PHYSICS 1012 — Solutions to Problem Set 2

Total – 94 marks — was due February 13, 2022

"I have no reason to believe that the human intellect is able to weave a system of physics out of its own resources without experimental labour. Whenever the attempt has been made it has resulted in an unnatural and self-contradictory mass of rubbish."
– **James Clerk Maxwell**

As always, in the following you need to show how you got an answer – i.e., show your work. If you don't you will get 0 marks for the problem. The text questions are given in the format "Chapter.Problem". For example, the 10.2 in question 1 refers to Text Chapter 10, Problem 2.

1. (3 marks) 10.2

- Initially $i_i = V_0/(r+R) = V_0/2r$. Then, after corrosion $r' = 3r$ and $i_a = V_0/(r'+R) = V_0/4r$ so $i_a = i_i/2$ (i.e., the current delivered is halved).

2. (6 marks) 10.22

   (a) $i = V_0/(r+R) = (12\text{ V})/(0.06\ \Omega) = 200\text{ A}(!)$

   (b) $\Delta V = iR = (200\text{ A})(.05\ \Omega) = 10\text{ V}$

   (c) $P = i^2 R = (200\text{ A})^2(0.05\ \Omega) = 2000\text{ W} = 2\text{ kW}$

   (d) Now $i = V_0/(r + r_c + R) = (12\text{ V})/(0.15\ \Omega) = 80\text{ A}$
   $\Delta V = iR = (80\text{ A})(.05\ \Omega) = 4\text{ V}$
   $P = i^2 R = (80\text{ A})^2(0.05\ \Omega) = 320\text{ W}$

3. (2 marks) 10.8

- No. Kirchoff's Current Law says that $i_1 + i_2 + i_3 = 0$ so at least one of them (though it can't be all) has to be negative (i.e., going in the opposite direction to that shown).

4. (6 marks) 10.42

- loop 1: $i_1 R_1 + i_2 R_2 - V_1 = 0$
  loop 2: $-i_2 R_2 - i_3 R_3 + i_4 R_4 + V_2 = 0$
  loop 3: $i_5 R_5 - V_2 = 0$

5. (6 marks) 10.72

   (a) Voltage division follows

$$V_{out} = V_{in} \frac{R_2}{R_1 + R_2}$$

(b) R_1 and R_2 are "in parallel" so the pair can be replaced by the equivalent resistance

$$R = \frac{R_2 R_L}{R_2 + R_L} \qquad \left(\text{i.e., } \frac{1}{R} = \frac{1}{R_2} + \frac{1}{R_L}\right)$$

so

$$V_{out} = V_{in}\frac{R}{R_1 + R} = V_{in}\frac{\frac{R_2 R_L}{R_2+R_L}}{R_1 + \frac{R_2 R_L}{R_2+R_L}} = V_{in}\left(\frac{R_2 R_L}{R_1 R_2 + R_1 R_L + R_2 R_L}\right)$$

6. (6 marks) 10.78 [Note that the answer to 10.72(a) may come in handy]

(a) $\Delta V = i_1 R_1 = (50\text{ mA})(40\ \Omega) = 2$ V

(b)

$$\Delta V_2 = \Delta V_1 \frac{R_2}{R_2 + R_3} = (2\text{ V})\frac{(5\Omega)}{(5\Omega + 15\Omega)} = \frac{1}{2}\text{ V}$$

7. (6 marks) 10.36

- First note that all of the resistors are in series so, for the purposes of calculating current, they can be replaced by the equivalent resistance $R = R_1 + R_2 + R_3 + R_4 + R_5$ = 60 kΩ. Similarly, the two batteries can replaced with 1 battery having a voltage of ΔV = 24 - 12 = 12 V (with the polarity of the 24 V battery). So $i = \Delta V/R$ = (12 V)/(60 kΩ) = (1/5) mA.

(a) $\Delta V_i = iR_i$ so $\Delta V_1 = iR_1$ = (0.2 mA)(10 kΩ) = 2 V = $\Delta V_3 = \Delta V_4 = \Delta V_5$. $\Delta V_2 = iR_2$ = (0.2 mA)(20 kΩ) = 4 V

(b) $P_{\text{supplied}} = \Delta V i$ = (12 V)(0.2 mA) = 2.4 mW while $P_{\text{dissipated}} = i^2 R$ = (0.2 mA)^2(60 kΩ) = (60/25) mW = 2.4 mW

8. (6 marks) 10.38 [Note that it is literally the same circuit as 10.37(b)]

- Apply KCL and KVL to get ΔV_1 = 18 V, i_2 = 1 A, and i_3 = 3 A

9. (6 marks) 10.40

- Apply KCL and KVL to get i_1 = 3/2 A, i_2 = 2 A, and i_3 = 1/2 A

10. (6 marks) 10.84

- First note that R_1, r_1 and R_5 are in series, as are r_2 and R_2 as well as R_3, r_3 and R_4. Similarly, ε_3 and ε_4 are in series although with opposite polarities leading to a single battery with ε = 30 V. Then apply KCL and KVL to find i_1 = -0.34 A, i_2 = 0.36, and i_3 = 0.02 A

11. (2 marks) 8.20

- $Q = C\Delta V = (8 \times 10^{-12}\text{ F})(5.5\text{ V}) = 4.4 \times 10^{-11}$ C

12. (2 marks) 8.22

- $Q = C\Delta V$ so

$$\Delta V = \frac{Q}{C} = \frac{(1.6 \times 10^{-4}\ \text{C})}{(8 \times 10^{-9}\ \text{F})} = \frac{1}{5} \times 10^5\ \text{V}\ = 20\ \text{kV}$$

13. (2 marks) 8.26

- $C = \varepsilon_0 A/d$ so

$$d = \frac{\varepsilon_0 A}{C} = \frac{(8.85\ \text{pF/m})(0.01\ \text{m}^2)}{(60\ \text{pF})} = 1.48\ \text{mm}$$

14. (4 marks) A parallel-plate, air-filled capacitor having area 35 cm^2 and spacing of 1.5 mm is charged to a potential difference of 500 V. Find (a) the capacitance, (b) the magnitude of the charge on each plate, (c) the stored energy, and (d) the electric field between the plates.

    (a) the capacitance
    - $C = \varepsilon_0 A/d = (8.85\ \text{pF/m})(3.5 \times 10^{-3}\ \text{m}^2)/(1.5 \times 10^{-3}\ \text{m}) = 20.7\ \text{pF}$

    (b) the magnitude of the charge on each plate
    - $q = C\Delta V = (2.07 \times 10^{-11}\ \text{F})(500\ \text{V}) = 1.03 \times 10^{-8}\ \text{C} = 10.3\ \text{nC}$

    (c) the stored energy
    - $U = C\Delta V^2/2 = (2.07 \times 10^{-11}\ \text{F})(500\ \text{V})^2/2\ 2.58\ \mu\text{J}$

    (d) the electric field between the plates.
    - $|\vec{E}| = \Delta V/d = (200\ \text{V})/(1.5 \times 10^{-3}\ \text{m}) = (1/3){\times}10^6\ \text{V/m} = 333.3\ \text{kN/C}$

15. (6 marks) A parallel-plate capacitor has plates of area A and separation d and is charged to a potential difference of ΔV_0. The charging battery is then disconnected (so the capacitor is electrically isolated) and the plates are pulled apart until their separation is $3d$. Derive expressions in terms of A, d, and ΔV_0 for (a) the new potential difference, (b) the initial and final stored energy, and (c) the work required to separate the plates.

- For the parallel plate capacitor $C = \varepsilon_0 A/d$. With $d' = 3d$ then $C' = \varepsilon_0 A/d' = \varepsilon_0 A/3d$ $= C/3$

    (a) the new potential difference
    - $\Delta V' = q/C' = 3q/C = 3\Delta V$

    (b) the initial and final stored energy
    - $U = C^{\Delta} V^2/2 = \varepsilon_0 A \Delta V^2/2d$ and $U' = C'(\Delta V')^2/2 = (C/3)(3\Delta V)^2/2 = 3U = 3\varepsilon_0 A \Delta V^2/2d$

    (c) the work required to separate the plates
    - The work required is U' - $U = 2U = \varepsilon_0 A \Delta V^2/d$

16. (6 marks) 8.54

(a)

$$|\vec{E}| = 2\left(\frac{\sigma}{2\varepsilon_0}\right) = \frac{\sigma}{\varepsilon_0} = \frac{(0.5 \times 10^{-3}\ \text{C/m}^2)}{(8.85 \times 10^{-12}\ \text{C}^2/\text{Nm}^2)} = 5.6 \times 10^7\ \text{V/m}$$

(b) $\Delta V = Et = (5.6 \times 10^7\ \text{V/m})(5.0 \times 10^{-9}\ \text{m}) = 0.28\ \text{V}$

(c) $u = U/V = (/2)\varepsilon_0 E^2$ where V is volume. Assume the cell volume given is enclosed by the outer cell wall so

$$V_{outer} = \frac{4}{3}\pi R_{outer}^3 \quad \Longrightarrow R_{outer} = \left(\frac{3V}{4\pi}\right)^{1/3} = \left(\frac{3(10^{-16}\ \text{m}^3)}{(4\pi)}\right)^{1/3} = 2.879 \times 10^{-6}\ \text{m}$$

By definition $R_{inner} = R_{outer} - t = 2.874 \times 10^{-6}$ m so

$$V_{inner} = \frac{4}{3}\pi R_{inner}^3 = \frac{4}{3}\pi(2.874 \times 10^{-6}\ \text{m})^3 = 9.94 \times 10^{-17}\ \text{m}^3$$

We would then have $V_{walls} = V_{outer} - V_{inner} = 6 \times 10^{-19}\ \text{m}^3$ and so

$$\begin{aligned} U &= Vu \\ &= V\left(\frac{1}{2}\varepsilon_0 E^2\right) \\ &= \frac{1}{2}(6 \times 10^{-19}\ \text{m}^3)(8.85 \times 10^{-12}\ \text{C}^2/\text{Nm}^2)(5.6 \times 10^7\ \text{N/C})^2 \\ &= 8.3 \times 10^{-15}\ \text{J} \ = \ 8.3\ \text{fJ} \end{aligned}$$

17. (2 marks) 10.17

- (Answer from text) "The time constant can be shortened by using a smaller resistor and/or a smaller capacitor. Care should be taken when reducing the resistance because the initial current will increase as the resistance decreases."

18. (2 marks) 10.50

- $f = 72\ \text{min}^{-1} = (6/5)$ Hz so $T = (5/6)$ s. Now

$$\tau = RC = T = \frac{5}{6}\ \text{s} \quad \Longrightarrow \quad R = \frac{T}{C} = \frac{5}{6(2.5 \times 10^{-8})} = \frac{1}{3} \times 10^8\ \Omega = 33.3\ \text{M}\Omega$$

19. (3 marks) 10.54

(a) $\tau = RC = 10.0$ ms and $C = 8.00\ \mu$F so

$$R = \frac{\tau}{C} = \frac{10^{-2}\ \text{s}}{8 \times 10^{-6}\ \text{F}} = .125 \times 10^4\ \Omega = 1.25\ \text{k}\Omega$$

(b) $V = V_0 e^{-t/\tau}$ so

$$\frac{-t}{\tau} = \ln\left(\frac{V}{V_0}\right) \quad \Longrightarrow \quad t = \tau \ln\left(\frac{V_0}{V}\right) = (10^{-2}\ \text{s})\ln\left(\frac{12\ \text{V}}{0.6\ \text{V}}\right) = 3.0{\times}10^{-2}\ \text{s} = 30.0\ \text{ms}$$

20. (12 marks) 10.94

- Assume the capacitor is initially uncharged and call the time when the switch closes $t = t_0 = 0$ so the potential drop across the capacitor will be $\Delta V_C(0) = 0$ V.

  (a) We have that $\Delta V_C = \Delta V_{R_1}$ so $\Delta V_C(0) = 0$ V means that $\Delta V_{R_1}(0) = 0$ V and all of the battery voltage is across R_2. Therefore,

  $$i_2(0) = \frac{V_1}{R_2} = \frac{24\ \text{V}}{30\ \text{k}\Omega} = \frac{4}{5}\ \text{mA}$$

  (b) When the switch has been closed for a long time then the capacitor is fully charged and no current flows to it. Therefore all the current flows through R_1 and R_2 in series and so

  $$i_2 = \frac{V_1}{R_1 + R_2} = \frac{24\ \text{V}}{40\ \text{k}\Omega} = \frac{3}{5}\ \text{mA}$$

  (c) Once the switch is opened then no current flows through the $R_2 + V_1$ branch and so the capacitor will discharge through R_1 with the time constant $\tau = R_1 C = (10$ k$\Omega)(10\ \mu$F$) = 0.1$ s.

  (d) While discharging the current goes like $i(t) = i_0 e^{-t/\tau}$. So, when $i = i_0/2$

  $$\frac{i}{i_0} = \frac{1}{2} = e^{-t/\tau} \quad \Longrightarrow \quad t = \tau \ln 2 = (0.1\ \text{s}) \ln 2 = 0.069\ \text{s} = 69.3\ \text{ms}$$