Chapter 5

Angular Motion

Circular motion, which is a special form of angular motion was initially thought to be "natural motion" (Galileo and Descartes in the early 17^{th} century). We have learned otherwise since then thanks to Newton (latter part of 17^{th} century). Our common experience with angular motion is much less than with linear motion, so our "intuition" about angular motion is not developed well. The first thing we need to tackle is how to describe it. As a starter we can try the ideas we used for linear motion. After all they may 'work', and we know the mathematics for them. So we shall try displacement, velocity, acceleration and force. However, we need to be cautious and to be prepared to adapt the ideas transplanted from linear motion. Shortly, we shall do a "theoretical analysis" of the motion of a mass moving in a circle. The analysis has two parts: 1) motion at a constant speed (NOT constant velocity) and 2) motion at a constant **magnitude** of acceleration (NOT constant acceleration). A sound understanding of the difference between a constant **scalar** and a constant **vector** is essential to grasp the relationship between the concepts and the mathematics in this experiment.

Objective

1. To determine the mass of an object moving in a circle through measurement of the centripetal force.
2. To determine the moment of inertia of a mechanical model of a diatomic molecule.

Apparatus Centripetal force apparatus, laboratory balance, Vernier caliper, stop watch.

In this lab we will exam two concepts: "Centripetal Force" and "Moment of Inertia".

5.1 Centripetal Force

5.1.1 Introduction

This experiment may seem simple but it is not. A good grasp of the principles and how the experiment is designed requires substantial thought. Please pay careful attention to:

- How we ensure the motion is in a circle.
- How we ensure there is a centripetal force of constant magnitude.
- How we acquire the required data.

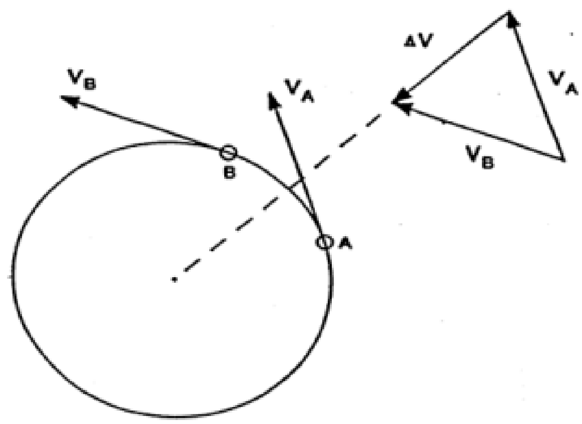


Figure 5.1: When a particle moves in a circle its velocity is always tangent to the circle.

As the particle moves from point A to B the velocity vector changes from $\vec{v_A}$ to $\vec{v_B}$ Hence an acceleration has to be present. If the **speed** is constant, then the velocity change is in the direction. That is, $\Delta\vec{v}$ is directed towards the centre of the circle So the acceleration defined as $\Delta\vec{v}/\Delta t$ has the same direction as vector $\Delta\vec{v}$. It can be shown that the size of this acceleration in circular motion, called radial acceleration a_R, is

$$a_R = \frac{v^2}{r} \tag{5.1}$$

where v, magnitude of the velocity $\vec{v}$, is the speed and r is the radius of the circle. If the particle makes one revolution in time T (period) then its speed is

$$v = \frac{2\pi r}{T} = \frac{\text{distance}}{\text{time}} = \frac{\text{circumference}}{\text{period}} \tag{5.2}$$

The period T is related to the frequency f (number of revolutions per second) by

$$f = \frac{1}{T} \tag{5.3}$$

Equation (5.2) can then be written as

$$v = 2\pi f r \tag{5.4}$$

or

$$v = \omega r \tag{5.5}$$

where $\omega = 2\pi f$ is called angular speed. Using the relation of equation (5.5), the size of the radial acceleration (equation (5.1)) can be written as

$$a_R = \omega^2 r \tag{5.6}$$

According to Newton's second law a particle can accelerate only when an unbalanced force acts upon it. So a force is needed to have circular motion. The required force is called a centripetal force and its magnitude is:

$$F_R = ma_R = m\frac{v^2}{r} = m\omega^2 r \tag{5.7}$$

We want to check that there is a centripetal acceleration equal to$\omega^2 r$ pointing radially inward when a mass moves around a circular path. How can we make this check? We shall use the relation $\vec{F} = m\vec{a}$, measure $\vec{F}$ and $\vec{a}$ then calculate m. Then we shall measure m on a laboratory balance to verify our calculation. If the two "agree", we will have a lot of confidence in our analysis.

5.1.2 Prelab Exercise 1

1. Draw a free-body diagram for mass M_b while in motion (Fig.5.2a). Identify the centripetal force. Assume that the mass hangs vertically.
2. Calculate the magnitude of the force exerted by the spring on mass M_b = 425 g, moving in a circle of radius $r = 19cm$, as shown in Fig. 5.2a. The mass makes 20 revolutions in 17 seconds. Determine the mass m , suspended over the pulley (Fig.5.2b), which stretch the spring by the same amount as during the rotation.

5.1.3 Experimental

You will use the apparatus shown below. (Underlined words tell you about key factors in the design of this experiment). A bob of mass M_b is suspended by a cord from one end of a horizontal bar. (What is the purpose of the counter mass shown in 5.2 a and b?). The bar is supported at the top of a vertical shaft. When the shaft is rotated by a motor the mass M_b has a tendency to swing outward, i.e. to move along a tangent (Fig. 5.2a).

This is opposed by a spring joining the mass with the shaft, i.e. the stretched spring supplied the required centripetal force. What is the direction of the force exerted by the spring on the mass? An adjustable pointer mounted on the base serves as a radius indicator. The force exerted by the spring on a moving mass can be measured when the system is at rest. This can be accomplished by attaching a string with a hook over a pulley and adding masses until the spring extension is the same as during the motion (Fig. 5.2b).

5.1.4 Measurements

1. Assemble the equipment as shown in Fig. 5.2a. Move the counter-mass to the end of the horizontal bar and find the balance between the bob and counter-mass. Fix the

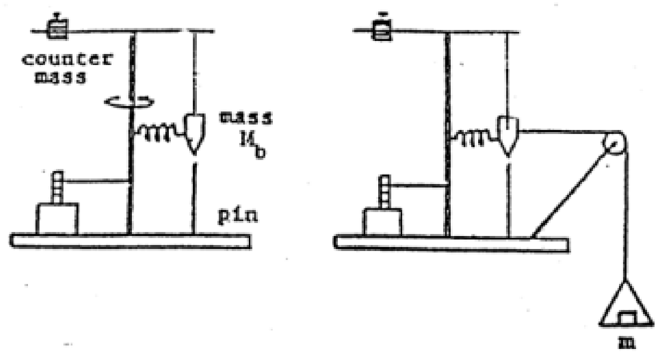


Fig. 2a
Rotating

Fig. 2b
Stationary

Figure 5.2: Angular Momentum Setup

screw joining the horizontal bar and the shaft. Switch on the motor, give a small push to the rotating mass and wait about 30s until **the speed is constant**. The speed can be changed by selecting different grooves on the motor shaft. The highest groove corresponds to the lowest speed. Select the groove which gives a **speed** such that the rotating bob hangs **vertically** with respect to the horizontal bar. Adjust the pointer so that the bob is directly over the pointer.

2. Measure the time t for about n = 20 revolutions. Repeat the time measurement three times and find the average and the average deviation as explained in Appendix A. In your calculations, the average deviation will serve as uncertainty δt.

3. The angular speed ω can be calculated using the formula

$$\omega = \frac{2\pi n}{t} \tag{5.8}$$

4. Measure the radius, r, of rotation from the tip of the pointer to the centre of the shaft. Don't forget to include the error, δr, associated with your measurement of r.

5. Assemble the equipment as shown in Fig. 5.2b. Add some masses to pull the bob over the pointer. Measure the total mass, m (including the mass of the hook which is 50g), and its uncertainty, δm using the balance. Determine the force (weight) $F = mg$.

6. Find the ratio $F/(r\omega^2)$. According to the equation (5.7) the ratio represents the mass of the bob M_b. Using (5.8) we have

$$M_b = \frac{F}{r\omega^2} = \frac{mgt^2}{4\pi^2 n^2 r}$$

Calculate the uncertainty δM_b

$$\frac{\delta M_b}{M_b} = \frac{\delta m}{m} + \frac{2\delta t}{t} + \frac{\delta r}{r}$$

take δt as the average deviation calculated in step 2 above, δm is the uncertainty in the measured mass, m, and δr is the error associated with the measurement of r. Note that quantities n and g do not contribute to the uncertainty.

Record the result in the form $(M_b \pm \delta M_b)$

7. Remove the bob and determine its mass $M_{b,balance}$ on a laboratory balance.

8. Verify if measurements of M_b and $M_{b,balance}$ are consistent, that is, if $|M_b - M_{b,balance}| \leq \delta M_b$.

5.1.5 Questions

1. If you were to increase the frequency of rotation, how would the radius r and the centripetal force F change? Would both quantities change; only one or none? Justify your answer!

2. Identify the centripetal forces in the following two situations:

   (a) The Moon moving around the Earth

   (b) An electron moving around the nucleus.

5.2 Rotational Inertia (Moment of Inertia)

5.2.1 Introduction

Again, this part of the experiment has a deceptively simple appearance. You need to pay careful attention to how the ideas of displacement, velocity, acceleration, mass, and force are adapted to describe changes in this special case of angular motion. Pay careful attention to:

- the rotational variables analogous to displacement, velocity, and acceleration that are used to describe angular motion

- the difference between a constant magnitude for acceleration or force versus a constant acceleration or force vector

In the first part of this lab we dealt with the rotation of a single mass. We neglected the dimensions of the rotating mass and treated it as a poiint particle. In the second part of the lab you will analyze the rotation of an extended **rigid** body (i.e., one consisting of many particles). Each element of a rigid body rotates with the same angular speed ω since each point of the body makes the same number of rotations in a given time. The definition of angular speed ω is:

$$\omega = \frac{d\theta}{dt}$$

where θ is the angular distance measured in radians. This is similar to the definition for linear speed v:

$$v = \frac{dx}{dt}$$

where x = linear distance. Similar to the magnitude of the linear acceleration $a = dv/dt$, we can define the magnitude of the angular acceleration α:

$$\alpha = \frac{d\omega}{dt} \tag{5.9}$$

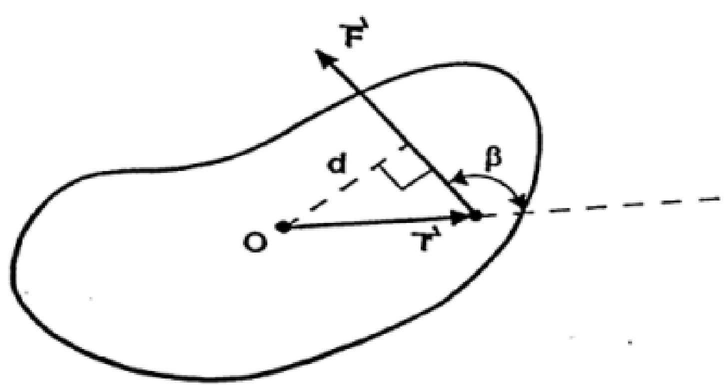
Both the angular speed and the magnitude of the angular acceleration are the same for **all particles** of a rigid body. Usually there is a simple relation between the magnitudes of the angular and linear accelerations

$$a = r\alpha \tag{5.10}$$

where r is the distance from the body to the axis of rotation. We will delay discussion of the circumstances for which $a \neq r\alpha$. The angular acceleration of the rigid body is proportional to the quantity called torque. Torque is a measure of the ability of a force to rotate the body about a certain axis. The magnitude of torque τ is defined as

$$\tau = rF \sin\beta = Fd$$

where the symbols are as shown in the figure below



$\vec{F}$ is the force acting on the body, and d is the perpendicular distance from the line of action of force $\vec{F}$ to the axis of rotation. The force $\vec{F}$ can rotate the body more effectively if d is bigger, and hence τ is bigger. Torque is related to the angular acceleration.

$$\vec{\tau} = I\vec{\alpha}$$

where I is what is called the moment of inertia. This relation is a rotational analogue of Newton's second law for linear motion ($\vec{F} = m\vec{a}$) with F, m, and a replaced by τ, I, and α. The moment of inertia of a particle of mass M a distance R from the axis of rotation is:

$$I = MR^2 \tag{5.11}$$

The moment of inertia of an extended body is found by dividing the body into small elements and finding the sum of the moment of inertia of each element

$$I = \sum_i M_i R_i^2 \tag{5.12}$$

where R_i is the perpendicular distance from M_i to the axis of rotation. Thus the moment of inertia of an extended body depends upon the size and shape of the body, and the location of the axis of rotation.

5.2.2 Prelab Exercise 2

Read through Experimental section before answering these questions.

1. Is the tension in the cord Fig. 5.3 equal to mg when the hanging mass drops? Draw a free-body force diagram for the hanging mass and write the equation of motion (Newton's II Law) for it.

2. Susan observed that when mass $m = 500g$ is suspended (see Fig. 5.3), the system acquires very high speed after a few rotations. This can make the experiment dangerous. Trying to reduce the speed of the system, Susan increases the distance of the two masses M_1 and M_2 from the rotating shaft. Does this make sense? Explain.

5.2.3 Experimental

We have a model for angular motion based on analogies from linear motion. Now we need to check the predictions with a carefully controlled experiment. In particular we want to examine torque, angular acceleration, moment of inertia and how they are related. Again, pay close attention to the way in which the ideas discussed are built into the design of the experiment.

Make changes to the equipment used in part A of the lab (Fig. 5.3). Remove the spring, the crossbar with the hanging mass and the belt from the motor shaft. Use the long thin crossbar. With one aluminum and one brass mass, set the masses as far apart as possible on the crossbar (**securely fastened** into a depression on the bar). The rotation is only about one point. To make this centre-of-mass coincide with the shaft, one person should hold the apparatus so that the shaft is tipped horizontal and free to rotate. The other person can then move the crossbar back and forth relative to the shaft until the balance point is found. Then clamp it **firmly** to the shaft. A nylon string with a mass on one end is attached to the shaft of the rotational apparatus. The shaft is rotated so that the string *winds around it* until m is close to the pulley. The stop watch should be started simultaneous to releasing the mass m. The stopwatch is stopped at the instant that the system completes n rotations.

In the first part, a uniform angular velocity was maintained in each run. In this experiment, the angular velocity *increases* during a run because m is *accelerating*. This is uniform acceleration. [How do you decide that $\vec{a}$ is constant?]

To simplify the analysis of the situation described above, friction and the mass of the pulley, which are small, will be ignored. [How does this simplify the analysis?]

Suppose m falls with a constant acceleration $\vec{a}$. The net force acting on m is $m\vec{a}$; if the size of the tension in the string is T_s, then

$$ma = mg - T_s \tag{5.13}$$

Be sure you know why this is true. Since the pulley requires a negligible force to turn it,

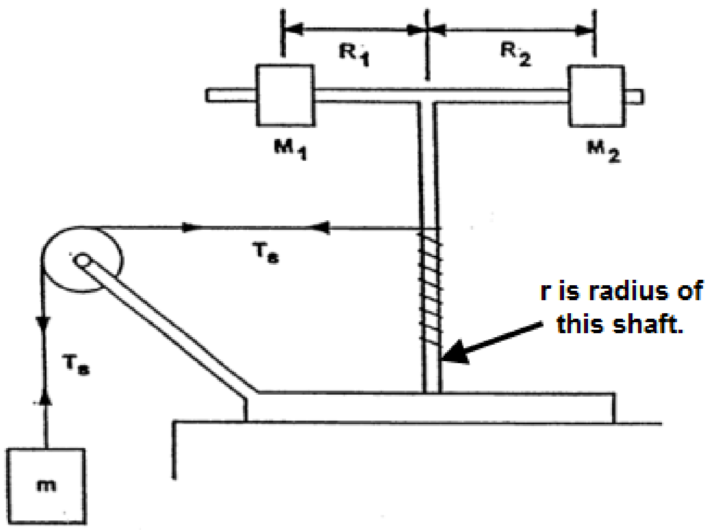


Figure 5.3: Schematic of Experiment

the tension in the string is the same throughout. The taut string applies a constant torque $\vec{\tau}$ to the central shaft of radius r, causing the system to rotate.

$$\tau = T_s r \text{ is the magnitude of the torque} \tag{5.14}$$

Note that mg is not explicitly involved. Recall that the analogue of Newton's second law, $\vec{F} = m\vec{a}$, for rotational motion is $\vec{\tau} = I\vec{\alpha}$. In this particular case $\vec{\tau} = I\vec{\alpha}$ are constant. Substituting from 5.13 and 5.14 gives the relationship

$$T_s = \frac{I\alpha}{r} = mg - ma \tag{5.15}$$

The magnitude of the linear acceleration can be related to the magnitude of the angular acceleration α by considering the point where the string just leaves the shaft. In this case $a = r\alpha$ where r is the radius of the shaft. If the system starts from rest and completes n revolutions ($= 2\pi n$ radians) in t seconds, the magnitude of the angular acceleration is given by

$$2\pi n = \frac{1}{2}\alpha t^2 \quad (\text{Compare } x = \frac{1}{2}at^2) \tag{5.16}$$

Substituting for a and α in 5.15 gives an expression for I which only involves easily measured quantities.

$$I = mr\left(\frac{t^2 g}{4\pi n} - r\right) \tag{5.17}$$

In the above formula, m is the hanging mass in Fig.5.3, and r is the radius of the shaft.

Note that this I is the moment of inertia of the entire rotating system, i.e. the sum of the moments of inertia of all the component parts.

5.2.4 Measurement and Calculations

You are now going to attempt to determine the rotational intertia of two masses orbiting about an axis. However, our system is comprised of more than just the masses- it has a support rod for the masses, a rotating shaft, some bearings. Describe below is a method of cancelling out the intertia of the unwanted components, allowing us to measure the intertia of just the masses.

1. Ensure the drive belt connecting the motor to the rotating shaft used in Part I is removed.

2. Measure the two masses on the scale provided.

3. Measure the radius r of the shaft using the Vernier caliper.

4. Set up the experiment as shown in Fig. 5.3 with the balance-point of the cross bar attached to the rotating shaft. Be sure that the screw securing each mass is tightened into an indent on the cross bar.

5. Measure the position (R_1 and R_2) of the masses with respect to the axis of rotation. Briefly describe your method of measuring R_1 and R_2. How well do you know where the centre of the masses is?

6. **Have the TA check your setup to ensure that everything is secured properly.**

7. To apply a known torque to the shaft (to cause rotation) we will use a third mass $m = 500g$ at the end of the nylon string. Gravity acting on this third mass supplies a torque $\vec{\tau}$ to the system about the shaft.

8. Release the mass and measure the time t for n (about 10) rotations of the system. You now have data which allows you to compute the rotational inertia of the entire system I_{system}.

9. Now, remove masses M_1 and M_2, but NOT the cross bar.

10. Repeat measurements performed in step 8 on the remaining part of the apparatus in order to determine the rotational inertia of the support system only I_{support}. This time, use a smaller third mass, $m = 50g$ to the end of the nylon thread so it does not spin too fast.

11. Determine the rotational inertia I_{system} for the whole system and the rotational inertia $I_{support}$ for the support system using equation (5.17). The difference ($I_{\text{system}} - I_{\text{support}}$) is the rotational inertia of the two masses M_1 and M_2 alone without the support rod $I_{2\ \text{MassesMeasured}}$. This is a useful model for any diatomic molecule or binary star system

12. Calculate the expected rotational inertia of two masses using the equation below (see Fig. 5.3).

$$I_{2\ \text{Masses Expected}} = M_1 R_1^2 + M_2 R_2^2 \tag{5.18}$$

13. Calculate the percentage difference between rotational inertias $I_{2\ \mathrm{MassesMeasured}}$ and $I_{2\ \mathrm{MassesExpected}}$. If the two different calculations of $I_{2\ \mathrm{Masses}}$ agree then we can feel pretty confident about the validity of the relationship $\vec{\tau} = I\vec{\alpha}$.

5.2.5 Questions

1. Did you neglect any frictional forces when calculating the rotational inertia? If so, what kind?
2. In Fig.5.3 , the cord wrapped around the shaft should form only one layer. Which of the following quantities: torque applied to the shaft, angular acceleration of the system, rotational inertia of the system, acceleration of the falling mass, are changed if more than one layer is formed by the cord? Explain why.

END OF LAB

Was this lab useful, instructive, and did it work well? If not, send an email to thatlabsucked@gmail.com and tell us your issues. example subject: *PHYS1010 Linear Motion monday 2:30.*