PHYSICS 1012 — Equation Sheet for Test #2

"*Never memorize something you can look up.*" – Albert Einstein

Constants and Conversion Factors

$$
\begin{aligned}
1\ \text{m} = 10^6\ \mu\text{m} = 10^9\ \text{nm} &= 10^{10}\ \mathring{A} = 10^{12}\ \text{pm} = 10^{15}\ \text{fm} \\
1\ \text{GeV} = 10^3\ \text{MeV} &= 10^6\ \text{keV} = 10^9\ \text{eV} = 1.6022 \times 10^{-10}\ \text{J} \\
e &= 1.6022 \times 10^{-19}\ \text{C} \\
\varepsilon_0 &= 8.85418781762 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2 \\
1/4\pi\varepsilon_0 &= 8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2 \\
\mu_0 &= 4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A} \\
c &= 2.99792458 \times 10^8\ \text{m/s} \sim 3.0 \times 10^8\ \text{m/s} \\
k_B &= 1.380649 \times 10^{-23}\ \text{J}/^\circ\text{K} \\
N_A &= 6.02214076 \times 10^{23}\ \text{per mole} \\
R &= 8.3144\ \text{J}/^\circ\text{K}\cdot\text{mol} \\
\sigma &= 5.67 \times 10^{-8}\ \text{J/s}\cdot\text{m}^2\cdot{}^\circ\text{K}^4 \\
m_e\ \text{(the electron mass)} &= 9.11 \times 10^{-31}\ \text{kg} \\
M_p\ \text{(the proton mass)} &= 1836 m_e
\end{aligned}
$$

Work:

$$W = \Delta U = -\int \vec{F} \cdot d\vec{s} \qquad \Delta U = Q\Delta V$$

Magnetic Field:

Around a long, thin, straight current-carrying wire:

$$|\vec{B}| = \frac{\mu_0 i}{2\pi d}$$

Along the axis of a thin, current-carrying ring of radius R:

$$|\vec{B}| = \frac{\mu_0 i R^2}{2(R^2 + d^2)^{3/2}}$$

Inside an ideal solenoid:

$$|\vec{B}| = \mu_0 n i$$

Ampère's Law:

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{enclosed}}$$

Magnetic Flux and Faraday's Law of Induction:

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

Induced Electric Field:

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

Inductors:

$$\Delta V_L = L\frac{di}{dt} \qquad\qquad U = \frac{1}{2}Li^2$$

AC Circuits and Complex Impedance:

$$\Delta V = V_0 \cos \omega t \qquad \Longrightarrow \qquad \Delta V = V_0 e^{j\omega t}$$

$$\begin{aligned} \Delta V &= iZ \\ Z_R &= R \\ Z_C &= \frac{1}{j\omega C} \\ Z_L &= j\omega L \end{aligned}$$

$$A_V = \left| \frac{\Delta V_{out}}{\Delta V_{in}} \right|$$

Optics

Reflection and Refraction

Reflection: $\theta_r = \theta_i$
Index of Refraction: $n = c/v \qquad c = f\lambda \quad v = f\lambda_n \quad \lambda_n = \lambda/n$
Dispersion: n is a function of λ
Snell's Law:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Critical angle:

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right) \text{ for } n_1 > n_2$$

Double Slit Interference

Constructive:

$$\Delta l = d\sin\theta = m\lambda \quad \text{with} \;\; m = 0, \pm 1, \pm 2, \pm 3, \ldots$$

Destructive:

$$\Delta l = d\sin\theta = \left(m + \frac{1}{2}\right)\lambda \quad \text{with} \;\; m = 0, \pm 1, \pm 2, \pm 3, \ldots$$

Distance from central maximum to m^{th} bright fringe:

$$y_m = \frac{m\lambda D}{d}$$

Diffraction

Single Slit Destructive Interference:

$$a\sin\theta = m\lambda \text{ with } m = \pm 1, \pm 2, \pm 3, \ldots$$

Rayleigh Criterion for Circular Aperatures: $\theta = 1.22\frac{\lambda}{D}$
Bragg Equation: $m\lambda = 2d\sin\theta$ with $m = 1, 2, 3, \ldots$

Table 1: Index of Refraction

Substance	n
Air	1.000293
Water	1.333
Ice (at 0°C)	1.309
Glass, crown	1.52
Glass, flint	1.66
Diamond	2.419

Volumes and Areas:

Area of a circle of radius R and area of the side of a cylinder of length ℓ and radius R;

$$A = \pi R^2 \qquad\qquad A = 2\pi R\ell$$

Volume and surface area of a sphere of radius R;

$$V = \frac{4}{3}\pi R^3 \qquad\qquad A = 4\pi R^2$$

Vectors, Trigonometry, Derivatives, Complex Numbers

We will use Cartesian coordinates throughout.

Vectors

$$\begin{aligned}
\vec{A} &= |\vec{A}|\hat{A} \;=\; A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k} \\
|\vec{A}| &= \sqrt{A_x^2 + A_y^2 + A_z^2} \\
\hat{A} &= \frac{\vec{A}}{|\vec{A}|} \;=\; \frac{\vec{A}}{\sqrt{A_x^2 + A_y^2 + A_z^2}} \\
\vec{C} &= C_x\hat{\imath} + C_y\hat{\jmath} + C_z\hat{k} \\
&= \vec{A} + \vec{B} \\
&= (A_x + B_x)\hat{\imath} + (A_y + B_y)\hat{\jmath} + (A_z + B_z)\hat{k} \\
\vec{A}\cdot\vec{B} &= |\vec{A}||\vec{B}|\cos\theta \\
&= (A_xB_x + A_yB_y + A_zB_z) \\
\vec{A}\times\vec{B} &= |\vec{A}||\vec{B}|\sin\theta\hat{n} \\
&\quad \text{where } \hat{n} \text{ is the unit direction vector normal to the } \vec{A}\vec{B} \text{ plane} \\
&= (A_yB_z - A_zB_y)\hat{\imath} + (A_zB_x - A_xB_z)\hat{\jmath} + (A_xB_y - A_yB_x)\hat{k}
\end{aligned}$$

Derivatives

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}x} A\cos bx &= -Ab\sin bx \\
\frac{\mathrm{d}}{\mathrm{d}x} A\sin bx &= Ab\cos bx \\
\frac{\mathrm{d}Ax^{\pm b}}{\mathrm{d}x} &= \pm Abx^{\pm b-1} \\
\frac{\mathrm{d}Ae^{\pm bx}}{\mathrm{d}x} &= \pm Abe^{\pm bx}
\end{aligned}$$

Trigonometry

$$\begin{aligned}
\sin^2\theta + \cos^2\theta &= 1 \\
\sin 2\theta &= 2\sin\theta\cos\theta \\
\cos 2\theta &= \cos^2\theta - \sin^2\theta \\
\sin(\theta \pm \phi) &= \sin\theta\cos\phi \pm \cos\theta\sin\phi \\
\cos(\theta \pm \phi) &= \cos\theta\cos\phi \mp \sin\theta\sin\phi
\end{aligned}$$

Complex Numbers

$$\begin{aligned}
j &= \sqrt{-1} \\
Z_\pm &= a \pm bj \\
&= |Z_\pm|(\cos\theta \pm j\sin\theta) \\
&= |Z_\pm|e^{\pm j\theta} \\
|Z| &= \sqrt{Z^*Z} = \sqrt{a^2+b^2} \\
\theta &= \pm\tan^{-1}\left(\frac{b}{a}\right)
\end{aligned}$$

Given $P = a + bj$ and $Q = c + dj$ then for $L = P \pm Q = |L|e^{j\theta_L}$, $M = PQ = |M|e^{j\theta M}$, and $N = P/Q = |N|e^{j\theta N}$:

$$\begin{aligned}
L &= (a \pm c) + (b \pm d)j \\
|L| &= \sqrt{(a \pm c)^2 + (b \pm d)^2} \\
\theta_L &= \tan^{-1}\left(\frac{b \pm d}{a \pm c}\right) \\
|M| &= |P||Q| \\
\theta_M &= \theta_P + \theta_Q \\
|N| &= |P|/|Q| \\
\theta_M &= \theta_P - \theta_Q
\end{aligned}$$