ESS5203.03 - Turbulence and Diffusion in the Atmospheric Boundary-Layer: Winter 2020

<u>Moments, Correlations, and Spectra</u>

See: Garrat, Section 2.1 and 3.5; and Kaimal and Finnigan, Chapter 2.

Turbulence is described in terms of moments, correlations, and spectra.

0) Units and converstions (frequencies, non-dimensionalization, Taylor's hypothesis)

Wavenumber (κ); Wavelength (λ); Frequency (f); Normalized frequency (n);
$\kappa = \frac{2\pi}{\lambda} = \frac{w}{\overline{u}} = \frac{2\pi f}{\overline{u}} \qquad n = fz/\overline{u}$
Length scale (Λ); Time scale ($\mathcal{T}$)
$x = \overline{u}\, t \qquad \Lambda = \overline{u}\, \mathcal{T}$

1) Moments (mean, variance, co-variance).

$s^n = \int_{\infty}^{\infty} (x-c)^n f(x) dx$
$\overline{s} = \frac{1}{T}\int_0^T s\, dt$ or $\overline{s} = \frac{1}{L}\int_0^L s\, dx$ (note: $\overline{s'} = 0$)
$\overline{s'^2} = \frac{1}{T}\int_0^T (s')^2 dt$ or $\overline{w's'} = \frac{1}{T}\int_0^T w's'\, dt$
(Here L and T are spatial and temporal lengths of the measurement series.)

2) Correlations (auto-, cross-). Times and length scales.

$\rho_s(\tau) = \frac{\overline{s'(t)s'(t+\tau)}}{\overline{s'^2}}$
$\rho_s(\tau) = \frac{\frac{1}{T}\int_0^T s'(t)s'(t+\tau)dt}{\overline{s'^2}}$ and $\rho_{ws}(\tau) = \frac{\frac{1}{T}\int_0^T w'(t)s'(t+\tau)dt}{\overline{w's'}}$
$\mathcal{T} = \int_0^{\infty} \rho_s(\tau) d\tau$ (Here $\mathcal{T}$ is time scale with corresponding length scale $\Lambda = \overline{u}\, \mathcal{T}$.)

3) Spectra (Energy or Power, One-dimensional). Fourier transforms.

$$R \begin{cases} \overline{s'^2} = \int_0^{\infty} \Phi_{\mathrm{ss}}(f)\, df \\ \overline{w's'} = \int_0^{\infty} \Phi_{\mathrm{ws}}(f)\, df \end{cases}$$
$E(\kappa) = \int_{-\infty}^{\infty} R(x) \exp(-i\, \kappa\, x)\, dx$
$R(x) = \int_{-\infty}^{\infty} E(\kappa) \exp(i\, \kappa\, x)\, d\kappa$

4) Discussion of the energy spectra of turbulence (Fig 2.1 in both texts).

Synoptic and Mesoscale ⇒ Spectral Gap ⇒ Energy Containing Range ⇒ Inertial Subrange ⇒ Dissipation Range.

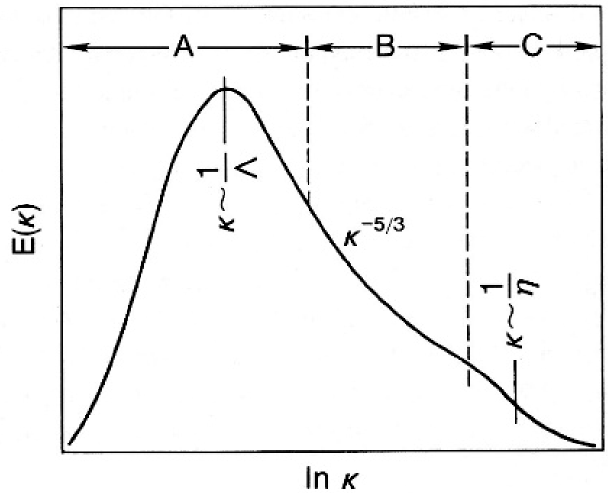

Fig. 2.1. Schematic of energy spectrum in the atmospheric boundary layer showing distinct regions of energy production (A) and dissipation (C) and the inertial subrange (B), where both energy production and dissipation are negligible. Λ is the integral scale of turbulence and η is the Kolmogorov microscale.

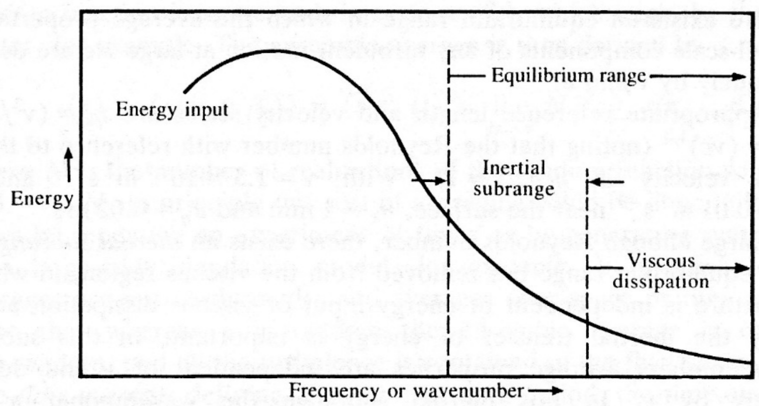

Fig. 2.1 Schematic representation of the energy spectrum of turbulence.