PHYSICS 1012 Electric Fields

"*Never memorize something you can look up.*" – Albert Einstein

Constants and Conversion Factors

$$1 \text{ m} = 10^6 \ \mu\text{m} = 10^9 \text{ nm} = 10^{10} \ \mathring{A} = 10^{12} \text{ pm} = 10^{15} \text{ fm}$$
$$1 \text{ GeV} = 10^3 \text{ MeV} = 10^6 \text{ keV} = 10^9 \text{ eV} = 1.6022 \times 10^{-10} \text{ J}$$
$$e = 1.6022 \times 10^{-19} \text{ C}$$
$$\varepsilon_0 = 8.85418781762 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2$$
$$1/4\pi\varepsilon_0 = 8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$$
$$\mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}$$
$$m_e \text{ (the electron mass)} = 9.11 \times 10^{-31} \text{ kg}$$
$$M_p \text{ (the proton mass)} = 1836 m_e$$

Coulomb's Law:

$$\vec{F} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{|\vec{r}|^2} \hat{r}$$

The Electric Field:

$$\vec{F}(x, y, z) = Q\vec{E}(x, y, z)$$

For a point charge;

$$|\vec{E}| = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2}$$

For an infinite uniformly charged line;

$$|\vec{E}| = \frac{\lambda}{2\pi\varepsilon_0 \ell}$$

For an infinite uniformly charged sheet;

$$|\vec{E}| = \frac{\sigma}{2\varepsilon_0}$$

At a distance ℓ away along the bisector of an electric dipole ($\vec{p} = q\vec{d}$);

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{-\vec{p}}{[\ell^2 + (d/2)^2]^{3/2}}$$

For a dipole $\vec{p}$ **in** an external field $\vec{E}$; $\vec{\tau} = \vec{p} \times \vec{E}$

Gauss' Law:

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\varepsilon_0}$$

Volumes and Areas:

Area of a circle of radius R and area of the side of a cylinder of length ℓ and radius R;

$$A = \pi R^2 \qquad\qquad A = 2\pi R\ell$$

Volume and surface area of a sphere of radius R;

$$V = \frac{4}{3}\pi R^3 \qquad\qquad A = 4\pi R^2$$

Vectors, Trigonometry, Derivatives, Complex Numbers

We will use Cartesian coordinates throughout.

Vectors

$$\begin{aligned}
\vec{A} &= |\vec{A}|\hat{A} \;=\; A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k} \\
|\vec{A}| &= \sqrt{A_x^2 + A_y^2 + A_z^2} \\
\hat{A} &= \frac{\vec{A}}{|\vec{A}|} \;=\; \frac{\vec{A}}{\sqrt{A_x^2 + A_y^2 + A_z^2}} \\
\vec{C} &= C_x\hat{\imath} + C_y\hat{\jmath} + C_z\hat{k} \\
&= \vec{A} + \vec{B} \\
&= (A_x + B_x)\hat{\imath} + (A_y + B_y)\hat{\jmath} + (A_z + B_z)\hat{k} \\
\vec{A} \cdot \vec{B} &= |\vec{A}||\vec{B}|\cos\theta \\
&= (A_xB_x + A_yB_y + A_zB_z) \\
\vec{A} \times \vec{B} &= |\vec{A}||\vec{B}|\sin\theta\hat{n} \\
&\quad \text{where } \hat{n} \text{ is the unit direction vector normal to the } \vec{A}\vec{B} \text{ plane} \\
&= (A_yB_z - A_zB_y)\hat{\imath} + (A_zB_x - A_xB_z)\hat{\jmath} + (A_xB_y - A_yB_x)\hat{k}
\end{aligned}$$

Derivatives

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}x} A\cos bx &= -Ab\sin bx \\
\frac{\mathrm{d}}{\mathrm{d}x} A\sin bx &= Ab\cos bx \\
\frac{\mathrm{d}Ax^{\pm b}}{\mathrm{d}x} &= \pm Abx^{\pm b-1} \\
\frac{\mathrm{d}Ae^{\pm bx}}{\mathrm{d}x} &= \pm Abe^{\pm bx}
\end{aligned}$$

Trigonometry

$$\begin{aligned}
\sin^2\theta + \cos^2\theta &= 1 \\
\sin 2\theta &= 2\sin\theta\cos\theta \\
\cos 2\theta &= \cos^2\theta - \sin^2\theta \\
\sin(\theta \pm \phi) &= \sin\theta\cos\phi \pm \cos\theta\sin\phi \\
\cos(\theta \pm \phi) &= \cos\theta\cos\phi \mp \sin\theta\sin\phi \\
\sin\theta \pm \sin\phi &= 2\sin\left[\frac{1}{2}(\theta \pm \phi)\right]\cos\left[\frac{1}{2}(\theta \mp \phi)\right] \\
\cos\theta + \cos\phi &= 2\cos\left[\frac{1}{2}(\theta + \phi)\right]\cos\left[\frac{1}{2}(\theta - \phi)\right] \\
\cos\theta - \cos\phi &= -2\sin\left[\frac{1}{2}(\theta + \phi)\right]\sin\left[\frac{1}{2}(\theta - \phi)\right]
\end{aligned}$$

Complex Numbers

$$\begin{aligned}
j &= \sqrt{-1} \\
Z &= a \pm bj \\
&= |Z|(\cos\theta \pm j\sin\theta) \\
&= |Z|e^{\pm j\theta} \\
|Z| &= \sqrt{a^2 + b^2} \\
\theta &= \pm\tan^{-1}\left(\frac{b}{a}\right)
\end{aligned}$$