PHYSICS 1011 — Problem Set 5

Total – 84 marks — due November 28, 2021

Nullius in verba ('take nobody's word for it') – motto of the **Royal Society**

1. (4 marks) Four equal masses m are located at the corners of a square of side L, connected by essentially massless rods. Find the moment of inertia of this system about an axis

   (a) that coincides with one side

   (b) that bisects two opposite sides

   (c) Would you expect I_a to be different from I_b and, if so, why?

2. (5 marks) The chamber of a rock-tumbling machine is a hollow cylinder with mass 120 g and radius 8.5 cm. The chamber is closed by end caps in the form of uniform circular disks, each of mass 33 g. Find

   (a) the moment of inertia of the chamber about its central axis

   (b) the torque needed to give the chamber an angular acceleration of 3.3 rad/s.

3. (5 marks) A 108 g Frisbee is 24 cm in diameter and has half its mass spread uniformly in the disk and the other half concentrated in the rim.

   (a) What's the Frisbee's moment of inertia?

   (b) With a quarter-turn flick of the wrist, a student sets the Frisbee rotating at 550 rpm. What's the magnitude of the torque, assumed constant, that the student applied?

4. (3 marks) A solid 2.4 kg sphere is rolling at 5.0 m/s. Find

   (a) its translational kinetic energy

   (b) its rotational kinetic energy

5. (4 marks) The moment of inertia of a thin rod of mass M and length L about a perpendicular axis is $ML^2/9$. Where's the axis?

6. (4 marks) A marble rolls down an incline, and when it's halfway down it's going at 1.12 m/s. Find

   (a) its starting height

   (b) its speed at the bottom

7. (4 marks) A wheel with total mass 14.7 kg consists of a solid wooden disk with a thin metal rim bonded to the disk's edge. It starts from rest and rolls down an incline of height 1.00 m. At the bottom it's going at 3.38 m/s. Find the masses of the disk and the rim.

8. (6 marks) A solid ball of mass M and radius R starts at rest at height h above the bottom of the path in Figure 1. It rolls without slipping down the left side. The right side of the path, starting at the bottom, is frictionless. To what height does the ball rise on the right?

9. (2 marks) A gymnast with a moment of inertia of 62 kg·m^2 is tumbling head over heels with angular momentum 470 kg·m^2/s. What's their angular speed?

10. (3 marks) A potter's wheel with moment of inertia of 6.40 kg·m^2 is spinning freely at 19.0 rpm. The potter drops a 3.2 kg lump of clay onto the wheel, where it sticks 46.0 cm from the rotation axis. What's the wheel's subsequent angular speed?

11. (5 marks) A 1150 kg car rounds a circular turn of radius 125 m, toward the left, on a horizontal road. Its angular momentum about the centre of the turn has magnitude 2.86×10^6 kg·m^2/s. What are

    (a) the direction of the car's angular momentum?

    (b) its speed?

    (c) the magnitude of its angular momentum about the centre of the turn, once it's exited the turn and is on a straight stretch of road?

12. (4 marks) A particle of mass m moves in a straight line at constant speed v. Show that its angular momentum about a point located a perpendicular distance b from its line of motion is mvb regardless of where the particle is on its line of motion. Stated differently, show that the angular momentum is independent of time (i.e., has a constant value).

13. (3 marks) Two identical 1800 kg cars are traveling in opposite directions at 83 km/hr on a straight two-lane highway. Each car's centre-of-mass is 3.2 m from the centre of the highway. What are the magnitude and direction of the angular momentum of the system containing the two cars about a point on the centre line of the highway? (Hint: you may want to use the result of the previous question.)

14. (3 marks) A 622-g basketball with 24.0 cm diameter is suspended by a wire and is undergoing torsional oscillations at 1.87 Hz. Find the torsional constant of the wire.

15. (5 marks) A simple pendulum is swinging with period $T = 2.62$ s and amplitude 8.85°. Find the following quantities where the pendulum's kinetic and potential energies are equal:

    (a) the angle the pendulum makes with the vertical

    (b) the speed of the pendulum bob.

16. (6 marks) A pendulum consists of a 320 g solid ball 15.0 cm in diameter suspended by an essentially massless string 80.0 cm long. Calculate the period of this pendulum treating it first as a simple pendulum and then as a physical pendulum. What's the error in the simple-pendulum approximation? (Hint: remember the Parallel-axis Theorem.)

17. (6 marks) A hollow ball of diameter D is suspended from a string of negligible mass whose length is equal to the ball's diameter. The string is attached to the surface of the ball. Find an expression for the period of this physical pendulum in the small-amplitude approximation.

18. (4 marks) A portion of a roller coaster track is described by the equation $h = 0.94x$-$0.010x^2$, where h is the height and x is the horizontal postion, both in metres.

    (a) Find a point where the roller-coaster car could be in static equilibrium on the track.

    (b) Is this equilibrium stable or unstable?

19. (4 marks) A potential energy function in two dimensions is given by $U(x) = a(x^2 - y^2)$ where x and y measure position in m and a is a positive constant with the units of $\mathrm{J/m^2}$.

    (a) Show that this function has an equilibrium at the origin.

    (b) Is the quilibrium stable against small displacements in the x direction? What the y direction?

20. (4 marks) The potential energy as a function of position for a particle is given by:

$$U(x) = U_0 \left(\frac{x^3}{x_0^3} + a\frac{x^2}{x_0^2} + 4\frac{x}{x_0} \right)$$

    where x_0 and a are constants. For what values of a will there be two static equilibria? Comment on the stability of these equilibria.

Flif flif flif flif very fast
is the noise the birds make
running over us.
A poet would say 'fluttering',
or
'see-sawing with sun on their wings'.
But all it is
is flif flif flif flif very fast.

"*Song to Alfred Hitchcock and Wilkinson*" by **Michal Ondaatje**

(I include this because I think it is one of the best poems about science ever written.)

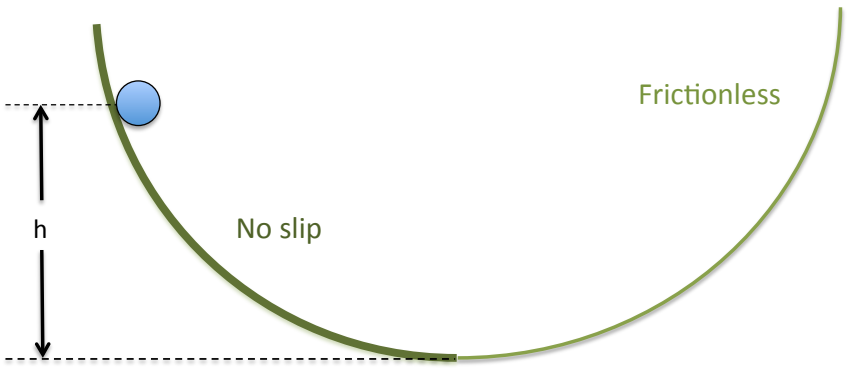

Figure 1: Figure for question #8

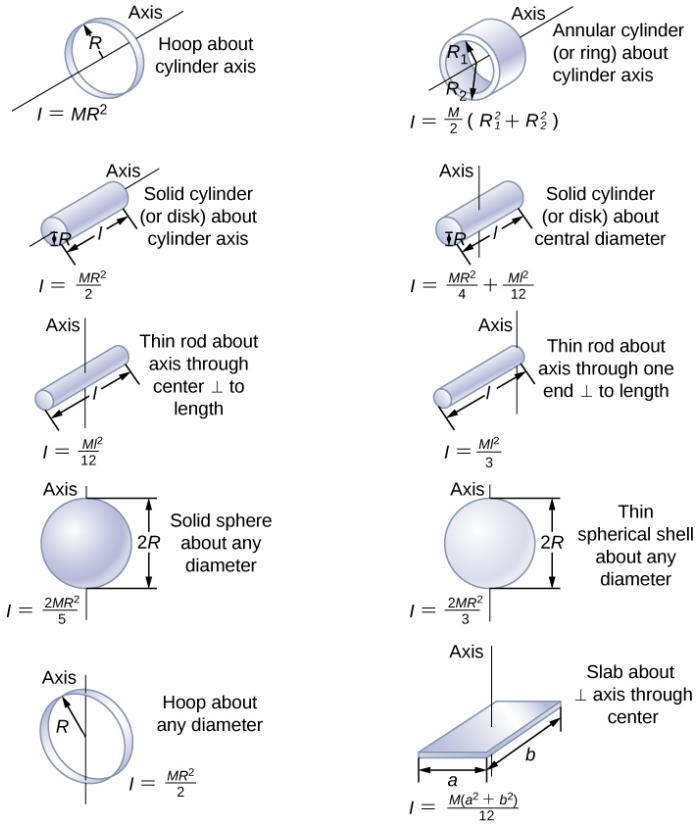

Figure 2: Moments of Inertia of Common Shaped Objects