PHYSICS 1012 — Solutions to Problem Set 3

Total – 76 marks — was due February 24, 2022

"A seven-year-old of my acquaintance claimed that the last number of all was 23,000. 'What about 23,000 and one?' she was asked. After a pause: 'Well, I was close.'"
– From the book "The Nothing That Is: A Natural History of Zero" by **Robert Kaplan**

As always, in the following you need to show how you got an answer – i.e., show your work. If you don't you will get 0 marks for the problem. The text questions are given in the format "Chapter.Problem". For example, 11.18 in question 1 refers to Text Chapter 11, Problem 18.

1. (6 marks) 11.18

   (a) to the left

   (b) out of the page

   (c) up

2. (5 marks) The magnitude of the Earth's magnetic field is about 0.5 gauss near Earth's surface. What's the maximum possible force on an electron near the Earth's surface with kinetic energy of 1 keV? How does this compare to the gravitational force on the electron near the Earth's surface?

- Given the kinetic energy the electron's speed is then:

$$v = \sqrt{\frac{2K}{m_e}} = \sqrt{\frac{2(10^3 \text{ eV})(1.602 \times 10^{-19} \text{ J/eV})}{(9.11 \times 10^{-31} \text{ kg})}} = 1.87 \times 10^7 \text{ m/s}$$

  The maximum is going to happen when $\vec{v}$ is perpendicular to $\vec{B}$ so we'll have:

$$|\vec{F}| = e|\vec{v}||\vec{B}| = (1.602 \times 10^{-19} \text{ C})(1.87 \times 10^7 \text{ m/s})(0.5 \times 10^{-4} \text{ T}) = 1.5 \times 10^{-16} \text{ N}$$

  This is to be compared to the weight of the electron at the Earth's surface:

$$W = |\vec{F}_G| = m_e g = (9.11 \times 10^{-31} \text{ kg})(9.81 \text{ m/s}^2) = 8.9 \times 10^{-30} \text{ N}$$

3. (4 marks) 11.32

- The object would have kinetic energy $K = qV$ after having been accelerated from rest through an electric potential difference of V V. The particle's speed is then:

$$K = qV = \frac{1}{2}mv^2 \Longrightarrow v = \sqrt{\frac{2qV}{m}}$$

If the particle moves in a plane perpendicular to B then $|\vec{F}| = qvB$ so

$$\begin{aligned} F &= qvB = \frac{mv^2}{R} \\ R &= \frac{mv}{qB} = \frac{m}{qB}\sqrt{\frac{2qV}{m}} \\ R &= \sqrt{\frac{2mV}{qB^2}} \end{aligned}$$

4. (8 marks) A 10 keV electron is moving in a circular orbit of radius 1 m in a plane at right angles to a uniform magnetic field. What is:

   (a) the speed of the electron?

   - Assuming it is non-relativistic then $K_e = m_e v_e^2/2$ so

   $$v_e = \sqrt{\frac{2K_e}{m_e}} = c\sqrt{\frac{2K_e}{m_e c^2}} = c\sqrt{\frac{(2)(10\text{ keV})}{(511\text{ keV})}} = c\sqrt{\frac{20}{511}} = 0.198c = 5.93 \times 10^7 \text{ m/s}$$

   (With $v_e \sim 0.2c$ the electron is just barely non-relativistic)

   (b) the magnitude of the magnetic field?

   - For a charged particle in a circular orbit $p = qBR = m_e v_e$ so for the electron

   $$B = \frac{m_e v_e}{eR} = \frac{(9.11 \times 10^{-31}\text{ kg})(5.93 \times 10^7 \text{ m/s})}{(1.602 \times 10^{-19}\text{ C})(1\text{ m})} = 3.4 \times 10^{-4}\text{ T (3.4 Gauss)}$$

   (c) the frequency of revolution?

   - The frequency of revolution is $f = v/2\pi R$ so

   $$f = \left(\frac{1}{2\pi R}\right)\sqrt{\frac{2K}{m_e}} = \left[\frac{1}{2\pi(1\text{ m})}\right](3 \times 10^8 \text{ m/s})\sqrt{\frac{20}{511}} = 9.4 \times 10^6 \text{ Hz} = 9.4\text{MHz}$$

   (d) the period of motion?

   - $T = 1/f = 1.06 \times 10^{-7}$ s $= 106$ ns

5. (8 marks) A cyclotron is needed to accelerate protons to $(1/30)c$ (i.e., one-thirtieth the speed of light). The cyclotron magnetic field is 1 T.

   (a) What is the minimum radius the cyclotron can have?

   - $p = qBR$ so

   $$R = \frac{p}{qB} = \frac{m_p v}{eB} = \frac{(1836)(9.11 \times 10^{-31}\text{ kg})(3 \times 10^8 \text{ m/s})}{(1.602 \times 10^{-19}\text{ C})(1\text{ T})(30)} = 0.104 \text{ m}$$

(b) What energy would an α particle and a deuteron[1] have if produced from the same cyclotron?

- Recall that $K = p^2/2m = q^2B^2R^2/2m$. So, with $q_\alpha = 2q_p = 2e$, $m_\alpha \approx 4m_p$ and $q_d = q_p = e$, $m_d \approx 2m_p$, we have

$$\begin{aligned}
K_p &= \frac{e^2B^2R^2}{2m_p} \\
&= \frac{(1.602\times10^{-19}\ \text{C})^2(1\ \text{T})^2(.104\ \text{m})^2}{2(1836)(9.11\times10^{-31}\ \text{kg})} \\
&= 8.3\times10^{-14}\ \text{J} \ = \ 518.0\ \text{keV} \\
K_\alpha &= \frac{(2e)^2B^2R^2}{2(4m_p)} \\
&= \frac{e^2B^2R^2}{2m_p} \\
&= K_p \ = \ 518.0\ \text{keV} \\
K_d &= \frac{e^2B^2R^2}{2(2m_p)} \\
&= K_p/2 \ = \ 259.0\ \text{keV}
\end{aligned}$$

6. (6 marks) 11.34

   (a) to the left

   (b) out of the page

   (c) up

7. (5 marks) A wire carrying 15 A makes a 25° angle with a uniform magnetic field. The magnetic force per unit length on the wire is 0.31 N/m. Find:

   (a) The magnitude of the magnetic field.

   - We have $\vec{F} = i\vec{L}\times\vec{B}$ so

$$|\vec{B}| = \left(\frac{|\vec{F}|}{L}\right)\left(\frac{1}{i\sin\theta}\right) = (0.31\ \text{N/m})\left(\frac{1}{(15\ \text{A})\sin(25°)}\right) = 49\ \text{mT}$$

   (b) the maximum force per unit length that could be acheived by reorienting the wire.

   - The maximum force per unit length would occur for $\theta = 90°$ so

$$\frac{|\vec{F}|}{L} = (15\ \text{A})(4.9\times10^{-2}\ \text{T})\sin(90°) = 0.73\ \text{N/m}$$

[1] An alpha particle is the bound state of 2 protons and 2 neutrons (i.e., the He nucleus) so has $q_\alpha = +2e$ and $M_\alpha \approx 4M_p$ while the deuteron is the bound state of 1 proton and 1 neutron.

8. (4 marks) 11.36

- $\vec{F} = i\vec{L} \times \vec{B}$ so for $\vec{L} \perp \vec{B}$ we have

$$\frac{F}{L} = iB = (2.0 \times 10^4 \text{ A})(3.0 \times 10^{-5} \text{ T}) = 0.6 \text{ N/m}$$

9. (4 marks) 11.48 (a) and (b) but not (c)

   (a) We have $B = 2.5$ T and $E_H = 1.5$ mV/m so

$$v_d = \frac{E_H}{B} = \frac{(1.5 \times 10^{-3} \text{ V/m})}{(2.5 \text{ T})} = 6.0 \times 10^{-4} \text{ m/s}$$

   (b) The current is

$$\begin{aligned} i &= nev_d A \\ &= (8.0 \times 10^{28} \text{ e}^-/\text{m}^3)(1.6 \times 10^{-19} \text{ C/e}^-)(6.0 \times 10^{-4} \text{ m/s})(5.0 \times 10^{-6} \text{ m}^2) \\ &= 38.4 \text{ A} \end{aligned}$$

10. (4 marks) Variation on 11.49 – instead of the 100 A given in the text problem assume the strip carries a current of 25 A and the magnetic field is 3.0 T instead of 1.5 T as given in the text problem.

- We are given $i = 25$ A, $B = 3.0$ T, $\ell = 2.0$ cm, $h = 2.0$ mm (so $A = 4 \times 10^{-5}$ m) and assume $n = 8.0 \times 10^{28}$ electrons/m^3 as in the previous question. Then:

$$\begin{aligned} \Delta V_H &= \frac{iB\ell}{neA} \\ &= \frac{(25 \text{ A})(3.0 \text{ T})(2.0 \times 10^{-2} \text{ m})}{(8.0 \times 10^{28} \text{ e}^-/\text{m}^3)(1.6 \times 10^{-19} \text{ C/e}^-)(4.0 \times 10^{-5} \text{ m}^2)} \\ &= 2.9 \ \mu V \end{aligned}$$

11. (8 marks) 11.54 (The Bainbridge mass spectrometer setup is shown in **Figure 11.19**)

- The velocity selector leads to $v = E/B$ while the radius of curvature is from the equation $p = mv = qB_0R$. Here it is singly charged Li ions so $q = e$. Therefore we have:

$$\begin{aligned} mv &= eB_0R \\ m\left(\frac{E}{B}\right) &= eB_0R \\ m &= \frac{eBB_0R}{E} \\ &= \frac{(1.60 \times 10^{-19} \text{ C})(0.500 \text{ T})(0.750 \text{ T})(2.32 \times 10^{-2} \text{ m})}{(1.2 \times 10^5 \text{ V/m})} \\ &= 1.16 \times 10^{-26} \text{ kg} \end{aligned}$$

12. (2 marks) What's the current in a long wire if the magnetic field strength (magnitude of the magnetic field) 1.2 cm from the wire's axis is 67 μT?

- It's a "long" wire so we use the experession for an infinitely long wire and solve for i.

$$|\vec{B}| = \frac{\mu_0 i}{2\pi d} \Longrightarrow i = \frac{2\pi d|\vec{B}|}{\mu_0} = \frac{(2\pi)(0.012 \text{ m})(6.7 \times 10^{-5} \text{ T})}{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})} = 4.0 \text{ A}$$

13. (6 marks) A single-turn wire loop is 2.0 cm in diameter and carries a 650 mA current. Find the magnetic field strength at (a) the loop centre and (b) on the loop axis, 20 cm from the centre.

   - From the equation sheet we have that the magnitude of the magnetic field along the axis of a current ring is:

$$|\vec{B}| = \frac{\mu_0 i R^2}{2(R^2 + d^2)^{3/2}}$$

   (a) at the loop centre $d = 0$ so

$$\begin{aligned} |\vec{B}| &= \frac{\mu_0 i}{2R} = \frac{\mu_0 i}{D} \\ &= \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(0.650 \text{ A})}{0.02 \text{ m}} \\ &= 4.1 \times 10^{-5} \text{ T} = 41 \ \mu\text{T} \end{aligned}$$

   (b) on the loop axis, 20 cm from the centre we have $d = 0.2$ m and R is still $D/2$ = 0.01 m so:

$$\begin{aligned} |\vec{B}| &= \frac{\mu_0 i R^2}{2(R^2 + d^2)^{3/2}} \\ &= \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(0.650 \text{ A})(0.01 \text{ m})^2}{2[(0.01 \text{ m})^2 + (0.2 \text{ m})^2]^{3/2}} \\ &= 5.1 \times 10^{-9} \text{ T} = 5.1 \text{ nT} \end{aligned}$$

14. (6 marks) A single-turn square loop 20.0 cm on a side carries a 1.5 A current.

   (a) What's the loop's magnetic dipole moment?

   - The magnetic dipole moment is $\vec{\mu} = Ni\vec{A}$ so

$$|\vec{\mu}| = NiA = (1)(1.5\text{A})(0.2\text{m})^2 = 6 \times 10^{-2} \text{A} \cdot \text{m}^2$$

   (b) What's the magnitude of the torque the loop would experience if it was in a 2.3 T magnetic field with the loop's magnetic dipole moment vector at 60° to the field?

   - We have $\vec{\tau} = \vec{\mu} \times \vec{B}$ so

$$\begin{aligned} |\vec{\tau}| &= |\vec{\mu}||\vec{B}| \sin\theta \\ &= (6 \times 10^{-2} \text{ A} \cdot \text{m}^2)(2.3 \text{ T}) \sin(60^\circ) \\ &= 0.12 \text{ N} \cdot \text{m} \end{aligned}$$