PHYSICS 1012 — Solutions to Problem Set 1

Total – 96 marks — was due January 30, 2022

"A scientist in his laboratory is not a mere technician: he is also a child confronting natural phenomena that impress him as though they were fairy tales." – **Marie Curie**

As always, in the following you need to show how you got an answer – i.e., show your work. If you don't you will get 0 marks for the problem.

1. (4 marks) Given that $|\vec{F}| = 2$ N between $q_1 =$ -50 nC and $q_2 = +1$ nC, how far apart are the two charges?

- We have

$$|\vec{F}| = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_2|}{r^2} \qquad \Longrightarrow \qquad r = \sqrt{\left(\frac{1}{4\pi\varepsilon_0}\right)\left(\frac{|q_1||q_2|}{|\vec{F}|}\right)}$$

so therefore

$$r = \sqrt{(8.99 \times 10^9 \text{ Nm}^2/\text{C}^2)(5 \times 10^{-8} \text{ C})(10^{-9} \text{ C})/(2 \text{ N})} = 4.74 \times 10^{-4} \text{ m} = 0.47 \text{ mm}$$

2. (8 marks) Text Chapter 5, Problem #52.

- We're meant to equate the gravitational force to the electric force assuming the Moon (mass $m = 7.348 \times 10^{22}$ kg) and Earth (mass $M = 5.972 \times 10^{24}$ kg) both had a negative charge -Q.

  (a) We have

$$F_G = G\frac{mM}{r^2} \qquad \text{and} \qquad F_E = \frac{1}{4\pi\varepsilon_0}\frac{(-Q)(-Q)}{r^2}$$

where r is the Moon-Earth distance. Equating them gives:

$$G\frac{mM}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{r^2}$$

and so:

$$\begin{aligned} Q &= \sqrt{\frac{GmM}{(1/4\pi\varepsilon_0)}} \\ &= \sqrt{\frac{(6.674 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(7.348 \times 10^{22} \text{ kg})(5.972 \times 10^{24} \text{ kg})}{(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)}} \\ &= \sqrt{\frac{(6.674)(7.348)(5.972)}{(8.99)} \times 10^{26}} \text{ C} \\ &= 5.71 \times 10^{13} \text{ C} \end{aligned}$$

(b) No, the distance doesn't matter since it drops out when we set $F_G = F_E$. That is, the answer to (a) had no dependence on r.

(c) Each electron has a charge of $-e = -1.602 \times 10^{-19}$ C so the charge -Q is equivalent to:

$$n_{e^-} = \frac{-Q}{-e} = \frac{(-5.708 \times 10^{13}\ \text{C})}{(-1.602 \times 10^{-19}\ \text{C})} = 3.56 \times 10^{32}\ \text{electrons}$$

3. (5 marks) Text Chapter 5, Problem #54.

- We have three charges in the problem as shown in Figure 1. They are: $q_1 = 50\ \mu$C located at $(0, 0)$ m, q_2 = -25 μC located at $(1, 0)$ m, and $q_3 = 20\ \mu$C located at $(d, 0)$ m. Why is q_3 placed to the right of q_2? The question asks where q_3 should be located so that the force on it is 0 N. Because of the signs of their electric charges, q_3 is repelled

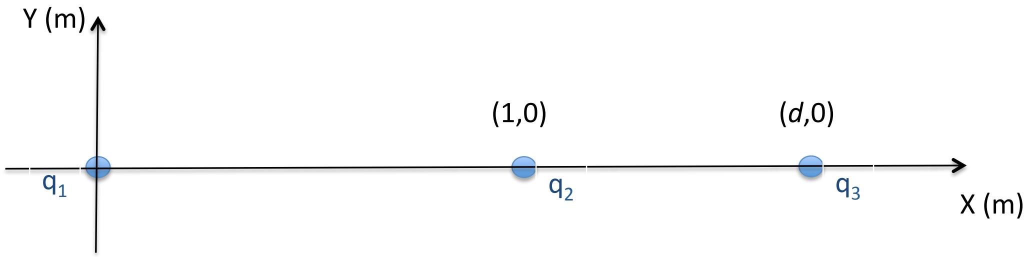

Figure 1: Figure for solution to question #3

by q_1 and is attracted to q_2. If q_3 was located between q_1 and q_2 then the forces on it due to the two other charges would both be directed to the right ($+x$ direction) and so you couldn't get zero net force. The forces on q_3 due to q_1 and q_2 are in opposite directions for either q_3 located to the left of q_1 or to the right of q_2. However, $|q_1| > |q_2|$ so to get a zero net force we need q_3 to be further from q_1 than q_2 and, hence, the arrangement as shown. The net force on q_3 is:

$$\vec{F}_3 = \vec{F}_{13} + \vec{F}_{23} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{d^2}\hat{i} + \frac{1}{4\pi\varepsilon_0}\frac{q_2 q_3}{(d-1)^2}\hat{i}$$

Setting $\vec{F}_3 = \vec{0}$ N gives:

$$\begin{aligned}
\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{d^2} + \frac{1}{4\pi\varepsilon_0}\frac{q_2 q_3}{(d-1)^2} &= 0 \\
\frac{q_1}{d^2} + \frac{q_2}{(d-1)^2} &= 0 \\
\left(\frac{q_1}{q_2}\right)(d-1)^2 + d^2 &= 0 \\
-2(d-1)^2 + d^2 &= 0 \\
-2d^2 + 4d - 2 + d^2 &= 0
\end{aligned}$$

Or, finally, $d^2 - 4d + 2 = 0$ which has the solutions:

$$d = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(2)}}{2(1)} = 2 \pm \sqrt{2}$$

The only valid solution is $d = (2+\sqrt{2})$ m since $d = (2-\sqrt{2})$ m < 1 m which would put q_3 between q_1 and q_2.

4. (6 marks) Assume the vector field $\vec{T}(x,y,z)$ is defined at every point in the space by the following equations: T_x = -2y, T_y = $x^2 + z^2$ and T_z = 6. What are $\vec{T}$(0,0,0), $\vec{T}$(-2,6,-3), and $\vec{T}$(2,-6,3)?

- $\vec{T}(0,0,0)$ = (0,0,6), $\vec{T}(-2,6,-3)$ = (-2)(6)$\hat{\imath}$ + $((-2)^2+(-3)^2)\hat{\jmath}$ +6$\hat{k}$ = (-12,13,6), and $\vec{T}(2,-6,3)$ = (-2)(-6)$\hat{\imath}$ + $(2^2+3^2)\hat{\jmath}$ +6$\hat{k}$ = (12,13,6).

5. (12 marks) A coordinate system (with the units of metres) is set up such that Q_1 = 10 fC is located at the origin. Three other charges are brought in close to Q_1. They are: Q_2 = -20 fC located at (2,0,0)m, Q_3 = 14 fC located at (0,0,2)m, and Q_4 = -12 fC located at (-2,0,-2)m. What is the electric field $\vec{E}$, at the location of Q_1, due to Q_2? Q_3? Q_4? Hence, what is the net force $\vec{F}$ on Q_1 and what is the magnitude of the force, $|\vec{F}|$, on Q_1?

- The distance vector $\vec{r}_n$ is drawn from Q_n to the origin so in this case $\hat{r}_2$ = -$\hat{\imath}$. And so we have;

$$\begin{aligned}
\vec{E}_2 &= -|\vec{E}_2|\ \hat{\imath} \\
&= -\left(\frac{1}{4\pi\varepsilon_0}\right)\left(\frac{Q_2}{|\vec{r}_2|^2}\right)\hat{\imath} \\
&= \frac{-(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(-2\times10^{-14}\ \mathrm{C})}{(2\ \mathrm{m})^2}\ \hat{\imath} \\
&= (4.5\times10^{-5}\ \mathrm{N/C})\ \hat{\imath}
\end{aligned}$$

Similarly, $\hat{r}_3$ = -$\hat{k}$ and

$$\begin{aligned}
\vec{E}_3 &= -|\vec{E}_3|\ \hat{k} \\
&= -\left(\frac{1}{4\pi\varepsilon_0}\right)\left(\frac{Q_3}{|\vec{r}_3|^2}\right)\hat{k} \\
&= \frac{-(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(1.4\times10^{-14}\ \mathrm{C})}{(2\ \mathrm{m})^2}\ \hat{k} \\
&= (-3.15\times10^{-5}\ \mathrm{N/C})\ \hat{k}
\end{aligned}$$

It is a little bit more complicated for Q_4 but still straightforward. The direction vector makes a 45° angle with the x and z axes so

$$\begin{aligned}
\vec{E}_4 &= |\vec{E}_4|\ [\cos(45°)\hat{\imath} + \cos(45°)\hat{k}] \\
&= \left(\frac{1}{4\pi\varepsilon_0}\right)\left(\frac{Q_4}{|\vec{r}_4|^2}\right)\left(\frac{\hat{\imath}}{\sqrt{2}}+\frac{\hat{k}}{\sqrt{2}}\right) \\
&= \frac{(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(-1.2\times10^{-14}\ \mathrm{C})}{[(-2^2+-2^2)\ \mathrm{m}]}\left(\frac{\hat{\imath}}{\sqrt{2}}+\frac{\hat{k}}{\sqrt{2}}\right) \\
&= (-0.95\times10^{-5}\ \mathrm{N/C})\ \hat{\imath} + (-0.95\times10^{-5}\ \mathrm{N/C})\ \hat{k}
\end{aligned}$$

And so

$$\begin{aligned}\vec{F}_1 &= Q_1(\vec{E}_2 + \vec{E}_3 + \vec{E}_4) \\ &= (4.5 \times 10^{-19}\ \text{N})\ \hat{\imath} + (-3.15 \times 10^{-19}\ \text{N})\ \hat{k} \\ &\quad +(-0.95 \times 10^{-19}\ \text{N})\ \hat{\imath} + (-0.95 \times 10^{-19}\ \text{N})\ \hat{k} \\ &= (3.5\ \hat{\imath} - 4.1\ \hat{k}) \times 10^{-19}\ \text{N}\end{aligned}$$

and $|\vec{F}_1| = 5.39 \times 10^{-19}$ N

6. (8 marks) A -5q charge and a +2q charge are a distance ℓ apart. At what location is the electric field zero N/C (i.e., $\vec{0}$ N/C)?

- The net electric field is $\vec{E} = \vec{E}_{-5q} + \vec{E}_{+2q}$. Define the x axis such that the two charges are located on it with the -5q at $x = 0$ and the +2q at $x = \ell$. It is clear from symmetry that the zero field point is going to lie along the x axis since any point off this axis (say at some y value) will never have the fields from the two sources cancel (i.e., there will always be a component in that direction). Further, $\vec{E}_{-5q}$ points towards the -5q charge while $\vec{E}_{+2q}$ will point away from the +2q charge. There are three regions of x where this has different consequences:

  (1) $x < 0$ where $\vec{E}_{-5q}$ points in the positive x direction and $\vec{E}_{+2q}$ points in the negative x direction.

  (2) $0 < x < \ell$ where both $\vec{E}_{-5q}$ and $\vec{E}_{+2q}$ points in the negative x direction.

  (3) $x > \ell$ where $\vec{E}_{-5q}$ points in the negative x direction and $\vec{E}_{+2q}$ points in the positive x direction.

  Clearly the two fields can't cancel in region (2) since they both point in the same direction. The zero field point can't be in region (1) because it would by definition be closer to the larger in magnitude -5q charge than the +2q charge and so would everywhere in the region have a $+x$ component. Hence it has to be in region (3). In this region we have (where we are only considering the x component so I dropped the vector signs):

$$E_x(x) = E_{-5q}(x) + E_{+2q}(x) = \frac{1}{4\pi\varepsilon_0}\frac{-5q}{x^2} + \frac{1}{4\pi\varepsilon_0}\frac{2q}{(x-\ell)^2}$$

  Just to be clear, this is the x component of the net field as a function of the x position. Setting $E_x(x) = 0$ leads to the condition:

$$\frac{5}{x^2} = \frac{2}{(x-\ell)^2}$$

  The solution is:

$$x = \frac{\sqrt{5}\ell}{\sqrt{5}-\sqrt{2}}$$

  Note that if we had put the +2q at the origin then we would have found:

$$x = \frac{\sqrt{2}\ell}{\sqrt{5}-\sqrt{2}}$$

The electric field lines will be like that of a distorted dipole, much tighter around the -5q than around the +2q but of course they will start on the +2q and end on the -5q.

7. (3 marks) A long, straight wire carries a chage density of 292 μC/m. Find the magnitude of the electric field 85.0 cm from the axis of the wire.

- For a long straight wire

$$\begin{aligned} |\vec{E}| &= \frac{\lambda}{2\pi\varepsilon_0 \ell} = \frac{2\lambda}{4\pi\varepsilon_0 \ell} \\ &= \frac{(2)(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(2.92 \times 10^{-4} \text{ C/m})}{(0.85 \text{ m})} \\ &= 6.2 \times 10^6 \text{ N/C} = 6.2 \text{ MV/m} \end{aligned}$$

8. (14 marks) There is a 25 pC point charge located on the x axis at x = -4 m and an electric dipole at the origin with $d = 2$ μm and an electric dipole moment of $\vec{p} = (10^{-12}$ C$\cdot$m$)\hat{\jmath}$. What is the electric field at (2,0,0)? Sketch the electric field components on the x-axis for -3 m $< x <$ -1 m? That is, sketch $E_x(x, 0, 0)$, $E_y(x, 0, 0)$, and $E_z(x, 0, 0)$ for the given x interval.

- As always the total electric field is the vector sum of the various electric fields in the problem. In this case there are two sources so $\vec{E}(2, 0, 0) = \vec{E}_Q(2, 0, 0) + \vec{E}_d(2, 0, 0)$. So

$$\begin{aligned} \vec{E}_Q(2,0,0) &= |\vec{E}|\hat{r} \\ &= \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(25 \times 10^{-12} \text{ C})}{(6 \text{ m})^2}\hat{\imath} \\ &= (6.25 \times 10^{-3} \text{ N/C})\hat{\imath} \end{aligned}$$

$$\begin{aligned} \vec{E}_d(2,0,0) &= |\vec{E}_d|\hat{E}_d \\ &= \left(\frac{1}{4\pi\varepsilon_0}\right)\left(\frac{-\vec{p}}{[2^2 + (2 \times 10^{-6}/2)^2]^{3/2}}\right) \\ &= -\frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(10^{-12} \text{ C} \cdot \text{m})}{(8 \text{ m}^3)}\hat{\jmath} = (-1.125 \times 10^{-3} \text{ N/C})\hat{\jmath} \end{aligned}$$

So

$$\vec{E}(2, 0, 0) = (6.25\hat{\imath} - 1.125\hat{\jmath}) \text{ mN/C}$$

The sketches are in a separate scanned document.

9. (6 marks) Text Chapter 6, Problem #30

- Recall that $\Phi_E = q_{\text{enclosed}}/\varepsilon_0$.

(S$_1$) q_{enclosed} = +3q + q + (-2q) + (-2q) = 0 so $\Phi_E = 0$

(S$_2$) q_{enclosed} = -2q so Φ_E = -2q/ε_0

(S$_3$) q_{enclosed} = +q so Φ_E = +q/ε_0

(S$_4$) q_{enclosed} = -2q + (-2q) = -4q so Φ_E = -4q/ε_0

(S$_5$) q_{enclosed} = -2q so Φ_E = -2q/ε_0

(S$_6$) $q_{\text{enclosed}} = +3q$ so $\Phi_E = +3q/\varepsilon_0$

10. (8 marks) Text Chapter 6, Problem #44

- We derived in class the formula for the electric field both inside ($r < R$) and outside ($r \geq R$) of a uniformly charged sphere of radius R and total charge Q. The formulae were:

$$\begin{aligned} \vec{E}(r) &= \frac{1}{4\pi\varepsilon_0}\frac{Qr}{R^3}\hat{r} \qquad r < R \\ \vec{E}(r) &= \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r} \qquad r \geq R \end{aligned}$$

We are given Q = -30 μC and R = 10.0 cm = 0.10 m.

(a) For r = 2.0 cm = 2.0×10^{-2} m (so $r < R$)

$$\begin{aligned} \vec{E}(r) &= \frac{1}{4\pi\varepsilon_0}\frac{Qr}{R^3}\hat{r} \\ E &= (8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)\frac{(-3.0 \times 10^{-5}\text{ C})(2.0 \times 10^{-2}\text{m})}{(0.10\text{ m})^3} \\ &= -5.4 \times 10^6 \text{ N/C} \;=\; -5.4 \text{ MN/C} \end{aligned}$$

(b) For r = 5.0 cm = 5.0×10^{-2} m (so $r < R$)

$$\begin{aligned} \vec{E}(r) &= \frac{1}{4\pi\varepsilon_0}\frac{Qr}{R^3}\hat{r} \\ E &= (8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)\frac{(-3.0 \times 10^{-5}\text{ C})(5.0 \times 10^{-2}\text{m})}{(0.10\text{ m})^3} \\ &= -1.35 \times 10^7 \text{ N/C} \;=\; -13.5 \text{ MN/C} \end{aligned}$$

(c) For r = 20.0 cm = 0.200 m (so $r > R$)

$$\begin{aligned} \vec{E}(r) &= \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r} \\ E &= (8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)\frac{(-3.0 \times 10^{-5}\text{ C})}{(0.20\text{ m})^2} \\ &= -6.7 \times 10^6 \text{ N/C} \;=\; -6.7 \text{ MN/C} \end{aligned}$$

11. (4 marks) Proton-beam therapy can be preferable to X-rays for cancer treatment because protons deliver most of their energy to the tumor, with less damage to healthy tissue. A medical cyclotron repeatedly passes the protons through a 15 kV potential difference.

- For a potential difference ΔV we have $\Delta K = q\Delta V$ so here $\Delta K = e\Delta V$ = 15 keV/pass = 2.4×10^{-15} J/pass

(a) How many passes are needed to bring the protons' kinetic energy to 1.2×10^{-11} J?

- The kinetic energy increases by 1.2×10^{-11} J in n passes so

$$n = \frac{1.2 \times 10^{-11}\ \text{J}}{2.4 \times 10^{-15}\ \text{J/pass}} = 0.5 \times 10^4\ \text{passes}$$

So it takes 5000 passes to increase the energy by 1.2×10^{-11} J

(b) What's that energy in eV?

- There are 1.6×10^{-19} J/eV so

$$\frac{1.2 \times 10^{-11}\ \text{J}}{1.6 \times 10^{-19}\ \text{J/eV}} = 0.75 \times 10^8\ \text{eV} = 75\ \text{MeV}$$

Note that the calculation is trivial if it was only done in electron-volts.

12. (6 marks) In the ALPHA antihydrogen experiment at CERN, the antiprotons travel to the experiment down a beam pipe. We need to reduce their energy so we can combine them with positrons to make antihydrogen. The antiprotons are introduced to a region with a 5 kV potential difference between the ends. How much energy is lost by each antiproton in the beam? How much energy would an alpha particle (a bare helium nucleus) gain if it was accelerated through this potential difference? Why does the antiproton lose energy but the alpha particle gains energy when in a region with such a potential difference across it?

- $\Delta E = \Delta K = q\Delta V$ where ΔV here is 5 kV. Now $q_{\bar{p}} = -e$ so $\Delta E_{\bar{p}} = -e(5\ \text{kV}) = -5$ keV. That is, each antiproton loses 5 keV of energy ($= 1.6 \times 10^{-16}$ J). The alpha particle charge (q_α) is $+2e$ so an alpha in the same field would gain $2e(5\ \text{kV}) = 10$ keV (or 1.6×10^{-15} J). The alpha particle and the antiproton are oppositely charged so when entering the same field, if one loses energy then the other gains energy.

13. (6 marks) Figure 2 shows a plot of electrical potential versus position along the x-axis. Make a plot of the x-component of the electric field for this situation.

- $E_x = -dV/dx$ and since these are all straight line segments it is $-\Delta V/\Delta x$. This gives the following:

  - o $E_x = 0$ V/m for $0 \leq x < 2$ m
  - o $E_x = 2$ V/m for $2 \leq x < 4$ m
  - o $E_x = -4$ V/m for $4 \leq x < 4.5$ m
  - o $E_x = 0$ V/m for $4.5 \leq x < 6$ m
  - o $E_x = -0.5$ V/m for $6 \leq x < 8$ m
  - o $E_x = 2$ V/m for $8 \leq x < 9$ m
  - o $E_x = 1$ V/m for $9 \leq x \leq 10$ m

14. (6 marks) Text Chapter 7, Problem #65

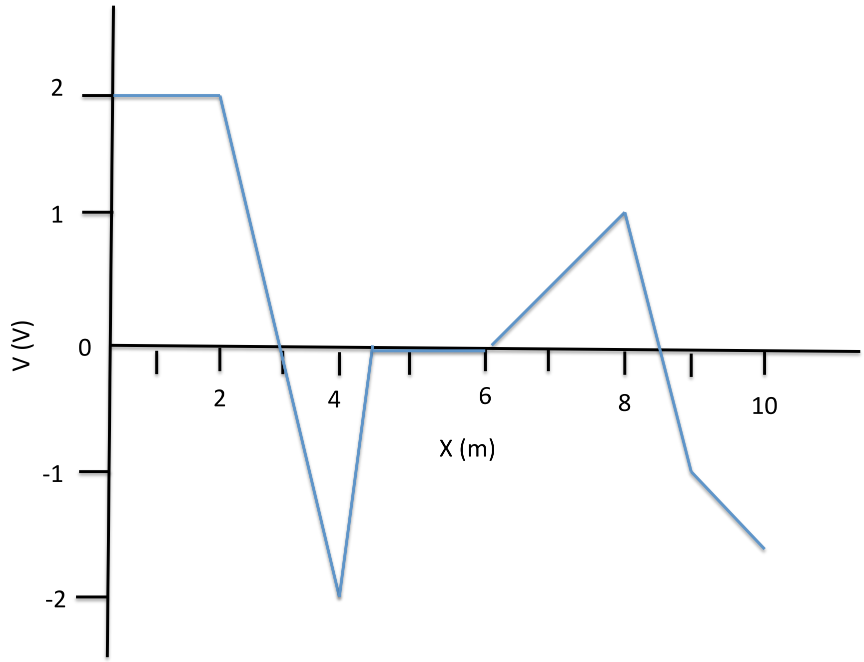

Figure 2: Figure for question #13

- For a conducting charged shell of radius R there are two regions: $r < R$ and $r > R$. The electric field for $r < R$ is 0 V/m so the potential is constant (and therefore ΔV = 0 V for any two points inside the shell). For $r > R$ the shell can be treated as a point charge so $V(r) = (1/4\pi\varepsilon_0)(q/r)$ where q is the total charge on the shell and we've taken $V = 0$ at $r = \infty$. Therefore, for $a \leq r \leq b$ we need only be concerned with the inner shell which can be treated as a point charge $+Q$ located at the origin and so

$$\Delta V_{ab} = \left(\frac{Q}{4\pi\varepsilon_0}\right)\left(\frac{1}{a} - \frac{1}{b}\right)$$