PHYSICS 1012 Test #1 Equation Sheet

"*Never memorize something you can look up.*" – Albert Einstein

Constants and Conversion Factors

$$\begin{aligned}
1\text{ m} = 10^6\ \mu\text{m} = 10^9\text{ nm} &= 10^{10}\ \mathring{A} = 10^{12}\text{ pm} = 10^{15}\text{ fm} \\
1\text{ GeV} = 10^3\text{ MeV} &= 10^6\text{ keV} = 10^9\text{ eV} = 1.6022\times10^{-10}\text{ J} \\
e &= 1.6022\times10^{-19}\text{ C} \\
\varepsilon_0 &= 8.85418781762\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2 \\
1/4\pi\varepsilon_0 &= 8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2 \\
\mu_0 &= 4\pi\times10^{-7}\text{ T}\cdot\text{m/A} \\
c &= 2.99792458\times10^8\text{ m/s}\sim3.0\times10^8\text{ m/s} \\
k_B &= 1.380649\times10^{-23}\text{ J/}^\circ\text{K} \\
N_A &= 6.02214076\times10^{23}\text{ per mole} \\
R &= 8.3144\text{ J/}^\circ\text{K}\cdot\text{mol} \\
\sigma &= 5.67\times10^{-8}\text{ J/s}\cdot\text{m}^2\cdot^\circ\text{K}^4 \\
m_e\text{ (the electron mass)} &= 9.11\times10^{-31}\text{ kg} \\
M_p\text{ (the proton mass)} &= 1836m_e
\end{aligned}$$

Work:

$$W=\Delta U=-\int\vec{F}\cdot d\vec{s}\qquad\qquad \Delta U=Q\Delta V$$

Coulomb's Law:

$$\vec{F}=\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{|\vec{r}|^2}\hat{r}$$

The Electric Field:

$$\vec{F}(x,y,z)=Q\vec{E}(x,y,z)$$

Energy density stored in an electric field: $u_E=\frac{1}{2}\varepsilon_0|\vec{E}|^2$

For a point charge;

$$|\vec{E}|=\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

For an infinite uniformly charged line;

$$|\vec{E}|=\frac{\lambda}{2\pi\varepsilon_0\ell}$$

For an infinite uniformly charged sheet;

$$|\vec{E}| = \frac{\sigma}{2\varepsilon_0}$$

At a distance ℓ away along the bisector of an electric dipole ($\vec{p} = q\vec{d}$);

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{-\vec{p}}{[\ell^2 + (d/2)^2]^{3/2}}$$

For a dipole $\vec{p}$ **in** an external field $\vec{E}$; $\quad \vec{\tau} = \vec{p} \times \vec{E}$

Gauss' Law:

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\varepsilon_0}$$

Electric Potential:

$$\Delta V = V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{s}$$

At a distance r from a point charge q with $V(r = \infty) = 0$;

$$V(r) = \frac{q}{4\pi\varepsilon_0 r}$$

Electrical Properties of Materials:

$$i = \frac{dq}{dt} = \int \vec{j} \cdot d\vec{A} \qquad \vec{j} = -en\vec{v}_d \qquad |\vec{j}| = \frac{i}{A}$$

For ohmic materials:

$$R = \rho\frac{L}{A} \qquad \Delta V = iR$$

DC Circuits: $P = \Delta V i = i^2 R = \Delta V^2/R$

Kirchoff's Laws

Current Law: At a junction $\sum i_{\text{in}} = \sum i_{\text{out}}$

Voltage Law: For a closed loop $\sum \Delta V = 0$

The equivalent resistance R for N resistors:

- On the same current path

$$R = \sum_{i=1}^{N} R_i$$

- Having the same potential difference across them

$$\frac{1}{R} = \sum_{i=1}^{N} \frac{1}{R_i}$$

Capacitors: $Q = C\Delta V$ $U = Q^2/2C$

Parallel-plate capacitor:

$$C = \varepsilon_0 \frac{A}{d}$$

With a dielectric filling the space between plates $C' = \kappa_e C$

The equivalent capacitance C for N capacitors:

- On the same current path

$$\frac{1}{C} = \sum_{i=1}^{N} \frac{1}{C_i}$$

- Having the same potential difference across them

$$C = \sum_{i=1}^{N} C_i$$

For a charging capacitor in series with a resistance R:

$$\Delta V(t) = \Delta V(t_0)\left(1 - e^{-(t-t_0)/\tau}\right)$$

$$\tau = RC$$

Magnetic Force:

On a moving charge at the instant it is at the location (x, y, z)

$$\vec{F}(x,y,z) = q\vec{v}(x,y,z) \times \vec{B}(x,y,z)$$

On a current-carrying wire;

$$\vec{F} = i\vec{L} \times \vec{B}$$

Torque on a current-carrying loop of N turns: $\vec{\tau} = Ni\vec{A} \times \vec{B}$

Magnetic Field:

Due to a moving charge:

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v} \times \hat{r}}{|\vec{r}|^2}$$

Around a long, thin, straight current-carrying wire:

$$|\vec{B}| = \frac{\mu_0 i}{2\pi d}$$

Along the axis of a thin, current-carrying ring of radius R:

$$|\vec{B}| = \frac{\mu_0 i R^2}{2(R^2 + d^2)^{3/2}}$$

Inside an ideal solenoid:

$$|\vec{B}| = \mu_0 n i$$

Magnetic Dipole Moment:

$$\vec{\mu} = i\vec{A} \qquad \vec{\tau} = \vec{\mu} \times \vec{B} \qquad U = -\vec{\mu} \cdot \vec{B}$$

Volumes and Areas:

Area of a circle of radius R and area of the side of a cylinder of length ℓ and radius R;

$$A = \pi R^2 \qquad\qquad A = 2\pi R\ell$$

Volume and surface area of a sphere of radius R;

$$V = \frac{4}{3}\pi R^3 \qquad\qquad A = 4\pi R^2$$

Vectors, Trigonometry, Derivatives, Complex Numbers

We will use Cartesian coordinates throughout.

Vectors

$$\begin{aligned}
\vec{A} &= |\vec{A}|\hat{A} \;=\; A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k} \\
|\vec{A}| &= \sqrt{A_x^2 + A_y^2 + A_z^2} \\
\hat{A} &= \frac{\vec{A}}{|\vec{A}|} \;=\; \frac{\vec{A}}{\sqrt{A_x^2 + A_y^2 + A_z^2}} \\
\vec{C} &= C_x\hat{\imath} + C_y\hat{\jmath} + C_z\hat{k} \\
&= \vec{A} + \vec{B} \\
&= (A_x + B_x)\hat{\imath} + (A_y + B_y)\hat{\jmath} + (A_z + B_z)\hat{k} \\
\vec{A}\cdot\vec{B} &= |\vec{A}||\vec{B}|\cos\theta \\
&= (A_xB_x + A_yB_y + A_zB_z) \\
\vec{A}\times\vec{B} &= |\vec{A}||\vec{B}|\sin\theta\hat{n} \\
&\quad \text{where } \hat{n} \text{ is the unit direction vector normal to the } \vec{A}\vec{B} \text{ plane} \\
&= (A_yB_z - A_zB_y)\hat{\imath} + (A_zB_x - A_xB_z)\hat{\jmath} + (A_xB_y - A_yB_x)\hat{k}
\end{aligned}$$

Derivatives

$$
\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}x} A\cos bx &= -Ab\sin bx \\
\frac{\mathrm{d}}{\mathrm{d}x} A\sin bx &= Ab\cos bx \\
\frac{\mathrm{d}Ax^{\pm b}}{\mathrm{d}x} &= \pm Abx^{\pm b-1} \\
\frac{\mathrm{d}Ae^{\pm bx}}{\mathrm{d}x} &= \pm Abe^{\pm bx}
\end{aligned}
$$

Trigonometry

$$
\begin{aligned}
\sin^2\theta + \cos^2\theta &= 1 \\
\sin 2\theta &= 2\sin\theta\cos\theta \\
\cos 2\theta &= \cos^2\theta - \sin^2\theta \\
\sin(\theta \pm \phi) &= \sin\theta\cos\phi \pm \cos\theta\sin\phi \\
\cos(\theta \pm \phi) &= \cos\theta\cos\phi \mp \sin\theta\sin\phi \\
\sin\theta \pm \sin\phi &= 2\sin\left[\frac{1}{2}(\theta \pm \phi)\right]\cos\left[\frac{1}{2}(\theta \mp \phi)\right] \\
\cos\theta + \cos\phi &= 2\cos\left[\frac{1}{2}(\theta + \phi)\right]\cos\left[\frac{1}{2}(\theta - \phi)\right] \\
\cos\theta - \cos\phi &= -2\sin\left[\frac{1}{2}(\theta + \phi)\right]\sin\left[\frac{1}{2}(\theta - \phi)\right]
\end{aligned}
$$