Chapter 4

Oscillatory Motion & Conservation of Energy

4.0.1 Prelab Exercises

A mass $m = 50$ g is suspended from a spring with spring constant $k = 8.0$ N/m.

1. Determine the force acting on the mass when it is displaced 8 cm from its equilibrium position. If the mass is then released, what is the period of oscillation? What is the frequency?

2. What is the expression for the uncertainty in the total energy E_{tot} of equation 4.6? Assume an uncertainty in k of δk, uncertainty in m of δm, uncertainty in x of δx and an uncertainty in v of δv. Assume g is perfectly known (i.e., $\delta g = 0$). Refer to the section on Uncertainty Calculations in the introduction, Section 0.2.2, for more details. (Hint: first compute the uncertainties of the three terms in the sum, then add the uncertainties in the sum together.)

4.1 Spring-Mass System

4.1.1 Introduction

In this part of the lab you will analyze a special kind of motion, called oscillatory motion. A good illustration of oscillatory motion is the motion of a mass attached to a spring. If the spring is displaced from the equilibrium position and then released, the mass oscillates up and down about the equilibrium position. If the frictional forces, which are always present, could be neglected the spring would oscillate for an indefinite period of time. That idealized, frictionless oscillatory motion of the mass on the spring is called simple harmonic motion (abbreviated SHM). There is a long list of systems to which the ideal of SHM applies including: pendula, vibration of parts of musical instruments, vibration of atoms and molecules in solids, oscillation of an electric and a magnetic field in electromagnetic wave, time variation of voltage and current in alternating-current circuits. Simple harmonic

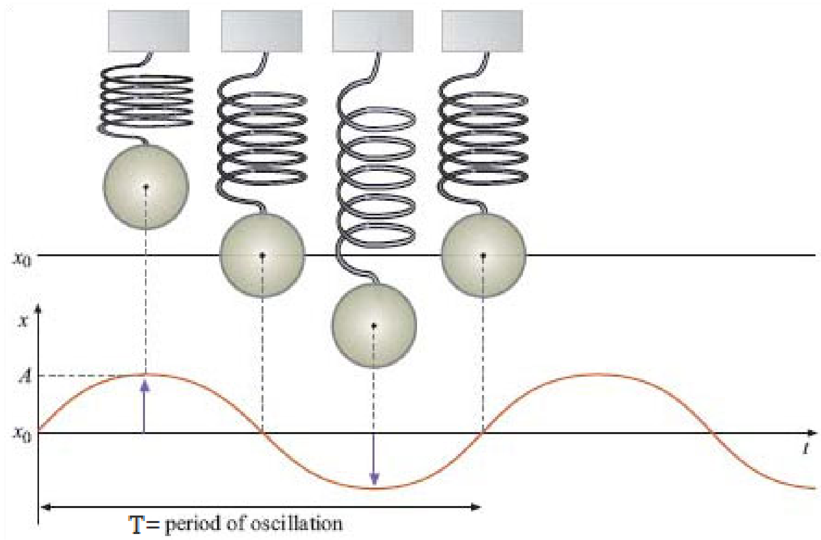

Figure 4.1: Motion of a mass attached to a spring.

motion in one dimension (call it x) can be represented by

$$x = A\cos\left(\frac{2\pi t}{T} + \theta_0\right) \tag{4.1}$$

where:

$x =$ the displacement from the equilibrium position ($x = 0$ is the equilibrium position)

$A =$ the Amplitude (maximum displacement from the equilibrium position)

$T =$ the period (time of one complete oscillation).

$\theta_0 =$ the phase constant (defines the displacement at $t = 0$).

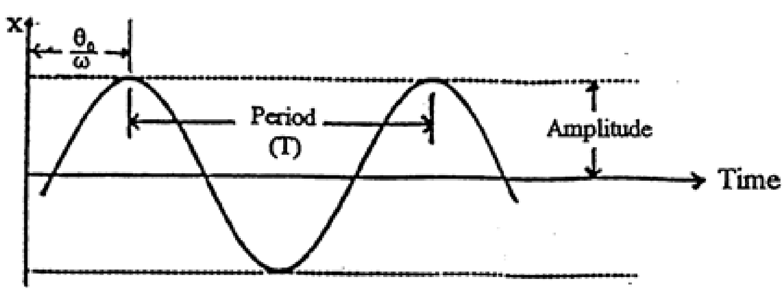

Figure 4.2: Illustration of meaning of parameters

The frequency f of the oscillatory motion is defined as the inverse of the period T

$$f = \frac{1}{T}$$

It is equal to the number of oscillations in 1 second. Recall that the unit of frequency is Hertz [Hz]. Using the angular frequency $\omega = 2\pi f$ (unit is radians/second) allows equation 4.1 to be rewritten in the more commonly used form

$$x = A\cos(\omega t + \theta_0) \tag{4.2}$$

So far we have described what SHM is but we have not explained what causes that kind of motion. When the mass connected to the spring is displaced, then the spring exerts a force on the mass in the opposite direction. The bigger the displacement, the bigger the force. Since this is just in one dimension $|\vec{F}| \equiv F$ and we then have $F = -kx$ where k is the so-called "spring constant." The negative sign expresses the fact that the force F has a direction opposite to the displacement. Using Newton's second law (where in one dimension $|\vec{a}| \equiv a$) we have

$$-kx = ma$$

Rearranging gives

$$a = -\frac{k}{m}x$$

but $a = d^2x/dt^2$ so we have

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x$$

The solution of the above differential equation is (be sceptical! check by differentiating twice)

$$x = A\cos\left(\sqrt{\frac{k}{m}}t + \theta_0\right) \tag{4.3}$$

Comparing equations 4.1 and 4.3 leads us to conclude

$$\frac{1}{T} = f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \tag{4.4}$$

That is, the frequency of oscillation depends on the mass m and the spring constant k, and not on the amplitude A.

The experimental setup is shown below:

Measurements and Calculations

1. Displace the mass $m = 500$ g suspended at the end of the spring about 10 to 15 cm from the equilibrium position and measure the time t for $N = 10$ oscillations. Repeat the measurement of the time t at least three times ($n = 3$). The uncertainty of your measurement is not given by the accuracy of the stopwatch but rather by your reaction time. The best estimate of the time t is the average of your three, or more,

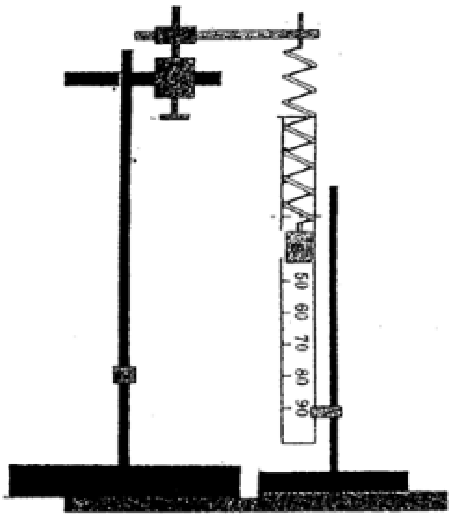

Figure 4.3: A mass is suspended at the end of a spring. When it is displaced downwards and released it will oscillate up and down.

measurements and the uncertainty δt is given by the average deviation (Appendix A). Record your results and calculations in the following table. A fillable copy of this table is included in Appendix D for your convenience. Cut or carefully tear it out the manual and include it in your lab report. Don't forget to include the spring number used in your apparatus.

i	Time for N=10 oscillations $t_i(s)$	Period $T_i = t_i/N(s)$	Average Period $\overline{T}(s)$	Average Deviation $\lvert T_i - \overline{T}\rvert(s)$
1				
2				
3				
avg		ΣT_i	$\Sigma T_i/n$	$\delta T = \frac{\Sigma\lvert T_i - \overline{T}\rvert}{n}$

Write your results in the form

$$(\overline{T} \pm \delta T)\ \text{s}$$

2. The spring constant k can be determined using the following formula (equation 4.4 , in which $T = 1/f$)

$$T = 2\pi\sqrt{\frac{m}{k}}$$
$$\text{or} \quad k = 4\pi^2\frac{m}{T^2}$$

Determine the spring constant k and its uncertainty δk. Based on the two rules described in Section 0.2.2 the uncertainty is calculated as:

$$\frac{\delta k}{k} = \frac{\delta m}{m} + \frac{2\delta T}{T}$$

The uncertainty δm is specified as the last significant digit written on the mass. For example, 10 g indicates $\delta m = 0.5$ g. If you use two masses 10 g + 5 g, then the total uncertainty is $\delta m = 1.0$ g.

Record the result in the form: $(k \pm \delta k)$ N/m

3. A different way to determine the spring constant is to measure the elongation of the spring when acted on by a known force, in this case, the force of gravity on a mass suspended from the spring $F_g = mg$. If the elongation of the spring is Δx when acted on by force mg, then $mg = k\Delta x$ or

$$k = \frac{mg}{\Delta x} \tag{4.5}$$

Its uncertainty is

$$\frac{\delta k}{k} = \frac{\delta m}{m} + \frac{\delta \Delta x}{\Delta x}$$

where we assumed that g is known relatively precisely.

With the 500 g mass suspended from the spring, measure the location of the bottom of this mass – this is your reference location x_1. Attached a second mass m to the 500 g mass and measure the new location of the bottom of the 500 g mass – this is reference location x_2. Δx is calculated the usual way $\Delta x = x_2 - x_1$. The mass m to use in equation 4.5 is the added mass only. Measure the spring constant using this technique and present the result in the form: $(k \pm \delta k)$ N/m.

4. Verify if the spring constants determined using two different methods are equal within the experimental uncertainty, that is, if the uncertainty intervals overlap. The uncertainty intervals overlap if the difference between the two spring constants, $|k_1 - k_2|$, is smaller than the sum of their two uncertainties $\delta k_1 + \delta k_2$.

4.1.2 Questions

1. Does the period T depend on the amplitude of the oscillation? Verify your answer experimentally.

2. Mass of the spring should be taken into account when analyzing oscillations of a spring-mass system. Using calculus, it can be shown that one third of the mass of the spring must be considered as a portion of the total mass. Designating the mass of the spring by m_s and the suspended mass by m

$$k = 4\pi^2 \frac{(m + m_s/3)}{T^2}$$

Measure the mass of the spring using a balance available in the lab. Was it justified to ignore mass of the spring? Explain.

3. Researchers try to design their experiments in such a way that the contribution of various measured quantities to the overall uncertainty are comparable. Which of the quantities: mass m, period T or elongation of the spring d, contributed most to the overall uncertainty δk? Propose a more precise way to measure this quantity.

4.2 Conservation of Energy

4.2.1 Introduction

The Law of Conservation of Energy states that Energy can be converted into different forms, but it cannot be created or destroyed. In an oscillating spring/mass system, energy can be transfered between: spring potential energy, gravitational potential energy, kinetic energy, energy lost as heat in the spring, and energy lost due to drag (viscous friction). In this experiment you will investigate this energy transfer.

The spring potential energy PE_{spring} is given by the expression:

$$PE_{\text{spring}} = \frac{1}{2}kx^2$$

where k is the spring constant of the spring, and x is the displacement from the equilibrium position.

Gravitational potential energy PE_{g} is given by:

$$PE_{\text{g}} = mgh = mgx$$

where m is the mass of the oscillating object, g is acceleration due to Earth's gravity, and h is the height with respect to some reference level. Since we have a vertically mounted spring, and we already called the displacement from the springs equilibrium x, this h is equal to x.

Kinetic Energy KE is given by

$$KE = \frac{1}{2}mv^2$$

where m is the mass of of the oscillating object, and v is the speed of the object.

The energy lost as heat in the spring and energy lost due to drag (viscous friction) occur in all systems, however, in the system under investigation are expected to be small, and for now, will be ignored (we have no easy way to measure them) . Therefore, the total energy of the system is:

$$E_{\text{tot}} = PE_{\text{spring}} + PE_{\text{g}} + KE = \frac{1}{2}kx^2 + mgx + \frac{1}{2}mv^2 \tag{4.6}$$

4.2.2 Setup and Data Collection

To collect data in this experiment, a sound probe (as was used in the Linear Motion Experiment) is used to measure the distance from the probe to the hanging mass and graphically display the experimental data. The software which interfaces with this probe is called LoggerPro. Run LoggerPro if it is not already running, and clear any old data (the easiest way is the brute-force method of closing LoggerPro, not saving the data when asked, and reopening.)

Ideally, we would have a spring which hangs without its coils compressed together. To achieve this situation, we will hang an initial mass onto the spring using the mass holder before we start. The mass holder weighs 50 g, and you should place a disk of 200 g onto the holder. This is the initial mass, m_i. This mass is needed simply to allow the spring to act more like an ideal spring. The level of the bottom of the mass hook will be the reference level for the gravitational potential energy and spring potential energy.

- We need to assign the present position of the spring as Zero. With the initial mass, m_i steady, select Experiment ⇒ Zero from the top menu.
- Now we will apply our oscillating mass to the spring. Chose a value which doesn't overly extend the spring, and place it on the mass holder. This is the added mass, m_{added}. The spring will now hang a lower position (do not re-zero the probe).
- To get the system to oscillate, pull the mass holder down by a few centimeters and release. Click on 

 to begin data collection. You will see a graph being plotted of the position vs time. Notice that it is a nice sinusoidal function (simple harmonic motion). Also notice there is a table on the left hand side of the screen of time, position, and velocity. These will be use for us to determine the energy.
- Print off two copies of the results, one for you, one for your partner.

4.2.3 Analysis

Based on the Law of Conservation of Energy, the total energy of the system should remain constant over time (for all rows in the table). What energies do we need to consider? From the list above, spring potential energy, gravitational potential energy and kinetic energy are certainly key for this experiment, and can be computed from the Position and Velocity values in the table. Energy lost as heat in the spring, and energy lost due to drag (viscous friction) are variables we cannot measure, but are likely quite small in this experiment, so we'll ignore them for now.To find if energy in conserved in this experiment, pick 5 times during the oscillations, preferably when the mass is at a variety of different positions and compute the spring potential energy, gravitational potential energy and kinetic energy for each.

Special Note When computing the Spring Potential Energy and the Gravitational Potential Energy, we only need to consider m_{added}, not the initial mass m_i required to extend the spring. (At all times, the extra Spring potential energy of m_i cancels the extra Gravitational Potential Energy of m_i) However, when computing the Kinetic Energy, we need to consider the total mass of $m_i + m_{\text{added}}$, since there is actually a larger mass moving than just m_{added}.

To get an idea of the Uncertainty in your measurement, calculate uncertainty for one line of your table. Assume the uncertainty in the Position reading is 1 mm, and the uncertainty in the velocity is 1 mm/s. Use a value of $k = 18.0 \pm 0.5$ N/m.

Question: Was energy conserved in this experiment?

Question: Did we need to include the heat lost in spring and work done by viscous drag in our analysis? Explain.

END OF LAB

Was this lab useful, instructive, and did it work well? If not, send an email to thatlabsucked@gmail.com and tell us your issues. example subject: *PHYS1010 Linear Motion monday 2:30.*