained and regarded as significant in any measurement, or in any calculation involving physical measurements. In the examples presented in the **Accuracy** paragraph above, 4.6 mm has two significant figures, 27.42 cm has four significant figures.

The location of the decimal point has no relation to the number of significant figures. The reading 6.54 cm could be written as 65.4 mm or as 0.0654 m without changing the number of significant figures - three in each case.

The presence of a zero is sometimes troublesome. If it is used merely to indicate the location of the decimal point, it is not called a significant figure, as in 0.0654 m; if it is between two significant digits, as in a temperature reading of 20.5°C , it is always significant. A zero digit at the end of a number tends to be ambiguous. In the absence of specific information we cannot tell whether it is there because it is the best estimate or merely to locate the decimal point. In such cases the true situation should be expressed by writing the correct number of significant figures multiplied by the power of 10.

Thus a student measurement of the speed of light, 186,000 mi/s is best written as $1.86x10^5 mi/s$ to indicate that there are only three significant figures. The latter form is called standard notation, and involves a number between 1 and 10 multiplied by the appropriate power of 10. It is equally important to include the zero at the end of a number if it is significant. If reading a meter-stick, one estimates to a fraction of a millimeter, then a reading of 20.00 cm is written quite correctly. In such a case, valuable information would be thrown away if the reading were recorded as 20 cm. The recorded number should always express the degree of accuracy of the reading. In computations involving measured quantities, carry only those digits which are significant. Consider a rectangle whose sides are measured as 10.77 and 3.55 cm (the doubtful digits are in **bold**). When these lengths are added to find the perimeter the last digit in the answer will also be doubtful.

$$\begin{array}{r} 10.7\mathbf{7} \\ 10.7\mathbf{7} \\ 3.5\mathbf{5} \\ \pm 3.5\mathbf{5} \\ \hline 28.6\mathbf{4} \end{array}$$

When the lengths are multiplied to obtain the area, any operation by a doubtful digit results in a doubtful digit.

$$\begin{array}{r} 10.7\ \mathbf{7} \\ \times\, 3.5\ \mathbf{5} \\ \hline 0.0\ \mathbf{5385} \\ 5.38\mathbf{5} \\ 32.38\mathbf{1} \\ \hline 38.0\ \mathbf{2335} \end{array}$$

In the result only one doubtful digit is retained and the area is $38.2cm^2$. When rounding off a number with too many insignificant figures, retain the last digit unchanged if the first digit dropped $< 5$: increase it by 1 if the first digit dropped $\geq 5$.

In the lab this year, most of your calculations will probably be limited to three significant figures by the measuring devices. A calculator quickly produces about 8 mathematically significant figures, but, in your final answer, record only those figures that have physical significance.

### 0.2.2 Uncertainties (Errors)

A quantity measured or calculated by a scientists is only of value if he/she can attach to it quantitative limits within which he/she expects that it is accurate - that is, its uncertainty. An uncertainty of 50% or even 100% is a vast improvement over no knowledge at all: an accuracy of $\pm 10\%$ is a great improvement over $\pm 50\%$ and so on. In fact much of science is directed toward reducing the uncertainties in specific quantities of scientific interest.

The uncertainty in a reading or calculated value is technically called on error. The word has this precise meaning in science and carries no implication of mistake or sin. This manual uses the words **error** and **uncertainty** interchangeably.

A **Systematic Error** is one which always produces an error of the same sign. Systematic errors may be sub-divided into three groups: instrumental, personal and external. Corrections should be made for systematic errors when they are known to be present.

An **Instrumental Error** is caused by faulty or inaccurate apparatus; for example an undetected zero error in a scale or an incorrectly adjusted watch. If 0.2 mm has been worn off the end of a ruler, all readings will be 0.02 cm too high.

**Personal Errors** are due to some peculiarity or bias of the observer. Probably the most common source of personal error is the tendency to assume that the first reading taken is correct. A scientist must constantly be on guard against any bias of this nature and make each measurement as if it were completely isolated from all previous experience. Other personal errors may be due to fatigue, the position of the eye relative to a scale, etc.

**External Errors** are caused by external conditions (wind, temperature, humidity, vibration); examples are the expansion of a scale as the temperature rises or the swelling of a meter stick as humidity increases.

**Random Errors** Random errors occur as variations which are due to a large number of factors. Each factor adds its own contribution to the total error. Resulting error is a **matter of chance** and, therefore, positive and negative errors are equally probable. Because random errors are subject to the laws of chance, their effect in the experiment may be lessened by taking a large number of observations. A simplified statistical treatment of random errors is described in Appendix A of this manual.

**The Error Interval** If it is not practical or possible to repeat a measurement many times, the errors in measurement must be estimated differently.

Since the last digit recorded for a reading is only an estimation, there is some possibility of error in this digit due to the instrument itself and the judgement of the observer. Hence, the best that can be done is to assign some limits within which the observer believes the reading to be accurate.

A reading of 6.540 cm might imply that it lay between 6.538 and 6.542 cm. The reading would then be recorded as (6.540 ± 0.002 cm). The scales on most instruments are as finely divided by the manufacturer as it is practical to read. Hence, the error interval will probably be some fraction of the smallest readable division on the instrument; it might be 0.5 of a division, or perhaps 0.2 of a division. The error interval is a property of the instrument and the user, and will remain the same for all readings taken provided the scale is linear.

Remember that measurement of a quantity (such as length) also involves a zero reading, so the error in the quantity will be twice the reading error. **Note that it is essential to quote an error with every set of measurements.**

**Absolute Uncertainty** The estimation of an error interval gives what is called an "absolute" uncertainty. It has the same units as the measurement itself; e.g. (6.540 ± 0.002) cm. The absolute uncertainty 0.002 cm is recorded to one significant figure, which in turn, defines the last significant digit in a measurement.

When using meter ruler, micrometer screw, or Vernier caliper, you will be required to interpolate between scale divisions. For most of this type of instruments it is reasonable to estimate the absolute uncertainty as ± half of the smallest scale division.

When using weights, with mass written on them, the absolute uncertainty should be taken as half of the last significant digit. For example, for the mass m = 200 g, the absolute uncertainty is $\delta m = 0.5g$. When using digital instruments, such as digital multimeters, the absolute uncertainty is the sum of a reading error and an instrument error. The reading error is ± last stable digit displayed, for example, if the digital voltmeter reading is 11.6 V, the reading error is ± 0.1 V. The instrument error is specified by the manufacturer, which usually is 1.5% of the reading value. For example, if the digital voltmeter reading is 11.6 V, the instrument error is (0.015)(11.6 V) = 0.2 V. The total uncertainty is 0.1 V + 0.2 V = 0.3 V. The voltmeter reading should be recorded in the form: (11.6 ± 0.3) V.

**Relative and Percentage Uncertainties** Frequently a statement of the absolute uncertainty $\delta x$, is not as meaningful as a comparison of the size of the uncertainty with the size of the measurement itself, $x$. This comparison is expressed by a relative uncertainty:

$$\frac{\delta x}{x} \text{in the case above would be } \frac{0.03}{2.56} \simeq 0.01$$

or expressed as a percentage uncertainty:

$$\frac{0.03 \times 100}{2.56} \simeq 1\%$$

An uncertainty $\delta x = \pm 0.2cm$, for example, is much more important in a measurement of $2cm$ ($\delta x/x = 0.1$or 10%) than in a measurement of $2m$ ($\delta x/x = 0.2/200 = 0.001$or 0.1%). Relative and percentage uncertainties have no units.

**Uncertainties in Calculated Quantities** Very rarely are the measurements themselves the desired end-products of an experiment. Usually the measurements are used to calculate something. How are the uncertainties in the measurement compounded when these measurements are used in calculations?

Until you have studied differentials in math, you can calculate errors compounded in computation using the rules which follow. These are derived by a differential method (Appendix B).

**RULE 1: For Addition and Subtraction**

> **Whenever only addition and/or subtraction occur in a calculation the resultant absolute uncertainty in the answer is the sum of the absolute uncertainties of all the measured quantities occurring in the calculation.**

**Example:** Let $x = 2.66 \pm 0.02$ and $y = 1.79 \pm 0.02$. Find the magnitudes of the uncertainties of $(x + y)$ and $(x - y)$.

**Solution:**

$$\begin{array}{r} (\mathbf{x}+\mathbf{y}) \\ 2.66 \pm 0.02 \\ \underline{+1.79 \pm 0.02} \\ 4.45 \pm 0.04 \end{array}$$

$$\begin{array}{r} (\mathbf{x}-\mathbf{y}) \\ 2.66 \pm 0.02 \\ \underline{-1.79 \pm 0.02} \\ 0.87 \pm 0.04 \end{array}$$

This is reasonable since

$$\begin{aligned} x = 2.66 \pm 0.02 &\quad \rightarrow \quad 2.64 \leq x \leq 2.68 \\ y = 1.79 \pm 0.02 &\quad \rightarrow \quad 1.77 \leq y \leq 1.81 \end{aligned}$$

Adding and subtracting in the most unfavourable ways to obtain the maximum possible uncertainty gives

$$4.41 \leq (x + y) \leq 4.49$$
$$0.83 \leq (x - y) \leq 0.91$$

which can be expressed as $4.45 \pm 0.04$ and $0.87 \pm 0.04$ as above.

**RULE 2: For Multiplication and Division**

> **Whenever only multiplication and/or division occur, the relative uncertainty of the product or quotient is equal to the sum of the relative uncertainties of each factor in the function.**

**Example:** Let $x = 2.66 \pm 0.02$ and $y = 1.79 \pm 0.02$. Find the magnitudes and the uncertainties of $z = (x \times y)$ and $w = (x/y)$.

**Solution:**

$$\mathbf{z = (x \times y) = 2.66 \times 1.79 = 4.7614}$$
$$\frac{\delta z}{z} = \frac{\delta x}{x} + \frac{\delta y}{y}$$
$$\frac{\delta z}{z} = \frac{0.02}{2.66} + \frac{0.02}{1.79}$$
$$\frac{\delta z}{z} = 0.0075 + 0.011$$
$$\frac{\delta z}{z} = 0.0185$$
$$\text{where } z = 4.7614 \text{ so,}$$
$$\delta z = 0.0185 \times 4.7614 = 0.088 = 0.09$$
$$\text{Therefore, } z = (4.76 \pm 0.09)$$

Please observe that the uncertainty $\delta z = 0.09$ was rounded to one significant figure.

Similarly,

$$\mathbf{w = (x/y) = 2.66/1.79 = 1.486}$$
$$\frac{\delta w}{w} = \frac{\delta x}{x} + \frac{\delta y}{y}$$
$$\frac{\delta w}{w} = \frac{0.02}{2.66} + \frac{0.02}{1.79}$$
$$\frac{\delta w}{w} = 0.0185$$
$$\text{where } w = 1.486 \text{ so,}$$
$$\delta w = 0.0185 \times 1.486 = 0.02749 = 0.03$$
$$\text{Therefore, } w = (1.48 \pm 0.03)$$

Where addition and/or subtraction and multiplication and/or divisions are all involved in one formula, the calculation is more complicated. The rules, above, should then be applied to one part of the function at a time and then combined.

**Example:** Find the uncertainty of the quantity given by the expression $Z = AB^2 + CD^2$ where $A, B, C, D$ are measured quantities, and $\delta A, \delta B, \delta C, \delta D$ are the corresponding absolute uncertainties.

**Solution:**

$$\text{Let } I_1 = AB^2 = A \times B \times B \text{ and } I_2 = CD^2 = C \times D \times D$$

$$\frac{\delta I_1}{I_1} = \frac{\delta A}{A} + \frac{\delta B}{B} + \frac{\delta B}{B} \quad , \quad \frac{\delta I_2}{I_2} = \frac{\delta C}{C} + \frac{\delta D}{D} + \frac{\delta D}{D}$$

$$\frac{\delta I_1}{I_1} = \frac{\delta A}{A} + \frac{2\delta B}{B} \quad , \quad \frac{\delta I_2}{I_2} = \frac{\delta C}{C} + \frac{2\delta D}{D}$$

$$\delta I_1 = (\frac{\delta A}{A} + \frac{2\delta B}{B})I_1 \quad , \quad \delta I_2 = (\frac{\delta C}{C} + \frac{2\delta D}{D})I_2$$

$$\delta I = \delta I_1 + \delta I_2 = (\frac{\delta A}{A} + \frac{2\delta B}{B})I_1 + (\frac{\delta C}{C} + \frac{2\delta D}{D})I_2$$

$$\delta I = (\frac{\delta A}{A} + \frac{2\delta B}{B})AB^2 + (\frac{\delta C}{C} + \frac{2\delta D}{D})CD^2$$

$$\delta I = B^2\delta A + 2AB\delta B + D^2\delta C + 2CD\delta D$$

In the calculation of uncertainties, it is generally assumed that whole numbers occurring in formulas have no uncertainty. Similarly, when using physical constants in formulas (such as $g$), one usually includes more significant figures than the measured quantities in the experiment, so that uncertainty of such physical constants is negligible.

Since uncertainties are only estimated, they should never be quoted to more than one or two significant figures. Also, because their accuracy is limited, approximations and simplifications can often be made which make their actual calculation much easier.

**RULE 3: For Multiplication/Division with constant**

> **Whenever multiplication with an exact/known number with no uncertainty occurs, the uncertainty is the product of the absolute value of known number times the uncertainty.**

**Example:** Find the uncertainty of the quantity given by the expression y = Bx, where $x = 2.66 \pm 0.02$ and B is exactly 4 with no uncertainty.

**Solution:**

$$\mathbf{y = Bx = 4 \times 2.66 = 10.64}$$
$$\delta y = |B| \times 0.02 = 0.08$$
$$\text{Therefore, } y = (10.64 \pm 0.08)$$

**Rules for Stating Uncertainties and Answers** Uncertainties are only estimated and as such they should be rounded to one significant figure. The only exception to this rule is when the leading digit in the uncertainty is 1. In this case two significant digits might be justified. For example, the uncertainty 0.14 rounded to one significant digit would be reduced very significantly.

The final answer of the measured or calculated quantity should have the last significant digit in the same decimal position as the uncertainty.

**Example:**

$$(26.3 \pm 0.5)s$$
$$(48 \pm 2)m$$
$$(36.82 \pm 0.06)N$$
$$(15.34 \pm 0.14) \times 10^2 kg$$

**Comparison** Occasionally in the lab you will be asked to **compare**.

1. several values for the same quantity which you have measured using different methods.
2. a value which you have measured or calculated with a standard value in a table.

In scientific terms the word **compare** means more than a comparison by eye alone - it means a mathematical comparison.

In case (1) the best way to compare the value is to calculate what percentage the average deviation is of the mean (Appendix A).

In case (2), if your measurement has an uncertainty associated with it, then you should see whether the range of your measured values including your uncertainty is consistent with the standard (accepted) value. You should ask the question does:

$$|\text{your value} - \text{standard value}| \leq \text{your uncertainty}$$

If the above it true, then measurement agrees with the standard value.

In case (2) if your measurement does not contain an uncertainty, it may be instructive to calculate the percentage difference.

$$\frac{|\text{your value} - \text{standard value}|}{\text{standard value}} \times 100\%$$

For more information on how errors and uncertainties are determined, please see Talyor, J.R. *An Introduction to Error Analysis: The study of uncertainties in physical measurements*, University Science Books, 1997.

## 0.3 Graphs

In this course it will frequently be necessary to plot a series of results on graph paper. A graph is often the most concise and meaningful way to display data. Plotting experimental data and deriving significant information from the resulting graph is rather different from the process of plotting the graph of a known analytic function. An experimental measurement is not exact, but rather is represented by a small range of possible values; e.g. $2.38 \leq x \leq 2.42$ which we usually express as $x = 2.40 \pm 0.02$; or $y = 1.42 \pm 0.03$.

On a graph we would represent the uncertainty by plotting the point as shown in Fig 2 where the two **error bars** cross at $(2.40, 1.42)$ and their lengths are $2 \times 0.02$ along the x direction and $2 \times 0.03$ along the y direction.

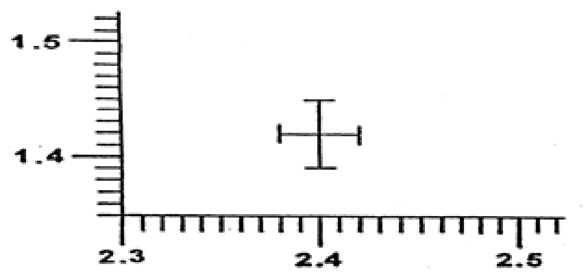


Figure 2: Plotted data point with Error Bar

Once the points are plotted, the task is to draw the best smooth curve (usually this will be a straight line) through the field of points. Due to the uncertainties the experimental points will never fall exactly on the analytic curve which you try to fit to them. Draw the curve which comes closest to the most points and in general lies within the error bars of all points.

Generally, one variable is under your control; and is known as the **independent variable**, by convention this is plotted on the horizontal, x axis. The second measured quantity will vary in some dependent way as you vary the first quantity and is called the **dependent variable**, and is plotted on the vertical, y axis.

The following general guidelines should be used when preparing graphs:

### 0.3.1 Labeling

Graphs should have some kind of title to give meaning to the data displayed. For example, *Variation of length of rubber band with load* is a meaningful title, where as *Length vs load* is not good enough. Axes should always be labelled with the name of the quantity being displayed and the units in which it is measured. e.g. Spring extension (mm).

### 0.3.2 Scales

Aim to spread your data out across as much of the graph paper as possible. If the value for one variable extended from, say, 90 to 212 (units) it would not be necessary to fit in a scale from 0 to 220 (units) but only from 90 to 220 units. Unless, of course, you do know, or want to know, something about the situation at the zero value.

Avoid the tendency to force the extrapolation of a graph through the origin even if intuition tells you it should go there. Limitations of apparatus and other side effects of which you are not aware cause unusual results close to the origin. Thus, by forcing your graph through the origin, you may be distorting it in the region of interest and thereby distorting the information which you wish to obtain. Draw your graph considering only the region in which measurements were taken. Extrapolations are often risky.

### 0.3.3 Plotting

Always plot *raw* figures if possible, which need not be in conventional units, e.g. a spark device makes points on a paper 5 times per second; when plotting, one axis could be labelled, *Time from start, (1/5 sec)*, rather than dividing all time values by 5 to give seconds. Convert to conventional units when the required information is taken from the graph.

Draw graphs with a sharpened pencil so that mistakes can be erased. A transparent ruler is particularly useful for deciding on the *best* straight line through experimentally determined points. Coordinates will be in an appropriately labelled table in your report and not beside the points so as not to clutter up the graph.

### 0.3.4 Straight Line Graphs

Without the aid of a computer, the one graph from which we can easily extract useful information is a straight line graph. If the relationship between two variables is in the standard straight line form, $y = mx + b$, a straight line graph of $y$ vs $x$ can be plotted and the constants $m$ and $b$ thereby determined: $m$ is the slope of the straight line and $b$ is the y-intercept.

Often it will not be convenient to start the x-axis at 0 and hence the y-intercept will not occur on the graph, but b can easily be determined, once m is known, by substituting the co-ordinates of one point on the line into the equation of the line (recall that each and every point on a line must satisfy the equation of the line).

You will often be required to plot experimental results in such a way as to obtain a straight line graph.

### 0.3.5 To Determine the Slope

Once the line has been drawn 3 you will probably need to calculate its slope to determine some information pertinent to the experiment.

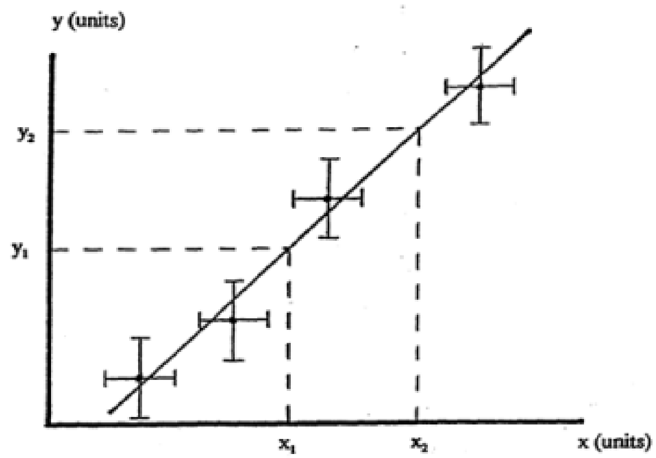


Figure 3: Determining the Slope from Plotted Data

From analytical geometry

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{rise}}{\text{run}}$$

Where $(x_1, y_1)$ and $(x_2, y_2)$ are points which lie **on** the line. Since your experimental points, in most cases, do not lie exactly on the line, do not use them for determining the slope. Instead, choose two arbitrary points lying (exactly) on the line **as far apart as possible** and determine the slope from them (draw in the rise and run or in some way indicate the points you have used).

Since the variables you plot will be physical quantities with units, the rise and run should be given units and the slope will also have units.

In the era of computers, another method is frequently used to calculate slope and the uncertainty in the slope. It is called the Method of Least Squares and is described below. For large amounts of data a computer is very useful, however for small numbers of data this method can be carried out by hand as well! This method also allows one to obtain the uncertainty in the slope.

### 0.3.6 The Method of Least Squares

The method of least squares is used when one wishes to fit a set of data to a given equation. We shall deal with the simplest of cases, the straight line.

Consider Fig.4. The mathematically best straight line through the set of data points is shown, together with the data. Not every data point falls on the line. There is a difference between the fitted $y$ value and the actual $y$ value. The purpose of least squares is to minimize these deviations over all the data points ($N$ points).

The equation of a straight line may be given by:

$$y = mx + b$$

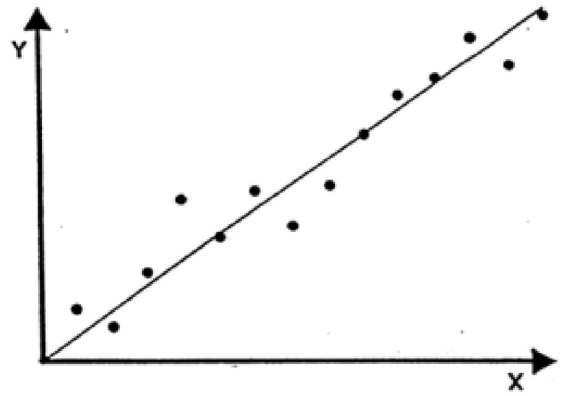


Figure 4: Data with a Linear Fit

The difference between the fitted $y$, on the straight line, and the actual $y_i$ may be given by:

$$b + mx_i - y_i$$

Summing the squares of these differences gives us the total square uncertainty, or (if divided by $N$) the variance.

$$\sigma = \sum (b + mx_i - y_i)^2$$

where $\sum$ means "the sum over all data points". It is the sum that we wish to minimize. We accomplish this by taking the derivative of $\sigma$ with respect to each of the individual parameters $b$ and $m$ and set these simultaneously equal to zero.

The derivatives with respect to $b$ only may be expressed as:

$$\frac{\partial \sigma}{\partial b} = 2 \sum (b + mx_i - y_i) = 0 \quad \text{follows from simple calculus}$$

The operator $\partial/\partial b$ means "take the derivative with respect to $b$ only and treat all other numbers as constant".

Expanding the summation, dropping the factor of 2, and moving the term with $y_i$ to the right of the equals sign gives:

$$\sum b + \sum mx_i = \sum y_i$$

the parameters $b$ and $m$ can be moved from inside the summation, as they do not change during the summations, i.e.:

$$bN + m \sum x_i = \sum y_i$$

The parameter $b$ is multiplied by $N$, the number of data points, as summing the constant, $b$, $N$ times is $bN$.

Similarly:

$$\frac{\partial \sigma}{\partial m} = 2\sum(bx_i + mx_i^2 - x_i y_i) = 0$$

and this equation may be manipulated as the one for $b$:

$$\sum bx_i = \sum mx_i^2 = \sum x_i y_i$$

to yeild

$$b\sum x_i + m\sum x_i^2 = \sum x_i y_i$$

solving equations gives:

$$\text{Intercept}: \quad b = \frac{(\sum x_i^2)(\sum y_i) - (\sum x_i y_i)(\sum x_i)}{N(\sum x_i^2) - (\sum x_i)^2}$$

$$\text{Slope}: \quad m = \frac{N(\sum x_i y_i) - (\sum x_i)(\sum y_i)}{N(\sum x_i^2) - (\sum x_i)^2}$$

In order to find the best straight line through a given set of data points, one needs to know the number of points, the sums $\sum x_i$, $\sum x_i^2$, $\sum y_i$ and $\sum x_i y_i$ then substitute these into the equations above. Note that the denominators in the equations are the same and, therefore, need only be calculated once.

The uncertainties of the slope and intercept are given by the following expressions:

$$\text{Uncertainty in Slope}: \quad \delta m = \sqrt{\frac{\sum[(y_i - \overline{y}) - m(x_i - \overline{x})]^2}{(N-2)\sum(x_i - \overline{x})^2}}$$

$$\text{where}: \quad \overline{x} = mean(x) = \frac{\sum x_i}{N} \quad \text{and} \quad \overline{y} = mean(y) = \frac{\sum y_i}{N}$$

$$\text{Uncertainty in Intercept}: \quad \delta b = \sqrt{\frac{[\sum(y_i - b - mx_i)^2](\sum x_i^2)}{N(\sum x_i^2) - (\sum x_i)^2}}$$

The method of least squares requires the evaluation of a few complicated expressions. To evaluate these expressions precisely and quickly, it is worth setting up a table or a computer program.

### 0.3.7 Logarithmic Graphs and Exponential Data

In some experiments, you will collect data which does not fit to straight line, rather to an exponential. Suppose variable $p$ and $q$ are related by the expression:

$$q = A_0 e^T p, \text{ where } A_0, T \text{ are constants, and } e \text{ is Euler's number}$$

Taking the natural logarithm of both sides gives

$$ln(q) = ln(A_0) + ln(e^T p)$$
$$ln(q) = ln(A_0) + Tp$$
compare this result to an equation for a straight line:
$$y = b + mx$$

Suppose now we collected data $x, y$ which had this expected exponential trend. If we were to plot $x$ vs $ln(y)$ then the results would be a straight line whose slope $m$ would be equal to the constant $T$, and whose y-intercept $b$ would be equal to the $ln(A_0)$. This is a convenient a powerful method of determining parameters of an exponential trend using normal graph paper. An alternative method would be to use Semi-Logarithmic graph paper, which has the y-axis as on a logarithmic scale and the x-axis on a linear scale. In this way, plotting the raw $x$ and $y$ data would yield a straight line.