PHYSICS 1011 — Solutions to Problem Set 1

Total – 89 marks — was due October 1

"In theory, there is no difference between theory and practice. But in practice, there is."
Yogi Berra

"The miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve."
Eugene Wigner – winner of the 1963 Nobel Prize for Physics.

Clearly in the following you need to show how you got an answer – i.e., show your work. If you don't you will get 0 marks for the problem.

1. (3 marks) Estimate, to an order of magnitude, the number of heartbeats in a typical human lifetime.

- This is an example of a "Fermi Question" where you need to break the problem down into a product of estimates to give a, typically, order of magnitude type answer. Here, the number of heartbeats in a typical human lifetime requires estimates of:

  i) a typical human lifetime, call it L, and estimate it to be 80 years

  ii) a typical human heart rate, call it R, and estimate it to be 70 beats/minute

  Then the total number of heartbeats (T) is;

$$\begin{aligned} T &= LR \\ &= (80 \text{ years})(70 \text{ beats/minute}) \\ &= (80 \text{ years})(\pi \times 10^7 \text{ seconds/year})(70 \text{ beats/minute})/(60 \text{ seconds/minute}) \\ &= ((8)(\pi)(7)/6) \times 10^8 \text{ beats} \\ &= 2.93 \times 10^9 \text{ beats} \end{aligned}$$

  So the estimated answer is a few billion beats in a typical human lifetime.

2. (3 marks) Estimate the area of skin on your body.

- As in problem 2 you have to make some approximations and then break it down into the product of estimates. All bodies are different so this would vary even from person to person. Okay, assume the cow is a sphere but that the human body is a rectangular prism of length $L = 1.5$ m, width W of 0.6 m, and thickness T of 0.2 m. Then the surface area would be $2(LW + LT + WT)$ which gives:

$$2[(1.5 \text{ m})(0.6 \text{ m}) + (1.5 \text{ m})(0.2 \text{ m}) + (0.6 \text{ m})(0.2 \text{ m})] = 2.64 \text{ m}^2$$

  So the answer is around 1 to 4 m^2.

3. (4 marks) It is found from a calculation that $Q = AB^2/\sqrt{C}$. What is the value of Q (with units and proper significant figures) for:

   (a) (2 marks) $A = 150.0$ kg^2, $B = 1.42 \times 10^{-2}$ m, and $C = .0250$ s^4?

   - $Q = 0.19129$ kg^{2}m^2/s^2 but keeping the most significant figures (which is 3 for B and C) gives $Q = 0.191$ kg^{2}m^2/s^2. And kg^{2}m^2/s^2 is units of momentum squared.

   (b) (2 marks) $A = 150$ kg^{-2}, $B = 1.42 \times 10^{-2}$ m^{-1}, and $C = .025$ s^{-4}?

   - The answer is the same but the units of Q are now kg^{2}m^{2}s^2

4. (5 marks) Text Chapter 1, Problem 50

- Using the notation of M for mass, L for distance, and T is time.

   (a) V is in L^3 and $\pi r^2 h$ is $L^2 \times L = L^3$ which is dimensionally consistent.

   (b) A is in L^2 and $2\pi r^2 + 2\pi r h$ is $L^2 + L \times L$ or L^2 which is dimensionally consistent.

   (c) V is in L^3 and $0.5bh$ is $L \times L = L^2$ which is not dimensionally consistent.

   (d) V is in L^3 and πd^2 is L^2 which is not dimensionally consistent.

   (e) V is in L^3 and $\pi d^3/6$ is L^3 which is dimensionally consistent.

5. (2 marks) Text Chapter 1, Problem 68

- 0.50 cm over 20 m is 2.5×10^{-4} or 0.025%

6. (16 marks) You have two vectors: $\vec{A}$ = (2,-1,3) and $\vec{B}$ = -3$\hat{\imath}$-2$\hat{\jmath}$+1$\hat{k}$. Write out the expression for and then evaluate:

   (a) $|\vec{A}|$ (2 marks)

   - $|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2} = \sqrt{(2)^2 + (-1)^2 + (3)^2} = \sqrt{4+1+9} = \sqrt{14} = 3.74$

   (b) $\hat{A}$ (2 marks)

   - $$\hat{Q} = \frac{\vec{A}}{|\vec{A}|} = \frac{2\hat{\imath} - 1\hat{\jmath} + 3\hat{k}}{\sqrt{14}} = (0.53, -0.27, 0.82)$$

   (c) the angle $\vec{A}$ makes with the y axis (2 marks)

   - $\vec{A} \cdot \hat{\jmath} = |\vec{A}| \cos\theta_{Ay}$ so
   $$\theta_{Ay} = \cos^{-1}\left(\frac{\vec{A} \cdot \hat{\jmath}}{|\vec{A}|}\right) = \cos^{-1}\left(\frac{-1}{\sqrt{14}}\right) = 105.5^\circ$$

   (d) $\vec{B}$-$\vec{A}$ (call it $\vec{C}$) (2 marks)

   - $\vec{C} = (B_x - A_x, B_y - A_y, B_z - A_z)$ = [(-3-2),(-2+1),(1-3)] = (-5,-1,-2)

   (e) $|\vec{C}|$ (2 marks)

   - $|\vec{C}| = \sqrt{(-5)^2 + (-1)^2 + (-2)^2} = \sqrt{30} = 5.48$

   (f) $\vec{A} \cdot \vec{B}$ (2 marks)

- $\vec{A}\cdot\vec{B}$ = (2)(-3)+(-1)(-2)+(3)(1) = -1

(g) the angle between $\vec{A}$ and $\vec{B}$ (2 marks)

- $\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta_{AB}$ so

$$\theta_{AB} = \frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|} = \frac{-1}{(\sqrt{14})(\sqrt{(-3^2)+(-2)^2+(1)^1}} = \frac{-1}{(\sqrt{14})(\sqrt{14})} = \frac{-1}{14} = 94.1^\circ$$

(h) $\vec{C}\cdot\vec{A}$ (2 marks)

- There are a number of ways to answer this question. Here's two.
  (1) $\vec{C}\cdot\vec{A} = C_xA_x + C_yA_y + C_zA_z$ = (-5)(2)+(-1)(-1)+(3)(-2) = -15
  (2) $\vec{C}\cdot\vec{A} = (\vec{B}-\vec{A})\cdot\vec{A} = \vec{B}\cdot\vec{A}$ - $\vec{A}\cdot\vec{A} = \vec{A}\cdot\vec{B}$ - $|\vec{A}|^2$ = -1 - $(\sqrt{14})^2$ = -15

7. (3 marks) Text Chapter 2, Problem 40

- We're given $\vec{F} = (-2980.0\hat{\imath} + 8200.0\hat{\jmath})$N where $\hat{\imath}$ and $\hat{\jmath}$ denote east and north (so the force has components of 2980.0 N west and 8200.0 N north). So

$$|\vec{F}| = \sqrt{(-2980.0)^2 + (8200.0)^2} = 8724.7 \text{ N}$$

That is, the pull is 8724.7 N 70° north of west.

8. (2 marks) The sum of two vectors, $\vec{A}+\vec{B}$, is perpendicular to their difference, $\vec{A}-\vec{B}$. How do the vectors' magnitudes compare?

- If the two vectors are perpendicular then their dot product is 0. That is,

$$(\vec{A}+\vec{B})\cdot(\vec{A}-\vec{B}) = \vec{A}\cdot\vec{A} - \vec{A}\cdot\vec{B} + \vec{B}\cdot\vec{A} - \vec{B}\cdot\vec{B} = 0 \quad (1)$$

Now $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$ so -$\vec{A}\cdot\vec{B}$+$\vec{B}\cdot\vec{A} = 0$ and, by definition, $\vec{A}\cdot\vec{A} = |\vec{A}|^2$ and $\vec{B}\cdot\vec{B} = |\vec{B}|^2$. Equation (1) therefore becomes $|\vec{A}|^2 - |\vec{B}|^2 = 0$ giving the answer $|\vec{A}|$=$|\vec{B}|$.

9. (4 marks) For the two vectors $\vec{G} = 2\hat{\imath}$-3$\hat{\jmath}$+1$\hat{k}$ and $\vec{H} = 2\hat{\imath}$+1$\hat{\jmath}$-1$\hat{k}$ what is $\vec{G}\cdot\vec{H}$? What is the angle between $\vec{G}$ and $\vec{H}$?

- $\vec{G}\cdot\vec{H}$ = (2)(2)+(-3)(1)+(1)(-1) = 4 - 3 -1 = 0. The angle between them is 90° since the dot product is zero.

10. (3 marks) Text Chapter 2, Problem 26

- Call the line of pull the x-axis, then:

$$\vec{F} = [2(196) - 4(98) - 5(62) + 3(150) - 250]\hat{\imath} \text{ N} = (-110 \text{ N})\hat{\imath}$$

So the net pull is 110 N to the left.

11. (3 marks) Text Chapter 2, Problem 38

  - You are $\sqrt{18.0^2 + 25.0^2}$ m or 30.8 m from your starting point.
  - The displacement vector is (-18.0 m)$\hat{\imath}$ + (25.0 m)$\hat{\jmath}$.

- The displacement is 54° north of west.

12. (4 marks) Text Chapter 2, Problem 50

- Calling east the $+x$ direction and north the $+y$ direction, the position vector after the first part of the journey is

$$\vec{r}_1 = 40(\cos 60°\hat{\imath} + \sin 60°\hat{\jmath}) \text{ km}$$

and for the second part of the journey it is

$$\vec{r}_2 = 30(\cos 15°\hat{\imath} + \sin 15°\hat{\jmath}) \text{ km}$$

Therefore the displacement vector is

$$\begin{aligned} \vec{d} &= \vec{r}_1 + \vec{r}_2 \\ &= [(40\cos 60° + 30\cos 15°)\hat{\imath} + (40\sin 60° + 30\sin 15°)\hat{\jmath}] \text{ km} \\ &= (49.0\hat{\imath} + 42.4\hat{\jmath}) \text{ km} \end{aligned}$$

The distance from the start is $|\vec{d}| = 64.8$ km and geographic direction of its displacement vector is 40.9° north of east.

13. (4 marks) Text Chapter 2, Problem 60

- Call the line AB the y-axis with the direction towards A being the $+y$ direction. The two force vectors, left and right, and the resultant force pulling on the barge are then:

$$\begin{aligned} \vec{F}_L &= 5000(-\sin 12°\hat{\imath} + \cos 12°\hat{\jmath}) \\ &= -1040\hat{\imath} + 4890\hat{\jmath} \\ \vec{F}_R &= 4000(\sin 15°\hat{\imath} + \cos 15°\hat{\jmath}) \\ &= 1035\hat{\imath} + 3864\hat{\jmath} \\ \vec{F} &= \vec{F}_L + \vec{F}_R \\ &= -4.3\hat{\imath} + 8754\hat{\jmath} \end{aligned}$$

So the magnitude of the resultant pull is 8754 units of force, 0.03° (essentially zero degrees) to the left.

14. (4 marks) Variation on Text Chapter 3, Problem 27
The position of a particle moving along the x-axis is given by $x(t) = 3.0 - 2.0t$ m.

(a) At what time does the particle cross the origin?

- The origin is at $x = 0$ m so we would have $3.0 - 2.0t = 0$. The particle would therefore be at the origin at $t = 1.5$ s.

(b) What is the displacement of the particle between $t = 2.0$ s and $t = 4.0$ s?

- $x(2) = 3.0 - 2.0(2.0) = -1.0$ m and $x(4) = 3.0 - 2.0(4.0) = -5.0$ m. Therefore the displacement is 4.0 m (in the negative direction).

15. (4 marks) Text Chapter 3, Problem 30

- If we call going to the right going in the $+x$ direction then the woodchuck first runs with a velocity of $\vec{v}_1 = [(20\text{ m})/(5\text{ s})]\hat{\imath} = (4\text{ m/s})\hat{\imath}$ and then $\vec{v}_2 = [(10\text{ m})/(3\text{ s})](-\hat{\imath}) =$ (-10/3 m/s)$\hat{\imath}$.

  (a) The average velocity is:

$$\frac{(\vec{v}_1\Delta t_1 + \vec{v}_2\Delta t_2)}{(\Delta t_1 + \Delta t_2)} = \left[\frac{(4\text{ m/s})(5\text{ s}) - (10/3\text{ m/s})(3\text{ s})}{(5\text{ s} + 3\text{ s})}\right]\hat{\imath} = (1.25\text{ m/s})\hat{\imath}$$

  (b) The average speed is:

$$\frac{(|\vec{v}_1|\Delta t_1 + |\vec{v}_2|\Delta t_2)}{(\Delta t_1 + \Delta t_2)} = \frac{(4\text{ m/s})(5\text{ s}) + (10/3\text{ m/s})(3\text{ s})}{(5\text{ s} + 3\text{ s})} = 3.75\text{ m/s}$$

16. (6 marks) Variation on Text Chapter 3, Problem 35
Same questions but the particle moves along the x-axis according to $x(t) = 26t - (4/3)t^3$.

- $v(t) = dx(t)/dt =$ 26-4t^2 m/s so $\vec{v}(t) = (26 - 4t^2)\hat{\imath}$ m/s

  (a) What is the instantaneous velocity at $t = 2$ s and $t = 3$ s? (Remember that velocity is a vector)

  - $\vec{v}(2) = [26 - 4(2^2)]\hat{\imath} = 10\hat{\imath}$ m/s and $\vec{v}(3) = [26 - 4(3^2)]\hat{\imath} = -10\hat{\imath}$ m/s

  (b) What is the instantaneous speed at these times?

  - 10 m/s

  (c) What is the average velocity between $t = 2$ s and $t = 3$ s?

  - $\vec{0}$ m/s

17. (6 marks) You're windsurfing at 6.28 m/s when a gust hits, accelerating your sailboard at 0.714 m/s^2 at 48.8° to your original direction. If the gust lasts 5.42 s, what's the magnitude of the board's displacement during this time?

- We're given that $|v_0| = v_0 = 6.28$ m/s. If we take the direction of v_0 to be the x direction then $\vec{v}_0 = v_0\hat{\imath}$ m/s or $v_{0x} = 6.28$ m/s and $v_{0y} = v_{0z} = 0$ m/s. If we define the motion to be in the x-y plane then the acceleration vector makes an angle of 48.8° with the x axis so $a_x = |\vec{a}|\cos\theta = a\cos\theta =$(0.714 m/s^2)$\cos 48.8° = 0.4703$ m/s^2 and, similarly, $a_y = |\vec{a}|\sin\theta = 0.5372$ m/s^2. If we take the origin as the point at which the gust hit (i.e., $x_0 = 0$ m and $y_0 = 0$ m) plus the fact we are given $t - t_0 = 5.42$ s then we have:

$$\begin{aligned}
\vec{r}(t) &= \vec{r}_0 + \vec{v}_0[t - t_0] + \frac{1}{2}\vec{a}[t - t_0]^2 \\
x(t) &= x_0 + v_{0x}[t - t_0] + \frac{1}{2}a_x[t - t_0]^2 \\
&= 0 + (6.28\text{ m/s})(5.42\text{ s}) + \frac{1}{2}(0.4703\text{ m/s}^2)(5.42\text{ s})^2 \\
&= 40.945\text{ m}
\end{aligned}$$

$$
\begin{aligned}
y(t) &= y_0 + v_{0y}[t-t_0] + \frac{1}{2}a_y[t-t_0]^2 \\
&= 0 + (0 \text{ m/s})(5.42 \text{ s}) + \frac{1}{2}(0.5372 \text{ m/s}^2)(5.42 \text{ s})^2 \\
&= 7.891 \text{ m}
\end{aligned}
$$

So $|\vec{r}| = \sqrt{x^2+y^2} = 41.7$ m.

18. (6 marks) A hockey puck glides across the ice at 32.5 m/s, when a player whacks it with her hockey stick, giving it an acceleration at 64.3° to its original direction. The acceleration lasts 50.3 ms and the puck's displacement during this time is 1.76 m. Find the magnitude of the puck's acceleration.

- This is very similar to the previous problem except we are given different inputs. Again assuming the initial direction as the x direction and the puck's acceleration began at the origin then:

$$
\begin{aligned}
\vec{r}(t) &= \vec{r}_0 + \vec{v}_0[t-t_0] + \frac{1}{2}\vec{a}[t-t_0]^2 \\
x(t) &= v_0[t-t_0] + \frac{1}{2}a\cos\theta[t-t_0]^2 \\
y(t) &= \frac{1}{2}a\sin\theta[t-t_0]^2
\end{aligned}
$$

and so

$$
\begin{aligned}
|\vec{r}| = r &= \sqrt{x^2+y^2} \\
&= \sqrt{(v_0[t-t_0] + \frac{1}{2}a\cos\theta[t-t_0]^2)^2 + (\frac{1}{2}a\sin\theta[t-t_0]^2)^2} \\
&= \sqrt{v_0^2[t-t_0]^2 + \frac{1}{4}a^2[t-t_0]^4 + v_0 a[t-t_0]^3\cos\theta}
\end{aligned}
$$

Rearranging yields:

$$
a^2 + \left(\frac{4v_0\cos\theta}{[t-t_0]}\right)a - \frac{4(r^2 - v_0^2[t-t_0]^2)}{[t-t_0]^4} = 0
$$

Taking the positive solution to the quadratic equation with $v_0 = 32.5$ m/s, $\theta = 64.3°$, $[t-t_0] = 50.3$ ms $= 0.0503$ s, and $r = 1.76$ m yields $a = 201.0$ m/s^2.

19. (3 marks) A jetline touches down at 275 km/hr on a 1.2-km-long runway. What's the minimum safe value for the magnitude of its acceleration (in m/s^2) as it slows down?

- You can use just the formula 3.14 from the text

$$
v^2 = v_0^2 + 2a(x-x_0)
$$

with $v = 0$ km/hr, $x - x_0 = 1.2$ km, and $v_0 = 275$ km/hr to find:

$$
a = \frac{-v_0^2}{2(x-x_0)} = \frac{(275 \text{ km/hr})^2(1000 \text{ m/km})^2/(3600 \text{ s/hr})^2}{2(1.2 \text{ km})(1000 \text{ m/km})} = -2.43 \text{ m/s}^2
$$

Or, since that's no fun, we can use the equations for $x(t)$ and $v(t)$ like so:

$$x(t) = x_0 + v_0\Delta t + \frac{1}{2}a\Delta t^2$$

so

$$x - x_0 = v_0\Delta t + \frac{1}{2}a\Delta t^2 \tag{2}$$

And $v(t) = v_0 + a\Delta t$ so $-v_0 = a\Delta t$ and

$$\Delta t = -\frac{v_0}{a} \tag{3}$$

Plugging (3) into (2) gives

$$\begin{aligned} x - x_0 &= v_0\left(\frac{-v_0}{a}\right) + \frac{1}{2}a\left(\frac{-v_0}{a}\right)^2 \\ a &= \frac{-v_0^2}{2(x - x_0)} \end{aligned}$$

as above.

20. (4 marks) An online retailer makes deliveries by drone, and packages the goods so they can withstand an impact at up to 10.0 m/s.

    (a) What's the maximum height from which the drone can safely drop a package?

    - Yes, I'm getting lazy by now so we'll use equation 3.14 from the text with $v_0 = 0$ m/s, $a = g =$ -9.81 m/s^2, and $v = 10$ m/s.

$$\begin{aligned} v^2 &= v_0^2 + 2a\Delta x \\ \Delta x &= \frac{-v_0^2}{2a} \\ &= \frac{-(10 \text{ m/s})^2}{2(-9.81 \text{ m/s})} \\ &= 5.1 \text{ m} \end{aligned}$$

    (b) How long is the package in the air?

    - $v = v_0 + a\Delta t$ so

$$\Delta t = \frac{-v_0}{a} = \frac{-(10 \text{ m/s})}{(-9.81 \text{ m/s}^2)} = 1.02 \text{ s}$$