# ESS5203.03 - Turbulence and Diffusion - W2017 <br> Final Exam, 18 April 2017, 2-5 pm. 

Aids allowed; Notes on four sides of $8.5 \times 11$ inch sheets of paper are permitted. Calculators may be used.

## ANSWER 4 QUESTIONS IN SECTION A AND 3 FROM SECTION B

Section $A \quad$ These questions require reasonably short (usually less than 1 page) answers. Use equations as necessary but add words of explanation! Answer 4 of these 5 questions. (40\% total)

Section B Answer 3 out of these 5 questions only ( $20 \%$ per question)

## Notes:

1) We use $\langle\ldots\rangle$ below to denote an average, ensemble, time or space according to context.
2) In the absence of any body forces, the Navier - Stokes equations for a viscous fluid in an inertial frame of reference can be written,

$$
\rho\left(\frac{\partial u_{i}}{\partial t}+u_{j} \frac{\partial u_{i}}{\partial x_{j}}\right)=-\frac{\partial p}{\partial x_{i}}+\mu \frac{\partial^{2} u_{i}}{\partial x_{j} \partial x_{j}}
$$

while the continuity equation for an incompressible fluid is,

$$
\frac{\partial u_{j}}{\partial x_{j}}=0
$$

3) The turbulent kinetic energy equation for an incompressible fluid can be written as,

$$
\frac{\partial E}{\partial t}+U_{j} \frac{\partial E}{\partial x_{j}}=-\left\langle u_{i} u_{j}\right\rangle \frac{\partial U_{i}}{\partial x_{j}}+\frac{g\left\langle u_{3} \theta\right\rangle}{\theta}-\frac{\partial\left\langle e u_{j}\right\rangle}{\partial x_{j}}-\frac{1}{\rho} \frac{\partial\left\langle p u_{j}\right\rangle}{\partial x_{i}}-\varepsilon+v \frac{\partial^{2} E}{\partial x_{j} \partial x_{j}}
$$

where $x_{3}$ is in the vertical direction, $U_{i}$ and $u_{i}$ are mean and fluctuating velocity components, $E$ and $e$ are the mean and fluctuating parts of the turbulent kinetic energy per unit mass, $p$ is the fluctuating pressure and $\varepsilon$ represents the viscous dissipation rate (per unit mass).
4) The basic equation for a Gaussian plume downwind $(x)$ of a point source at the origin $(0,0,0)$ is

$$
C(x, y, z)=\left(\frac{Q}{2 \pi U \sigma_{y} \sigma_{z}}\right) \exp \left[-\frac{1}{2}\left(\frac{y}{\sigma_{y}}\right)^{2}\right] \exp \left[-\frac{1}{2}\left(\frac{z}{\sigma_{z}}\right)^{2}\right]
$$

where symbols have their usual meanings.
5) For air at 1000 mb and $T=15^{\circ} \mathrm{C}$, take $\rho=1.2 \mathrm{~kg} \mathrm{~m}^{-3}, c_{p}=1007 \mathrm{~J} \mathrm{~kg}^{-1} \mathrm{~K}^{-1}$ and $v=1.5 \times 10^{-5} \mathrm{~m}^{2} \mathrm{~s}^{-1}$.
6) The Obukhov length, $L=-\frac{\rho c_{p} \theta_{\nu} u_{*}^{3}}{k g H}$ where $k=0.4$ is the Karman constant and heat flux, $H$ is measured positive upwards.

## SECTION A

A1 Make a carefully labelled sketch of a wind hodograph for a neutral barotropic ABL over horizontally homogeneous terrain with roughness length about 0.1 m . Assume a geostrophic wind of $10 \mathrm{~ms}^{-1}$. In particular mark points on the hodograph corresponding, roughly, to wind vectors at $10 \mathrm{~m}, 100 \mathrm{~m}$ and 1000 m . How different would the hodograph be in a mid-afternoon convective, well mixed, boundary layer with a capping inversion at 1000 m .

A2 Explain the concept of a "constant flux layer" and discuss its strengths and weaknesses. Show how dimensional analysis leads to a logarithmic wind profile in a neutrally stratified boundary layer and explain the concept of a "roughness length".

A3 Explain, with mathematical details, how inertial oscillations can explain low level jets in midlatitude nocturnal boundary layers over homogeneous, flat terrain

A4 Explain in detail any four reasons that a real, measured turbulent energy spectrum may deviate from the theoretical, idealized shape. Include sketches of the spectra to demonstrate the frequency or wavenumber ranges where these deviations are expected to occur. (Numerical values of the ranges are not necessary.)

A5 Describe how the Gaussian plume equation (Note 4) can be modified to account for the initial momentum and buoyancy of the gas released at the source. List (with some justification) the variables that would be required to parameterize this modification.

## SECTION B

B1 Monin-Obukhov Similarity Theory (MOST) implies that, in a constant flux, surface boundary layer, and for $z \gg z_{0}$, any turbulence statistic, $f$, can be represented as $f_{0} F(z / L)$ where $f_{0}$ is a quantity with the dimensions of $f, F$ is a dimensionless function and $L$ is the Obukhov length (see Note 6).
a) Explain the basis for the theory using dimensional analysis.
b) What are the implications for measurements of $\sigma_{w} / u_{*}$ from different experiments over different, but horizontally homogeneous, surfaces, all conducted in neutral stratification.

B2 Consider a plume of material emitted from a horizontal, cross wind, low level line source (such as traffic on a highway) at $(x, z)=(0, h)$ in a turbulent flow with uniform mean wind, $U$ in the $x$-direction. How is the result in Note (4) above modified in this line source situation and how can one account for effect of the ground when $\sigma_{z}$ is comparable to $h$. Note that $\int_{-\infty}^{\infty} e^{-a x^{2}} d x=\sqrt{\pi / a}$.

If you measure a ground level concentration of $\mathrm{PM}_{2.5}$ of $C=100 \mu \mathrm{~g} \mathrm{~m}^{-3}$ at a distance of 100 m from the road in a wind speed of $5 \mathrm{~ms}^{-1}$, normal to the roadway, you assume $h=0$ and you estimate $\sigma_{z}=5 \mathrm{~m}$ (neutral stratification) at this distance from the source, estimate the source strength (in $\mu \mathrm{g} \mathrm{m}^{-1} \mathrm{~s}^{-1}$ ) of emissions from traffic on the road. And if there were on average 1 vehicle every 10 m on the highway what would the average emission rate per vehicle be.

B3 Suppose you are asked to develop a surface flux parametrization scheme for a large scale numerical weather prediction model, i.e. to provide roughness lengths or drag and heat transfer coefficients for each grid square. What difficulties would you anticipate in the case of a grid square which was part lake, part forest and part city, i.e. how would you deal with terrain heterogeneity? [This was not covered in lectures, I am just asking for your ideas of how to best approach this open problem.]

B4 The vertical turbulent flux of a gas can be equated (analogous to Fick's law) as a product of the eddy diffusivity $(K)$ and the local concentration gradient. The eddy diffusivity is a function of friction velocity, height, and stability. Time-averaged measurements of concentration have been measured at six heights $C_{i}\left(z_{i}\right)$, where $i=1,2, \ldots 6$. Formulate an equation to estimate the vertical flux of the gas based on these concentration measurements. Assume a neutral stability for your solution.

B5 Derive the equation for the measurement of wind speed by sonic anemometers as a function of distance between sensors ( $d$ ), the two pulse times ( $t_{1}$ and $t_{2}$ ), and the speed of sound ( $c$ ). Use diagrams as needed.

