Chapter 3

Standing Waves

Objective

1. study transverse standing waves in a rope and longitudinal standing waves in a spring
2. study sound standing waves in a tube

Apparatus Mechanical wave driver, sine wave signal generator, orange string, spring, electronic balance, weights, microphone, oscilloscope, sound tube with speaker attached, measuring tape and stands

3.1 Introduction

Interference is a fundamental property of waves. It occurs when two or more waves coexist in the same medium and produce a resultant wave. A particularly interesting interference occurs when two identical waves (of the same amplitude and frequency/wavelength) travel in the same medium in opposite directions. The superposition of these two waves under certain conditions leads to standing waves. Standing waves can be created only in a medium of finite size, for example, in a rope or spring fixed at both ends or in a solid rod of a finite length. A simple harmonic wave traveling in the positive x-direction can be described by the following wave function:

$$y = A \sin(kx - \omega t)$$

In this wave equation A is amplitude, $k = 2\pi/\lambda$ is the wave vector (in rad/m), and $\omega = 2\pi f$ is angular frequency (in rad/s). It is important to distinguish angular frequency $\omega = 2\pi f$ and frequency $f = 1/T$, where T is the period. The wavelength λ, frequency f, and speed v of the wave are inter-related through the equation

$$v = \lambda f$$

The speed of mechanical waves depends on the properties of the medium. For example, the speed of a transverse wave in a rope is given by $v = \sqrt{T/\mu}$, where T is the tension in the rope and $\mu = m/L$ is called linear mass density; m is the mass of the rope and L its length.

The speed of the longitudinal wave moving in the spring is $v = \sqrt{kL/\mu}$, where k is the spring constant of the spring (in units of N/m), L is the length of the spring, and $\mu = m/L$ is called linear mass density of the spring (m is the mass of the spring and L its length).

When two identical waves travel in opposite directions, they interfere and a new resultant wave is created. According to the superposition principle the net displacement of the resultant wave is equal to the algebraic sum of the displacements due to the individual waves. If the displacements of the two waves are given by equations: $y_1 = A\sin(kx - \omega t)$ and $y_2 = A\sin(kx + \omega t)$ then the resultant wave is $y = y_1 + y_2 = A([\sin(kx - \omega t) + \sin(kx + \omega t)]$ Using the identity

$$\sin a + \sin b = 2\sin\frac{(a+b)}{2}\cos\frac{(a-b)}{2}$$

we obtain

$$y = 2A\sin(kx)\cos(\omega t)$$

The frequency of the resultant wave is the same as the frequency of the individual waves, but the amplitude $2A\sin(kx)$ is different and depends on x.

If the crests or troughs of the two waves traveling in opposite directions occur at the same positions, the waves interfere "constructively" and the resultant wave is bigger than the individual ones. When the crest of the first wave coincides with the trough of the second wave, then we have "destructive" interference between the waves.

The maximum amplitude occurs when $\sin(kx) = 1$, which means that

$$kx = \frac{n\pi}{2}, n = 1, 3, 5, ...$$

substituing

$$\begin{aligned} k &= \frac{2\pi}{\lambda} \\ x &= \frac{n\lambda}{4} \end{aligned}$$

The positions at which the amplitude is a maximum are called **antinodes** and are separated by $\lambda/2$.

The minimum amplitude occurs when $\sin(kx) = 0$, which leads to the condition

$$x = \frac{n\lambda}{2}, n = 1, 3, 5, ...$$

Points corresponding to zero displacement are called **nodes**.

3.1.1 Prelab Exercsie 1

Explain the differences between travelling and standing waves.
Please see: https://www.youtube.com/watch?v=7e67lv7-Wk4 for a short presentation.

3.2 Standing Wave in a Rope and a Spring

3.2.1 Transverse Standing waves in a rope

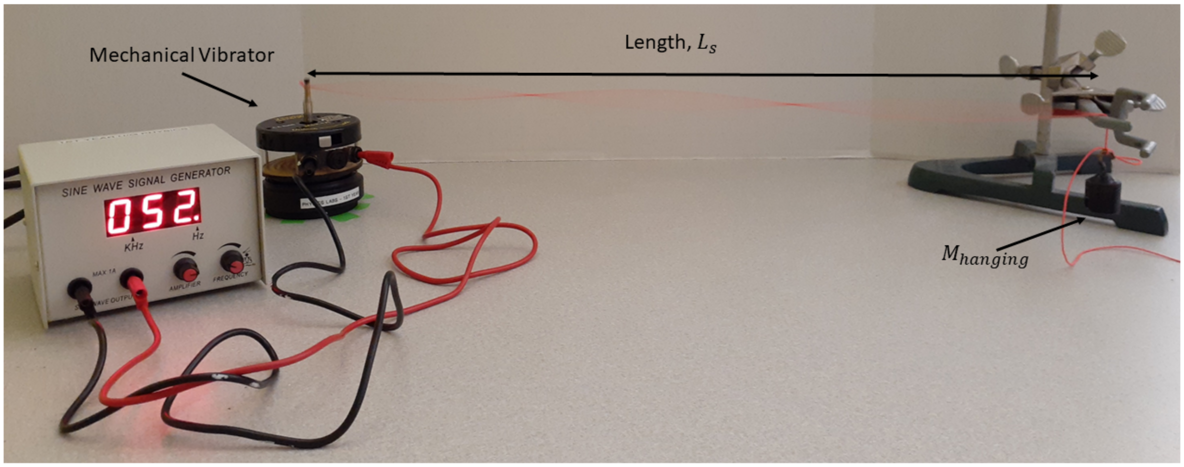

Figure 3.1: The experimental setup to produce standing waves in a rope

The suspended mass creates tension T in the rope, $T = M_{hanging}g$. A mechanical wave driver, whose frequency is controlled by the sine wave signal generator, creates a harmonic wave at one end. The wave is reflected and inverted at the fixed end and returns to the source where it is reflected again. If the mechanical wave driver sends out a new crest just as the reflected crest reaches it, the new and reflected waves will reinforce each other. The two identical waves traveling in opposite directions interfere and for certain discrete frequencies form a standing wave.

Create at least four standing waves and measure their wavelength (remember that the distance between two adjacent nodes is $\lambda/2$).The frequency of the standing wave is displayed on the signal generator.

Use the formula $v = \lambda f$ to determine speed v.

Record the results in the fillable Table D.8 provided in Appendix D and include it in your lab report.

Determine the mean speed of the wave and the deviation from the mean.

Calculate the speed of the wave using the formula $v = \sqrt{T/\mu_s}$, where tension $T = M_{\text{hanging}}g$ and linear mass density $\mu_s = m_{\text{string}}/L_s$. In the experiment the string is under tension (stretched). The linear mass density for the unstretched string, call it μ_u, will be provided.

To get the linear mass density for the stretched string, μ_s, perform the following procedure:

Measure the distance between the two pivot points of the stretched string ($L_s \pm \delta L_s$).

To obtain the mass of the stretched string use the following procedure:

1. Pinch the string at the end with the hanging mass.
2. Keeping hold of the string remove it from the holder.
3. Measure the length of string used but now in the unstretched state – call this $L_u \pm \delta L_u$. The mass of string used in the experiment is then $m = \mu_u L_u$ (don't forget to estimate the uncertainty in m due to the uncertainty in L_u and μ_u).

The μ_s to use in the expression for the velocity is equal to m/L_s. The expression for the uncertainty in v is:

$$\frac{\delta v}{v} = \frac{1}{2}\left(\frac{\delta T}{T} + \frac{\delta \mu_s}{\mu_s}\right)$$

Are the measured speed and calculated speed consistent within uncertainties?

3.2.2 Questions

1. Imagine that you create a second harmonic in a short and long rope (both of the same mass density and with the same mass suspended at the end). Which of the three quantities: λ, f and v, would be larger, smaller or the same for both rope?
2. How should the amplitude of the standing wave depend on the frequency of the wave? Does it match with you observations? Can you think of some reasons why or why not?

3.3 Longitudinal Standing Waves in a Spring

Rearrange the standing wave apparatus as shown in Figure 3.2. Create at least four standing waves in a spring. Measure their wavelengths and record frequencies for which they occur. Use the formula $v = \lambda f$ to determine speed v. Record the results in Table D.9 provided in Appendix D and include it in your lab report.

Determine the mean value of the speed and the deviation from the mean.

The formula for the speed is $v = \sqrt{kL/\mu}$ where L is the length of the spring used to obtain standing waves, k is the spring constant, and μ is the linear mass density of the spring. Using the fact that $\mu = m_{\text{spring}}/L$ we can rewrite this as

$$v = \sqrt{\frac{kL^2}{m_{\text{spring}}}}$$

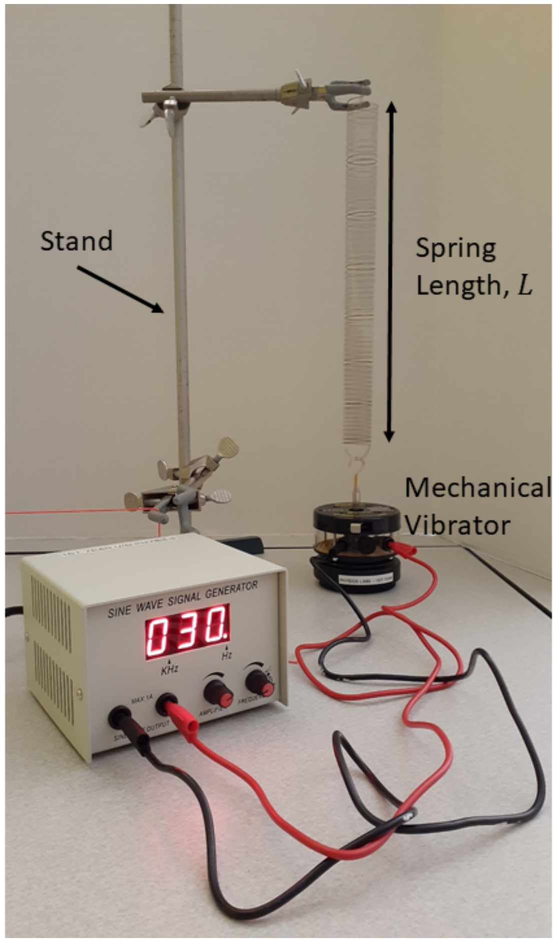

Figure 3.2: Apparatus for studying longitudinal waves in a spring

The values for k and m_{spring} will be provided. Measure L using a ruler or a measuring tape **while it is installed in the apparatus** and calculate the value for the speed. The expression for the uncertainty in v is:

$$\frac{\delta v}{v} = \frac{1}{2}\left(\frac{\delta k}{k} + \frac{2\delta L}{L} + \frac{\delta m_{\text{spring}}}{m_{\text{spring}}}\right)$$

Are the measured speed and calculated speed consistent within uncertainties?

3.3.1 Questions

Imagine that you create a third harmonic in two springs of the same length, and of the same linear mass density, but one soft and the other stiff. Which of the three quantities: λ, f and v, would be larger, smaller or the same for both springs?

3.4 Sound standing waves in the air tube

Sound waves are longitudinal waves. When a sound wave travels through the air, the air particles vibrate along the direction of motion of the wave. This results in series of high- and low-density regions of the air. The simplest kind of sound wave is called a "simple harmonic wave", which corresponds to a sinusoidal variation in the density, as shown in the figure below. Consider a simple harmonic wave generated by a loudspeaker, travelling

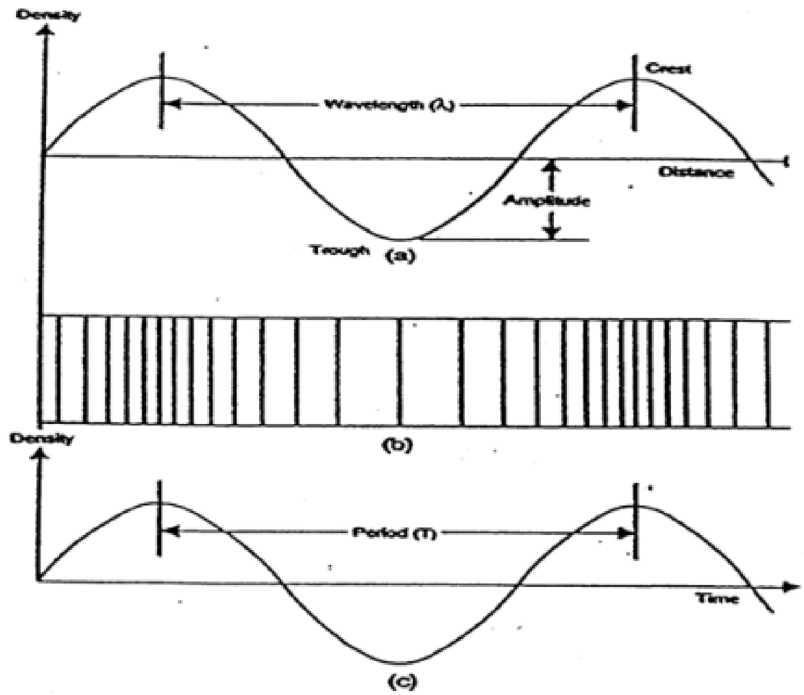

Figure 3.3: Illustration of transverse wave. Density is represented by variation of the shading lines (b). The higher density corresponds to more closely spaced lines. Note that density changes with distance (a) and time at a fixed position (c).

in a glass tube closed at one end. The wave will be reflected and inverted at the closed end of the tube. The reflected wave returns to the source and there it is reflected again. If the speaker is sending out a new crest just as the reflected crest reaches it, the new and reflected waves will reinforce each other and a standing wave will be produced.

The closed end of the tube is always a node, as the air particles at the closed end do not have the freedom to vibrate along the axis of the tube (sound is a longitudinal wave!). The open end of the tube is an antinode, as particles have complete freedom of motion. The wave which is created in the tube is called a **standing wave** and can occur only if the length of the tube is equal to an odd number of the quarter-wavelengths:

$$L = \frac{n\lambda}{4} \quad n = 1, 3, 5, ... \tag{3.1}$$

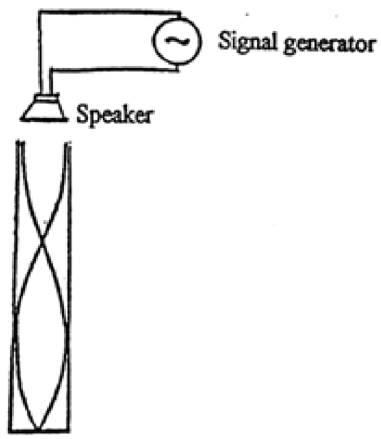

Figure 3.4: Acoustic standing wave in a tube

The wavelength λ, frequency f and the speed of sound v are inter-related via $v = \lambda f$. Note that the n in this equation is not the number of nodes, rather it is the number of quarter wavelengths. You will experimentally determine λ and since you will know f can then find the speed of sound v. Note how standing waves are located.

3.4.1 Prelab Exercises 2

A glass tube, open at one end, is filled with an unknown liquid. Using a sound wave with a frequency of (1200 $\pm$ 1) Hz, you set up the first standing wave and measure a lenght, L = (0.244 $\pm$ 0.001) m. Calcuate the speed of sound, including uncertainty, in the unknown material. Which liquid is the tube filled with given the speed of sound in (a) water is 1481 m/s, (b) acetic acid is 1173 m/s or (c) liquid Argon is 840 m/s.

3.4.2 Experimental

The experimental set up is shown in Figure 3.5.

The length of the air column in the glass tube may be varied by changing the position of the piston. A simple harmonic wave is produced by the loudspeaker attached at the open end of the glass tube. The loudspeaker is driven by the signal generator. A small microphone attached at the top of the tube is connected to an oscilloscope. The signal from the microphone is amplified and displayed on the screen of the oscilloscope. Resonance occurs when the maximum amplitude signal is seen on the oscilloscope.

3.4.3 Measurements and Calculations

1. Power on the oscilloscope, function generator, and microphone.
2. Adjust the frequency of the signal generator to about 700 Hz, and the amplitude to low level sound, about half of the maximum level.

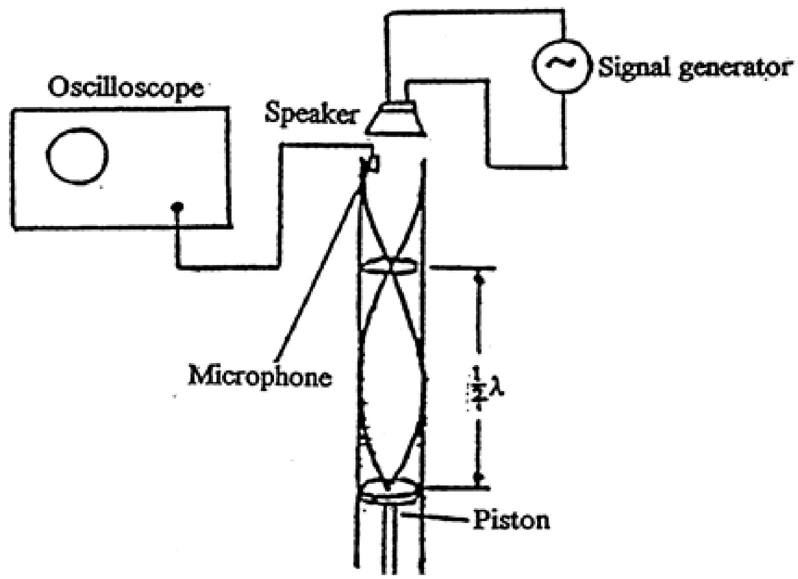

Figure 3.5: Experimental setup for studying acoustic standing waves.

3. Move the piston in the tube starting at the end closest to the speaker until you hear a substantial increase in the sound intensity (maximum amplitude on the screen of the oscilloscope). That corresponds to the first resonance (formation of a standing wave). Find other resonances in the tube.

4. Use the measuring tape to measure the resonance length for the piston to the open end of the tube. Record the resonance lengths in Table D.10 provided in Appendix D and include it in your lab report.

5. Draw wave envelopes, as in Fig. 3.5, for the first three standing waves obtained for frequency 700 Hz.

6. Adjust the frequency of the signal generator now to 1400 Hz and record all the positions of the piston for which a high intensity sound is detected at the open end of the tube. Enter your positions in the table and repeat the same calculations as before.

7. Calculate the average value of the velocity v and the average deviation σ (see Appendix A - Statistical Treatment of Random Errors). In your calculations use all the velocities you obtained for both frequencies since the speed of sound does not depend on the frequency.

8. Compute the percentage difference between the speed of sound you determined experimentally and the standard value for the speed of sound, 343 m/s (at room temperature and zero altitude).

3.4.4 Questions

1. Explain why speed of sound does not depend on the frequency of sound wave?
2. Explain how the speed of sound can be found keeping the piston at the same position and changing the frequency of the sound wave with the signal generator?

END OF LAB

Was this lab useful, instructive, and did it work well? If not, send an email to thatlabsucked@gmail.com and tell us your issues. In the subject line, be sure to reference the your course, the experiment, and session. example subject: *PHYS1010 Linear Motion monday 2:30*. We won't promise a response, but we will promise to read and consider all feedback.